Endogenous Communication in Complex Organizations*

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Abstract

We study the information flows that arise within an organization with local knowledge and payoff externalities. Our organization is modeled as a network game played by agents with asymmetric information. Before making decisions, agents can invest in pairwise communication. Both active communication (speaking) and passive communication (listening) are costly. Our main result is a close-form characterization of equilibrium communication intensities and decision functions for our class of network games. This result can be used to describe the determinants of pairwise communication, the overall influence of each agent, the ratio between active and passive communication, and the discrepancy between actual and efficient communication patterns. The analysis is also extended to organizations that contain teams of agents who share the same objective. Throughout the paper, we apply our results to two examples of organizations: a matrix-form firm and a network of professionals.

1 Introduction

As Arrow (1974) argues in his pioneering work on the limits of organization, one of the key activity of any organization – be it a firm, or any other formal or informal group of professionals – is internal communication.

Before making decisions, agents typically want to communicate with other agents. Think of the web of interactions that is present within a firm. When a division manager makes a decision, he knows that the consequence of his actions will depend on what other divisions will do. The manager may want to seek the views of other selected colleagues or let his views be known to them before proceeding. Similarly, his colleagues may want to consult with him or other colleagues before making their decisions. This generates a potentially complex

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†Toni passed away in November 2007. His friendship, energy and talents are sorely missed. While the ideas and the results contained in this paper are due to the three authors, only de Martí and Prat are responsible for any remaining errors or omissions.

communication network within the organization. Similarly, groups of professionals – such as researchers in the same discipline – have frequent more or less structured exchanges of ideas, ranging from informal conversations or the transmission of written material to presentations at conferences or seminars. Typically, one or more professionals inform their colleagues about some of their current projects. These communication activities can take up a considerable portion of the professionals' time.

Of course, communication is costly, in terms of physical resources but especially in terms of time. Agents will have to be selective when they decide who they talk to and who they listen to. As Arrow (1974) pointed out, the tradeoff between the cost of communication and the benefit of communication lies at the core of the agenda of organizations. We should expect the communication pattern that we observe to be the result of some – more or less coordinated – optimization process on the part of organization members.

At the same time, organizations are potentially complex structures, with a large number of members interacting in heterogeneous ways with each other. It appears natural in this setting to look at networks (Goyal 2007, Jackson 2008) as a tool to model information flows in a large class of organizational architectures. Here, we think of organizations as networks of agents with asymmetric information who can communicate with each other before making decisions. To the best of our knowledge, this is the first work that considers costly endogenous communication in network games with asymmetric information.

The model can be sketched as follows. There are a number of agents who face local uncertainty (for simplicity, local states are assumed to be mutually independent). Each agent observes the realization of his local state and must take an action. The payoff of each agent depends on his local state, his own action, and the action of other agents. The intensity of payoff interactions among agents is described by a directed graph. While our setup encompasses both negative and positive interactions, in most of the paper we restrict attention to positive complementarities only.

Before choosing his action, an agent can engage in communication. He can inform other agents about his own state of the world and he can gather information about other agents' state of the world.[1] Formally, the agent selects a vector of active communication intensities and a vector of passive communication intensities. The precision of the communication of one agent to another is then determined by how much the sender invests in active communication (talking) and how much the receiver invests in passive communication (listening). Both types of communication are costly, and the cost is an increasing and convex function of communication intensity.

At this point, we face two alternative modeling choices. Agents may choose communication intensities before or after they learn about their own state of the world. This depends on whether we think of communication investment as long-term or short-term.

The example of a short-term investment could be a visit. A firm invites representatives from another firm to visit their plants, as a chance for the visitors to learn more about the host firm. The active communication cost is sustained by the host firm (hospitality, time devoted to visitors, etc...), while the passive communication cost is sustained by the visitors (travel costs, time spent on visit, etc...). As a visit can be planned in a matter of weeks, it

[1] In our set-up, which allows for one round of communication only, there is no loss of generality in assuming that communication only relates to the observed state of the world. Things would be different if we allowed for more than one round.

is conceivable that the decision to organize a visit is taken after the two parties learn their respective states of the world (demand, production costs, etc...).

Instead the example of a longer term communication investment could be the appointment of a liaison officer. A firm invites another firm to second one of their employees to the first firm. The liaison officer would then be in a good position to report information about the host firm as it arises. Here, again, the host sustains an active communication cost while the liaison officer's firm sustains a passive communication cost. A secondment of this kind appears to require a longer time horizon and it is likely that both firms will receive new information after the investment is made.

The present paper considers both alternatives. Namely, it examines the case where agents select their vectors of communication intensities before they observe their private information and the case where they select them afterwards.[2]

Besides choosing a vector of active and passive communication intensities, each agent determines his action as a function of the signal he has received from other agents. Hence, an equilibrium of our game can be described as a directed graph of active and passive communication intensities together with a directed graph of "decision influences".

To provide a tractable framework we restrict attention to the standard normal-quadratic setting developed in team theory (Marschak and Radner 1972). Private information is normally distributed and payoffs are a quadratic function of all arguments. Communication cost is linear in the precision of the message.

We first obtain an equivalence result, that greatly simplifies our analysis. We consider a one-shot game where agents choose their communication intensity vectors and their decision functions simultaneously (without knowing their private information). Moreover, the decision functions are restricted to be linear. We show that the unique Nash equilibrium of this linear one-shot game corresponds to a perfect Bayesian equilibrium of the game where agents choose communication intensities before learning their private information as well as to an identical perfect Bayesian equilibrium of the game where agents choose communication intensities after learning their private information. The equivalence between the static and the dynamic version of the game hinges on the normal-quadratic structure of the network game under consideration. In general, we would expect signalling to occur because communication intensities should depend on the local state observed and the realization of the messages transmitted in the second period should provide information on what other players chose in the first period. But we show that this effect does not arise in the class of network games that we consider.

Armed with the equivalence result, we arrive at a characterization of the unique pure-strategy equilibrium, which constitutes the main result of the paper. The characterization takes a simple form. The problem can be split into n separate problems, where n is the number of agents. For each agent, we express the influence of that agent (how the decisions of all agents depend on signals received from that agent) as the product of the inverse of the interaction matrix and a vector which can be interpreted as a measure of the relative benefit of communication. The interaction matrix is the same for all agents: only the vector changes. The decision matrix that arises in equilibrium can be interpreted as the limit of a

[2] However, we abstract from the possibility of information manipulation. In both the example of the visit and the example of the liaison officer, the host firm might have ways to manipulate the information that the other firm gathers. [*add references to other papers, which use the same assumption]

sequence of matrices corresponding to increasingly high levels of interaction.

This tractable characterization leads to a number of comparative statics results about communication patterns in organizations:

1. Overall communication intensity within an organization is increasing in the interaction coefficients and on the amount of uncertainty that agents face, and it is decreasing in the cost of active and passive communication. The information flow between two particular agents is increasing in the payoff externality between the two agents.

2. We can define the global influence of an agent as the aggregate effect that his private information has on all other agents. We prove that global influence can be expressed in terms of Bonacich centrality (a measure of the centrality of a node developed by Bonacich 1987, and introduced to economics by Ballester et al 2006). We show that if an agent's becomes more important in terms of interactions, he becomes relatively more influential if and only if the cost of passive communication is sufficiently low relative to the cost of active communication. Unless listening is very expensive, agents who are central to the network invest less in listening to other people than other people invest in listening to them. The global influence of an agent is also increasing in the amount of uncertainty that the agent faces.

3. As in Dessein and Santos (2006), our agents face a tradeoff between adaptation (adapting their decision to the local state) and coordination (fitting in with the decisions of other agents). We show that the relation between the relative need for adaptation/coordination and communication intensity is U-shaped. This non-monotonicity is due to the fact that in our model it takes two to communicate. Hence, communication to and from an agent tends to zero when the agent cares only about coordination (no one is interested in speaking or listening to him) or only about adaptation (he is not interested in speaking to or listening to other people).

4. In general, with positive complementarities, all communication intensities are inefficiently low. More interestingly, if there are more than two agents, active communication is inefficiently low compared to passive communication. Our agents have a direct incentive to gather information because they can use it to make decisions while only an indirect incentive to send information to others in the hope that it will affect their decisions.

5. Our set-up can be extended to encompass *teams* of agents. Namely, we can assume that some subsets of agents share common goals (members of the same division or firm). We provide a full characterization of the equilibrium communication and decision networks. Agents in the same team communicate with each other and affect each other's decisions more than comparable agents in different teams.

Throughout the analysis, we use two leading economic examples. The first is a matrix-form organization, where agents are interpreted as units in a two-dimensional space (e.g. functional areas and country offices). We ask whether communication is stronger on one dimension or another and whether it is a better idea to group units into teams along one dimension or along the other.

The second example is a network of independent and similar professionals (or professional firms) who benefit from coordinating with each other. This second example uses a relative homogenous interaction matrix in order to explore the role of individual characteristics in determining information flows.

In the conclusion, we discuss the potential empirical value of our model. We argue that information about data about communication patterns in a network is in principle sufficient to identify the underlying interaction matrix.

The present paper aims to combine two strands of literature: network economics and organization economics. Our contribution with respect to the first is to allow for endogenous strategic communication among asymmetrically informed agents. Our contribution with respect to the second is to provide a flexible framework that can be used to analyze a large class of organizational forms.

The closest contributions in the network literature are Calvó-Armengol and de Martí (2008), which consider a normal-quadratic team-theoretical set-up and study the effect of communication among agents. The authors provide a full characterization of the decision functions and the equilibrium payoffs given a communication structure. Calvó-Armengol and de Martí also study the best communication structure is when the overall number of links among agents is bounded: they provide sufficient conditions for the optimal communication network to be a star or the maximum aggregate span network.

Morris and Shin (2007) also consider a normal-quadratic setup. In their model, with a continuum of agents, they allow for partial communication among predetermined groups. They analyze the welfare effects of public and semi-public information (derived from partial communication), complementing their previous analysis on the value of public information (Morris and Shin, 2002).

This paper also adopts a normal-quadratic specification, close to the one in Calvó-Armengol and de Martí. The key innovation here is of course that communication is endogenous. We also move away from a team-theoretical framework (now a special case, when all agents belong to the same team), we introduce the idea of communication intensity and we distinguish between active and passive communication.

Hagenbach and Koessler (2008) also consider, as we do, strategic endogenous communication in a network game with a normal-quadratic structure. Their focus is on costless, non verifiable information (cheap talk) when agents may have biases as in Crawford and Sobel (1982). They show that full information may not arise in equilibrium and they analyze various communication protocols. Our set-up is different in that we focus on costly and verifiable information. The kind of issues we ask is thus entirely different (and complementary).[3]

With regards to the literature on the formation of (communication) networks, Bloch and Dutta (2007) study the creation of communication networks with endogenous link strength. In their model, agents have a fixed resource, for example time, and have to decide how to allocate it to create connections with others. The benefits of a connection depends on the exposure decisions of both agents involved in it. Furthermore, in the spirit of the connections model introduced in Jackson and Wolinsky (1996), an agent obtains benefits of indirect connections through the more reliable path connecting them with each one of

[3]The only point of overlap is their result that, when information is fully verifiable, agents will want to communicate all they know. This corresponds to our set-up when communication costs go to zero.

the agents in the society. In this setup, both the equilibrium and the efficient networks are star-shaped, i.e., with one agent connected to all the rest of the population and all the rest connected only to this center.

Rogers (2007) analyzes another network formation game in which all agents have a limited resource available to spend building links with the rest of agents, but differs with the work of Bloch and Dutta in the structure of benefits. In Rogers (2007) the utility of an agent depends on the utility of each other agent with which he is directly connected. This recursive definition of utilities generates indirect effects that spread through indirect connections of any length. The author analyzes two games, one in which the dependency expresses that each agent gives utility to his connections, and another one in which the dependency expresses that each agent receives utility from his connections. In both cases, the Nash equilibria are characterized.

Our paper is also linked with the growing literature on games played in a network, in which players' payoffs are intimately related to the geometry of relations among them.[4] Ballester et al. (2006) analyze a class of complete information games with quadratic payoffs and pairwise dependent strategic complementarities. They show that in the equilibrium of these games the effort exerted by each agent strongly depends on his position of the network of relations. In particular, this effort is proportional to his Katz-Bonacich centrality measure (Bonacich, 1987), that measures his prominence derived from the direct and indirect connections in which he is involved. While our setup differs in a number of ways with theirs one, we also establish a close connection of individual decisions with the Katz-Bonacich centrality measure.

Chwe (2000) studies a collective action problem with communication. In particular, agents are connected in a network and they communicate to their neighbours their willingness to participate in an activity. The analysis provides a neat picture of how the network shapes individual decisions and helps or precludes coordination. Our work also analyzes a coordination game with incomplete information and communication, but in our case the sources of incomplete information and the specification of the coordination game are different and communication is endogenous.

Goyal and Galeotti (2008) is among the few works that analyze, as we do, at the same time the network formation process and the play of a game that depends on the network formed. The authors study a game in which payoffs depend on the, costly, information they acquire and gather from their neighbours in a network of relations. The analysis of this game in a fixed network is performed in Bramoullé and Kranton (2008), in which a set of varied possible equilibria are presented. The novelty in Goyal and Galeotti is that they allow agents to choose their connections. They show that the introduction of endogenous network structures induce a simpler core-periphery structure in the equilibrium formed. In particular, equilibrium networks show a core-periphery pattern in which a set of few individuals are highly connected with a high number of poorly connected agents. While their setup is different from ours, we share Goyal and Galeotti's goal of studying endogenous network formation.

[4] We analyze an incomplete information game played in a network. However, as usual in the literature, we assume full knowledge by all players on the realized network structure. For some facts about network games with incomplete information on the network structure we refer the interested reader to Galeotti et al. (2007).

In the second strand of literature, there are a number of papers which study endogenous communication in a variety of settings. Our approach to multi-person decision making under asymmetric information, as well as our normal-quadratic formulation, is inspired by Marschak and Radner's (1972) team theory. Some recent papers (Dessein and Santos 2006, Alonso et al. 2007, Rantakari 2007, Dessein et al 2006) explore decentralized decision making within organizations. Besides sharing their normal-quadratic set-up, we are also interested in the tradeoff between adaptation and coordination. We are closest to Dessein and Santos (2006), who analyze the role of endogenous communication. In their model, an agent can send a signal about his local state to the other agent, and the precision of the signal is endogenous.[5] They show the existence of complementarity between communication, adaptation, and job description: in particular, when communication costs decrease, the organization is more likely to adopt a new set of organizational practices that include broader tasks and more adaptation. The present paper is complementary to this literature: while it abstracts from a number of organizational dimensions, it provides a general framework to study endogenous information flows, which allows to draw a number of lessons on what communication networks we should expect to observe in a variety of complex organizational architectures.

The present work is close to Dewatripont and Tirole (2005), who analyze a model of endogenous costly communication between a sender and a receiver. As in our model, both active and passive communication are endogenous and costly, and there are positive externalities (it takes two to communicate). Dewatripont and Tirole's communication model has a number of features that are absent here, such as the presence of signaling and the possibility of sending "cues" – information about the sender's credibility. Obviously, our contribution is to extend endogenous communication to complex architectures. While our representation of pairwise communication is simpler, we believe it still captures Dewatripont and Tirole's insight about moral hazard in communication. For instance, their comparative statics results on congruence find a partial parallel in our Proposition 11.

Our work is also related to Van Zandt (2004), a model of endogenous costly communication where several agents can transmit information at the same time. This leads to screening costs on the part of receivers and the potential for "information overload". Van Zandt examines possible mechanisms for reducing overload – an important problem in modern organizations. Our paper abstracts from information overload, by assuming that receivers do not face a screening problem (they can always choose not to listen to a particular sender).

Following the seminal work of Radner (1993), the literature of organizational economics has also studied the role of networks in minimizing human limitations in information processing. The works of Bolton and Dewatripont (1994), Van Zandt (1999a), Garicano (2000), Guimerà et al. (2003), and Dodds et al. (2003) highlight the importance of hierarchies, and more general network structures, to diminish the costs related to processing information that ‡ows through the network of contacts. This literature is surveyed by Van Zandt (1999b) and Ioannides (2003). Our work is complementary to this one, and analyzes how individual payoff complementarities shape both the network structure of communication and the equilibrium actions.

[5] One technical difference is that Dessein and Santos' (2006) signals are either fully informative or uninformative, and precision is defined as the probability that the signal is informative. Here, instead, signals are normally distributed and the precision is the reciprocal of the variance.

Cremer, Garicano, and Prat (2007) formalize Arrow's (1974) idea of coding: the medium of communication used by a group of people (the organizational language) is endogenous and it determines communication costs. For analytical tractability, in the present model the communication medium is not modeled explicitly but it is represented by a communication cost function.[6]

Related work can also be found in political economy. Dewan and Myatt (2007) analyze the role of communication in the interplay of leaders and activists in political parties. Leaders are heterogeneous in two different skills: their ability to interpret which is the correct policy to promote, and the clarity of communication of his ideas to the activists. Activists seek to advocate for the correct policy by listening with different intensities to the party leaders. The authors show that, generally, clarity in communication is the leader's ability that induces higher influence on activists' opinion. Their interpretation of communication is close to the one we propose in our work: in a bayesian game with quadratic payoff functions and normally distributed signals, that represent the messages send and received, agents can affect the precision of these signals. On the other hand, the communication protocols and, therefore, the strategic effects of communication are different in the two models, as well as the questions that are analyzed.

The rest of the paper is organized as follows. The next section introduces the model. Section 3 discusses the equilibrium equivalence between our two original games and the linear one-shot game. Section 4 presents the equilibrium characterizatiomn theorem while section 5 uses it to study: determinants of overall communication intensity, determinants of relative communication intensities, global influence of agents, active vs passive communication, comparison with efficient communication patterns. Section 6 extends the model to teams of agents. The two leading examples are introduced with the model in section 2 and re-visited at various points in sections 4, 5, and 6. Section 7 concludes with a brief discussion of the scope for bringing this approach to the data. All proofs are in appendix.

2 Model

Consider a set of n agents. Agent i faces a local state of the world

$$\theta_i \sim \mathcal{N}(0, s_i),$$

where s_i denotes the precision of θ_i. The local states of different agents are mutually independent. Agent i observes only θ_i.

All agents engage in, pairwise, communication activity.[7] Agent i receives message y_{ij} from agent j, such that

$$y_{ij} = \theta_j + \varepsilon_{ij} + \eta_{ij},$$

[6] Cremer (1993) and Prat (2002) study costly endogenous information collection in a team-theoretic setting. Hellwig and Veldkamp (2008) examine optimal information choices in a strategic setting. The present paper is complementary in that it endogenizes communication rather than information collection.

[7] We assume that there exist channels to engage in communication among any two members inside the organization, and hence that all these channels are potentially used. While we believe this makes sense in a setup to analyze organizations, it is an strict assumption when dealing with other social arrangements, such as friendship networks. In Appendix E we show that in this last case linearity of equilibrium would be preserved and hence a similar analysis could be performed.

where ε_{ij} and η_{ij} are two normally distributed noise terms

$$\begin{aligned} \varepsilon_{ij} &\sim \mathcal{N}(0, r_{ij}), && \text{(active)} \\ \eta_{ij} &\sim \mathcal{N}(0, p_{ij}), && \text{(passive)} \end{aligned}$$

and r_{ij} (resp. p_{ij}) is the precision of ε_{ij} (resp. η_{ij}). We interpret ε_{ij} as the noise associated with passive communication (listening to a presentation, reading a report, visiting a plant, appointing a liaison officer) and η_{ij} as the noise associated with active communication (preparing a presentation, writing a report, hosting a visit, hosting a liaison officer).

Agent i chooses how much to invest in speaking with and listening to other players. Namely, he selects:

- The precision of the active communication part of all the signals he sends: $(r_{ji})_{j\neq i}$, for which he incurs cost $k_r^2 \sum_{j\neq i} r_{ji}$, where k_r is a parameter.
- The precision of the passive communication part of all the signals he receives, $(p_{ij})_{j\neq i}$, for which he incurs cost $k_p^2 \sum_{j\neq i} p_{ij}$, where k_p is a parameter.

We also assume that each precision term is bounded below by a very small number m: $r_{ji} \geq m$, $p_{ij} \geq m$. This avoids dominated equilibria where i does not speak to j because he does not expect j to listen and viceversa.[8]

After observing the local state θ_i and the vector of signals $(y_{ij})_{j\neq i}$, agent i chooses an action $a_i \in (-\infty, \infty)$.

This setup contains two implicit assumptions. First, agents do not observe the communication intensities chosen by other agents directly (i.e. the agent does not see how much effort the others put into writing their reports or into reading his reports; the opportunity cost of sending/hosting a particular liaison officer is unobservable). Second, when engaging in active communication, the agents cannot manipulate the signal they send (the report may be more or less clear but it cannot contain lies; the liaison officer cannot be bribed). While costly signaling and strategic misrepresentation are important aspects of organizational economics, the present paper must restrict attention to direct and non-manipulable information flows in order to keep the analysis tractable.

The payoff of agent i is quadratic:

$$u_i = -\left(d_{ii}(a_i - \theta_i)^2 + \sum_{j\neq i} d_{ij}(a_i - a_j)^2 + k_r^2 \sum_{j\neq i} r_{ji} + k_p^2 \sum_{j\neq i} p_{ij} \right), \tag{1}$$

where the term d_{ii} measures the importance of tailoring i's action to the local state and the term d_{ij} represents the interaction between the action taken by agent i and the action take by agent j. For the rest of the paper we assume that the interaction terms are positive ($d_{ij} \geq 0$ for all i and all j). However, all of our results are valid, as stated, if some – or even

[8] A natural question is whether in this model speaking and listening are strategic complements. The answer to this question is not straightforward at this stage and we postpone it to the discussion that follows the main result: 6 on page 16).

all – of the interaction terms are negative, as long as the d_{ii}'s are positive and sufficiently large.[9]

We consider two versions of this two-stage game. In the first version, agents invest in communication *before* observing their local state. Namely, the timeline is:

1. Agents simultaneously select their active and passive communication intensity vectors $(r_{ji})_{j\neq i}$ and $(p_{ij})_{j\neq i}$.
2. Agents observe their local state of the world θ_i.
3. Agents receive signals from other agents $(y_{ij})_{j\neq i}$.
4. Agents select their actions a_i.

We refer to the first version of the game as $\Gamma(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$, where $\boldsymbol{D} = (d_{ij})_{i,j}$, $\boldsymbol{k} = (k_r, k_p)$ and $\boldsymbol{s} = (s_i)_i$.

In the second version, called $\Gamma_\theta(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$, agents invest in communication *after* observing their local state. The timeline is as above, except that stages 1 and 2 are swapped. As mentioned in the introduction, the "before" version captures long-term investments in communication (such as seconding a liaison officer), while the "after" version is more appropriate to short-term investments (such as making a presentation).

2.1 Examples

This set-up can be applied to a number of situations. We introduce here two examples, which we will use throughout the rest of the paper to illustrate our results. Besides establishing a more concrete connection with organization economics, the examples are also useful to explore separately two different aspects of the model. The first example assumes that all agents are identical and focuses on the role of the interaction matrix. The second example does the opposite: it assumes a homogenous interaction matrix in order to focus on the role of agent heterogeneity.

2.1.1 A Matrix-Form Firm

Beginning with Chandler's (1962) analysis of some early examples of modern corporations, management scholars have been studying decentralized organizational structures in which a firm is divided into several divisions. One of the more studied structures is matrix management, where the units of an organization are arranged along two (or more) dimensions. Each division can have associated different attributes, such as its function inside the organization (marketing, product design, manufacturing,...) or the country in which it is established (see Galbraith, 1977, for an early definition of the Matrix-form organization, and the organizational challenges associated to it). We are going to model such kind of organization and try to extract a number of conclusions about communicational aspects of this particular structure of production. Examples of companies that have attributed part of their success

[9] In our notation, whenever a variable has two agent indices, such as y_{ij} or d_{ij}, the first index denotes the agent that is "directly affected", such as the receiver of a signal or the owner of the payoff.

to the Matrix form include ABB or 3M, among others (see, for example, Chi and Nystrom, 1998, for a general analysis of the Matrix form from a managerial perspective).

We interpret here agents as units of a multinational firm. Suppose that every i is associated to two attributes: function f_i and country c_i. There are n_f functions and n_c countries, so there are a total of $n_f n_c$ units.

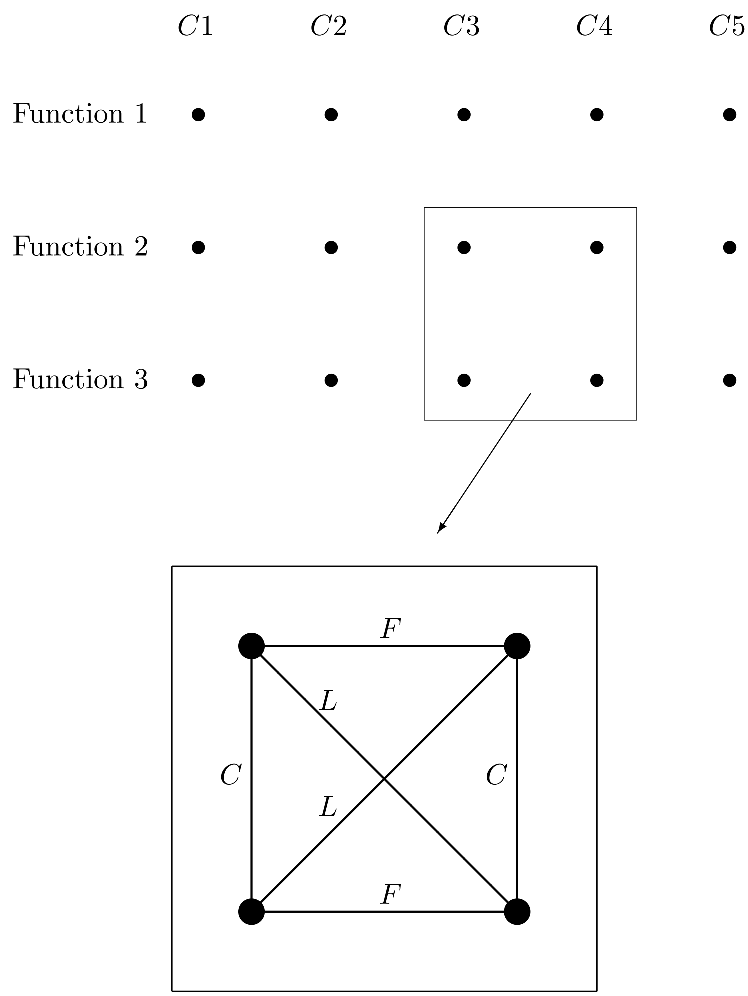
We assume that

$$d_{ij} = \begin{cases} G & \text{if } f_i = f_j \text{ and } c_i = c_j \\ F & \text{if } f_i = f_j \text{ and } c_i \neq c_j \\ C & \text{if } f_i \neq f_j \text{ and } c_i = c_j \\ L & \text{otherwise} \end{cases}$$

Moreover, $L < F < G$ and $L < C < G$: the interaction between units in the same country or in the same function is greater than the interaction between units who are in different countries and in different functions (but not as large as the adaptation term). For now, we do not make assumptions on the relative importance of national links as opposed to functional links.

To focus attention on the role of the interaction matrix, we assume that all units face that same level of uncertainty ($s_i = s$ for all i).

The figure below depicts a matrix-form firm with three functions and five countries.



2.1.2 A Network of Professionals

A number of important areas of economic activity – like medicine, law, and scientific research – tend to be structured as loose networks of professionals (Garicano and Santos 2004). These networks present two key featuures. First, each individual professional typically enjoy a high level of independence, especially when it comes to exercising his professional judgment, which is often enshrined in a charter. Second, despite their autonomy (or perhaps because of it), the professionals create active communication networks, which tend to be organized among common interests. Information flows take forms that are familiar to academic economists: emails, conversations, documents, presentations, etc. Our set-up can be specilized to illustrate such a loose communication network among peers.

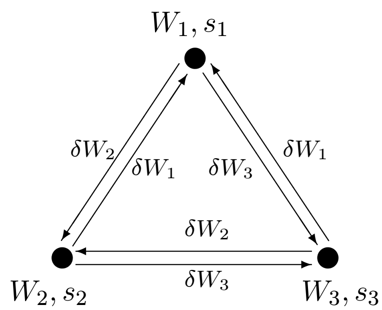
Agents are now n professionals (doctors, lawyers, academics, etc.) who face different private information (patients, cases, problems) and want to maintain a certain degree of coordination with their colleagues. In contrast to the previous example, we assume that the relative importance of interaction among professionals is the same, but that professionals differ in terms of information and payoff importance.

Namely, we assume:

$$d_{ij} = \begin{cases} W_i & \text{if } i = j \\ \delta W_i & \text{if } i \neq j \end{cases},$$

where $\delta \in (0, 1)$ measures the relative importance of coordination and W_i represents the "size" of agent i. The other important parameter will be the variance of professional i's local state, s_i.

The figure below depicts a network of three professionals.



3 Equilibrium Equivalence

In both versions of the game, $\Gamma(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ and $\Gamma_\theta(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$, the relevant solution concept is perfect Bayesian equilibrium. Games with quadratic payoffs have equilibria in linear strategies. In our case, we say that agent i uses a linear decision function if the action a_i that he selects can always be written as a linear function of his information:

$$a_i = b_{ii}\theta_i + \sum_{j \neq i} b_{ij} y_{ij}$$

Note that, while the b-terms are obviously not allowed to depend on the agent's information, they may be a (nonlinear) function of the communication intensities that agent i chooses in the first stage of the game. We define a *linear perfect Bayesian equilibrium* of $\Gamma(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ as a PBE where all agents use linear decision functions.

Remark 1 *In what follows, we are only going to consider interior equilibria of the two-stage games $\Gamma(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ and $\Gamma_\theta(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$. It is possible that, for some range of parameters of the primitives of our model, corner equilibria in which one or more agents choose the minimum possible precision ρ in either active or passive communication with someone else exist. However, under mild restrictions on the primitives of our model,[10] there exists an interior equilibrium (that we are going to characterize in few lines). In that case, corner equilibria are prevented. In any case, several of our comparative statics results would continue to hold when analyzing the behavior associated to a particular corner equilibria. The main difference would be on the fact that slight perturbations of a parameter may not alter*

[10] For example, if all d_{ij}s are strictly positive, for any values of k_r and k_p there exists $\rho > 0$ small enough such that existence of corner equilibria in both $\Gamma(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ and $\Gamma_\theta(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ is prevented.

at all the communication decisions of some agents. That would result in non-strict local monotonicities.

We can show:

Theorem 2 *The set of linear perfect Bayesian equilibria of the games* $\Gamma(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ *and* $\Gamma_\theta(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ *is the same and it is described by the following set of conditions:*

$$D_i b_{ii} = d_{ii} + \sum_{j \neq i} d_{ij} b_{ji} \qquad \text{for all } i \tag{2}$$

$$D_i b_{ij} = \frac{r_{ij} p_{ij}}{s_j r_{ij} + s_j p_{ij} + r_{ij} p_{ij}} \sum_{k \neq i} d_{ik} b_{kj} \qquad \text{for all } i, j \neq i \tag{3}$$

$$\frac{\sqrt{d_{ji}} b_{ij}}{k_r} = r_{ij} \qquad \text{for all } i, j \neq i \tag{4}$$

$$\frac{\sqrt{D_i} b_{ij}}{k_p} = p_{ij} \qquad \text{for all } i, j \neq i \tag{5}$$

To prove this result, it is useful to define two ancillary games. Starting from the dynamic game $\Gamma(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ introduced in the previous section, define the following linear one-shot game $\tilde{\Gamma}(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$. The set of players and the payoffs are the same as in $\Gamma(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$, but each agent i simultaneously selects:

- A vector $b_{i\cdot} = (b_{i1}, ..., b_{in})$, where all the b_{ij} are real numbers;
- Two vectors of communication variances: $(p_{ij})_{j \neq i}$ and $(r_{ji})_{j \neq i}$.

The signals are as before. Agent i's action is determined as a linear function of the state and the signals:

$$a_i = b_{ii}\theta_i + \sum_{j \neq i} b_{ij} y_{ij}$$

There are two differences between Γ and $\tilde{\Gamma}$: first, the former was a two-stage game with asymmetric information (the precisions chosen in the first stage are private information) while $\tilde{\Gamma}(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ is a one-shot game with symmetric information; second, the decision functions in $\tilde{\Gamma}(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ are restricted to be linear in the signals.

The proof of Theorem 2 in a set of three different lemmas.[11] We begin by showing that – thanks to the normal-quadratic assumption – the two versions of the one-shot game are equilibrium-equivalent:

Lemma 3 *Interior equilibria in* $\tilde{\Gamma}(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ *and* $\tilde{\Gamma}_\theta(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ *are characterized by the same system of first-order conditions, corresponding to (2) through (5).*

The second intermediate result finds a correspondence between linear equilibria of the one-shot game $\tilde{\Gamma}(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ and Perfect Bayesian equilibria of the two-stage game $\Gamma(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$:

[11] Proofs of all results can be found in the Appendix.

Lemma 4 *If* $\left(\hat{\boldsymbol{b}}, \hat{\boldsymbol{r}}, \hat{\boldsymbol{p}}\right)$ *is an interior equilibrium profile of* $\tilde{\Gamma}(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$*, then it is a linear perfect Bayesian equilibrium profile of* $\Gamma(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$*.*

The game $\Gamma(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ *has no other linear perfect Bayesian equilibrium.*

Finally, the third lemma provides the same correspondence between the one-shot game and the two-stage game for the case in which local information is know before decisions are taken:

Lemma 5 *If* $\left(\hat{\boldsymbol{b}}, \hat{\boldsymbol{r}}, \hat{\boldsymbol{p}}\right)$ *is an interior equilibrium profile of* $\tilde{\Gamma}_\theta(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$*, then it is a perfect Bayesian equilibrium profile of* $\Gamma_\theta(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$*.*

The game $\Gamma_\theta(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ *has no other linear perfect Bayesian equilibrium.*

In the next section, we will show by construction that the one-shot game $\tilde{\Gamma}(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ has a unique equilibrium and that such equilibrium is in pure strategies. Combined with Theorem 2, this will ensure that we can always find a linear pure-strategy equilibrium of game $\Gamma(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$.

One may wonder about the importance of the restriction to linear equilibria. Do $\Gamma(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ and $\Gamma_\theta(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ have equilibria where agents use strategies that are not linear in their signals? A similar question has arisen in other games with quadratic payoff functions, such as Morris and Shin (2002), Angeletos and Pavan (2007,2008), Dewan and Myatt (2008), and Calvó-Armengol and de Martí (2008).[12]

For our game we can prove the following uniqueness result. Consider $\Gamma(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ but assume that local states and actions are bounded above and below. Namely assume that $a_i \in [-\bar{a}, \bar{a}]$ and θ_i is distributed as a truncated normal distribution on $[-k\bar{a}, k\bar{a}]$, where $k < 1$. Call this new game $\Gamma^{\bar{a}}(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$. We can show that, as the bound $\bar{a}$ goes to infinity, the set of equilibria of the game $\Gamma^{\bar{a}}(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ contains (at most) one equilibrium and that this equilibrium corresponds to the linear equilibrium that we study here. The proof is available in the Appendix.

4 Equilibrium Communication Network

Given the equivalence result in Theorem 2, from now on we focus on the symmetric-information static game $\tilde{\Gamma}(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$. Unfortunately, Theorem 2 is not directly useful because it involves solving a system of $(3n-2)\,n$ equations, some of which are non-linear in the unknowns. As we shall see shortly, we can offer a more useful characterization.

[12] Uniqueness in the team-theoretic setting is proven in Marschak-Radner (1972, Theorem 5).

Calvó-Armengol and de Martí (2008) show that Marschak-Rdaner's line of proof extends to a strategic setting if the game admits a potential. Unfortunately, this does not apply to the game at hand ($\Gamma(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ has a potential only in the special case where $d_{ij} = d_{ji}$ for all pairs ij).

Angeletos and Pavan (2008) prove uniqueness by showing that in their economy the set of equilibria corresponds to the set of efficient allocations. A similar argument is used by Hellwig and Veldkamp (2008).

Dewan and Myatt (2008) prove uniqueness by restricting attention to strategies with non-explosive higher-order expectations.

We add two final pieces of notation. Let $\omega_{ij} = \frac{d_{ij}}{D_i}$, and let $\boldsymbol{\Omega}$ be the matrix with zeros in the diagonal and out-diagonal entries ω_{ij}. Let

$$h_{ij} = \begin{cases} \omega_{jj} & \text{if } i = j \\ -s_j \left(\frac{k_p}{\sqrt{D_i}} + \frac{k_r}{\sqrt{d_{ji}}} \right) & \text{otherwise} \end{cases}$$

Then we can show:

Theorem 6 *For any $(\boldsymbol{D}, \boldsymbol{s})$, if k_r and k_p are sufficiently low, the game $\tilde{\Gamma}(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ has a unique pure-strategy equilibrium:*

(i) Decisions are given by

$$\boldsymbol{b}_{\cdot j} = (\boldsymbol{I} - \boldsymbol{\Omega})^{-1} \cdot \boldsymbol{h}_{\cdot j} \qquad \textit{for all } j;$$

(ii) Active communication is

$$r_{ij} = \frac{\sqrt{d_{ji}} b_{ij}}{k_r} \qquad \textit{for all } i \neq j;$$

(iii) Passive communication is

$$p_{ij} = \frac{\sqrt{D_i} b_{ij}}{k_p} \qquad \textit{for all } i \neq j$$

The theorem offers a simple equilibrium characterization. Conditions (ii) and (iii) correspond to (4) and (5) and express each communication intensity as a linear function of just one decision coefficient.

Condition (i) is based on a characterization of $\boldsymbol{b}$ that is particularly tractable because: (a) It does not depend on communication intensities (it only depends on primitives); (b) It can be split into n systems of equations, one for each agent; (c) It is linear; (d) It uses a coefficient matrix that is the same for all n agents.

Each subsystem in condition (i) determines all the coefficients $(b_{ij})_{i=1,\ldots n}$ that relate to decisions taken by the n agents with respect to information originating from a certain agent j: namely, the signal y_{ij} if $i \neq j$ and the local state θ_j if $i = j$.

The matrix $(\boldsymbol{I} - \boldsymbol{\Omega})^{-1}$ can be understood as the result of adding an infinite sequence of effects corresponding to increasing orders of interaction:

$$\boldsymbol{b}_{\cdot j} = (\boldsymbol{I} - \boldsymbol{\Omega})^{-1} \cdot \boldsymbol{h}_{\cdot j} = \left(\boldsymbol{I} + \boldsymbol{\Omega} + \boldsymbol{\Omega}^2 + \ldots \right) \boldsymbol{h}_{\cdot j}$$

The first round, $\boldsymbol{b}^1_{\cdot j} = \boldsymbol{I} \cdot \boldsymbol{h}_{\cdot j}$ indicates the decision coefficients that the players would choose if they thought that the other players' decision coefficients were all zero. The second round $\boldsymbol{b}^2_{\cdot j} = (\boldsymbol{I} + \boldsymbol{\Omega}) \cdot \boldsymbol{h}_{\cdot j}$ yields the decision coefficients that the players would select if each of them thought that the other players chose decision coefficients as in $\boldsymbol{b}^1_{\cdot j}$. The iteration continues to infinity. As all the elements of $\boldsymbol{\Omega}$ are smaller than 1, the interaction effects become smaller and smaller, and the series converges.

The matrix $(\boldsymbol{I} - \boldsymbol{\Omega})^{-1}$ is the same for all subsystems because the propagation of interaction effects across agents goes through the same payoff matrix: a change in any part of j's decision function affects i through coefficient d_{ij}.

A remark about equilibrium strategies is in order. Equilibrium actions $a_i^* = b_{ii}\theta_i + \sum_{j\neq i} b_{ij}y_{ij}$ are not, in general, weighted averages of the information agents possess. Indeed, it can be shown that $\sum_{j=1}^{n} b_{ij} \leq 1$ with the inequality being strict unless communication is costless. More precisely:

Corollary 7 ***(Reaction to Communication)*** *Linear equilibrium actions $a_i^* = b_{ii}\theta_i + \sum b_{ij}y_{ij}$ satisfy that $\sum_j b_{ij} \leq 1$, with equality only in the case that $k_r = k_p = 0$. Moreover, $\sum_j b_{ij}$ is decreasing in k_r and k_p.*

The reason is simple. Agents use both prior and posterior information, the latter obtained from communication, to determine optimal, equilibrium, actions. When communication is cheap agents lean over posterior information, while when communication is very expensive and, thus, imprecise, they are more inclined to use prior information, meaning that they are biased towards the mean of prior distributions for local information. This mean in all cases is equal to 0. Agents react more to the information that communication reveals when it is cheap and precise. Otherwise they choose actions closer to 0, that are focal actions to resolve the coordination problem. The final weights depend both on communication costs and the magnitude of coordination motives.

One may wonder whether active communication and passive communication are strategic substitutes or complements. This question can be answered formally by examining the expected payoff function (**??**). Holding the decision coefficients $\boldsymbol{b}$ constant, the payoff function is additive in r_{ij} and p_{ij}: there are no direct strategic complementarities between active and passive precision. However, if the decision coefficients are taken as endogenous, r_{ij} and p_{ij} can be seen as complements – a fact that is evident from (ii) and (iii) in theorem 6.

4.1 Examples

To illustrate the use of Theorem 6, we apply it to the two examples introduced early.

4.1.1 Matrix-Form Firm (continued)

In the case of the matrix-form firm, we have:

Proposition 8 *In the matrix-form organization example:*

(i) The decision coefficients and the communication intensities can be written as follows:

$$b_{ij} = \begin{cases} b_G & \text{if } f_i = f_j \text{ and } c_i = c_j \\ b_F & \text{if } f_i = f_j \text{ and } c_i \neq c_j \\ b_C & \text{if } f_i \neq f_j \text{ and } c_i = c_j \\ b_L & \text{if } f_i \neq f_j \text{ and } c_i \neq c_j \end{cases}$$

and

$$(r_{ij}, p_{ij}) = \begin{cases} (r_F, p_F) & \text{if } f_i = f_j \text{ and } c_i \neq c_j \\ (r_C, p_C) & \text{if } f_i \neq f_j \text{ and } c_i = c_j \\ (r_L, p_L) & \text{if } f_i \neq f_j \text{ and } c_i \neq c_j \end{cases}$$

(ii) The decision coefficients are given by

$$\begin{aligned}
b_G &= H_G h_G + (n_F - 1) H_F h_F + (n_C - 1) H_C h_C + \Delta \\
b_F &= H_F h_G + (H_G + (n_F - 2) H_F) h_F + (n_C - 1) H_L h_C + (n_C - 1)(H_C - H_L) h_L + \Delta \\
b_C &= H_F h_G + (H_G + (n_C - 2) H_C) h_C + (n_F - 1) H_L h_F + (n_F - 1)(H_F - H_L) h_L + \Delta \\
b_L &= H_L h_G + ((n_F - 2) H_L + H_C) h_F + ((n_C - 2) H_L + H_F) h_C \\
&\quad + ((n_C - 2)(H_C - H_L) + (n_F - 2)(H_F - H_L) + (H_G - 2H_L)) h_L + \Delta
\end{aligned}$$

where H_G, H_F, H_C, and H_L are positive numbers (elements of the inverse of $(I - \Omega)$)

$$\Delta = (n_F - 1)(n_C - 1) H_L h_L$$

and,

$$\begin{array}{ll}
h_G = \frac{G}{W} & h_F = -s\left(\frac{k_p}{\sqrt{W}} + \frac{k_r}{\sqrt{F}}\right) \\
h_C = -s\left(\frac{k_p}{\sqrt{W}} + \frac{k_r}{\sqrt{C}}\right) & h_L = -s\left(\frac{k_p}{\sqrt{W}} + \frac{k_r}{\sqrt{L}}\right)
\end{array}$$

where $W = G + (n_F - 1) F + (n_C - 1) C + (n_F - 1)(n_C - 1) L$

(iii) The communication intensities are:

$$\begin{array}{lll}
r_F = \frac{\sqrt{F} b_F}{k_r} & r_C = \frac{\sqrt{C} b_C}{k_r} & r_L = \frac{\sqrt{L} b_L}{k_r} \\
p_F = \frac{\sqrt{W} b_F}{k_r} & p_C = \frac{\sqrt{W} b_C}{k_r} & p_L = \frac{\sqrt{W} b_L}{k_r}
\end{array}$$

The matrix-form firm displays a simple equilibrium information flow. Communication intensities are the same within a units that belong to the same function (for all units, for alal functions) and for units that belong to the same country. They are also between any two units that belong to different functions and different countries.

In turn, the decision coefficients can be expressed as a straightforward (but long) weighted sum of the four basic communication costs/benefits h_G, h_F, h_C, and h_L.

To illustrate the result, consider the following numerical example. Suppose that the firm operates in five countries ($n_C = 5$) and has three functions ($n_F = 3$). The interaction terms are: $G = 10$, $C = 2$, $F = 5$ and $L = 1$. The communication cost parameters are $k_r = k_p = 0.1$ and $s_i = 0.05$.

In this case, the weights in equilibrium actions are

$$\begin{array}{l}
b_G = 0.306\,77 \\
b_F = 4.475\,3 \times 10^{-2} \\
b_C = 4.\,067\,7 \times 10^{-2} \\
b_L = 2.\,404\,4 \times 10^{-2}
\end{array}$$

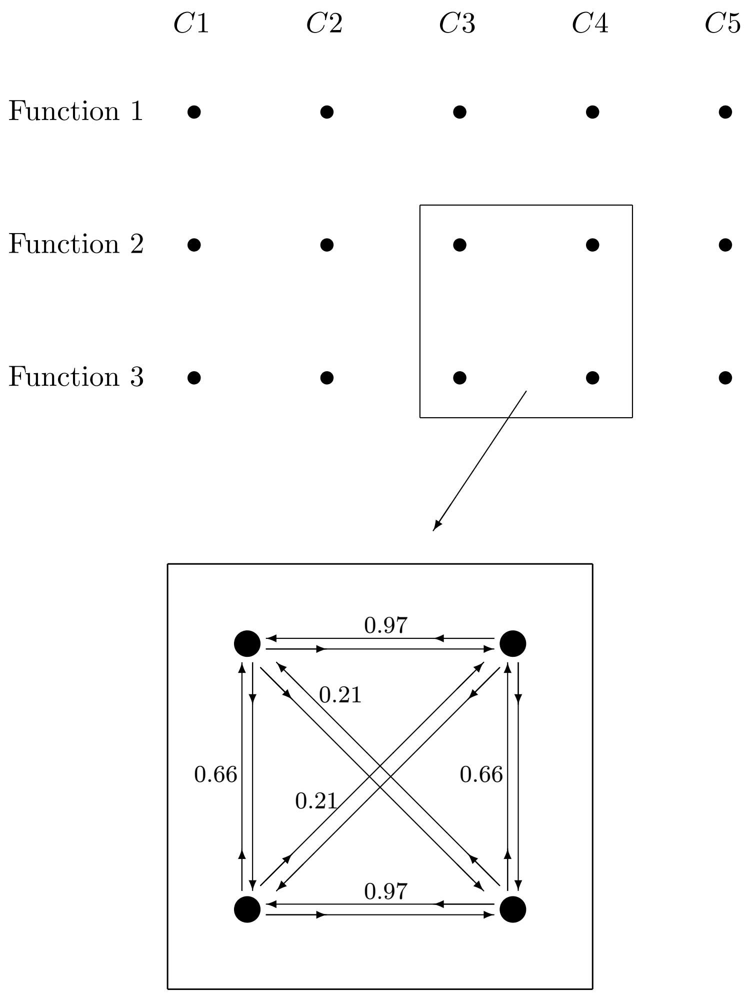
and equilibrium precisions of active and passive communication are

$$\begin{aligned}
r_F &= 1.\,328\,2;\ r_C = 0.811\,21;\ r_L = 0.240\,44 \\
p_F &= 3.\,563\,9;\ p_C = 3.\,441\,7;\ p_L = 1.\,442\,6
\end{aligned}$$

The overall communication precisions are

$$\begin{aligned}
\frac{r_F p_F}{r_F + p_F} &= 0.967\,60 \\
\frac{r_C p_C}{r_C + p_C} &= 0.656\,48 \\
\frac{r_L p_L}{r_L + p_L} &= 0.206\,09
\end{aligned}$$

The following figure provides a graphical representations of these overall equilibrium communication precisions



4.1.2 Network of Professionals (continued)

In our second example, Theorem 6 yields the following equilibrium decision and communication network:

Proposition 9 *The decision coefficients are*

$$b_{ij} = \begin{cases} \frac{1+\delta}{1+n\delta} - s_i \left(\frac{k_p(n-1)\delta}{\sqrt{(1+(n-1)\delta)W_i}} + \sum_{k\neq i} \frac{k_r\delta}{\sqrt{\delta W_k}} \right) & \text{if } i = j \\ \frac{\delta}{1+n\delta} - s_j \left(\frac{k_p(1+(n-1)\delta)}{\sqrt{(1+(n-1)\delta)W_i}} + \frac{k_r(1+\delta)}{\sqrt{\delta W_j}} + \sum_{k\neq\{i,j\}} \frac{k_r\delta}{\sqrt{\delta W_k}} \right) & \text{otherwise} \end{cases}$$

Active communication is

$$r_{ij} = \frac{\sqrt{\delta W_j} b_{ij}}{k_r} \qquad \text{for all } i \neq j;$$

Passive communication is

$$p_{ij} = \frac{\sqrt{W_i} b_{ij}}{k_p} \qquad \text{for all } i \neq j$$

The structure becomes even more simple if one assumes that agents differ only in the richness of their local environment (the opposite of precision s_i):

Corollary 10 *In the special case where $W_i = 1$ for all i,*

$$b_{ij} = \begin{cases} \frac{1+\delta}{1+n\delta} - s_i \frac{1+(n-1)\delta}{1+n\delta} \left(\frac{k_p}{\sqrt{(1+(n-1)\delta)}} + \frac{k_r}{\sqrt{\delta}} \right) (n-1)\,\delta & \text{if } i = j \\ A - Bs_j & \text{otherwise} \end{cases}$$

where

$$\begin{aligned} A &= \frac{1+\delta}{1+n\delta} \\ B &= \left(\frac{k_p}{\sqrt{(1+(n-1)\,\delta)}} + \frac{k_r}{\sqrt{\delta}} \right) (1+(n-1)\,\delta) \end{aligned}$$

and

$$\begin{aligned} r_{ij} &= \frac{\sqrt{\delta}}{k_r} (A - Bs_j) \\ p_{ij} &= \frac{1}{k_p} (A - Bs_j) \end{aligned}$$

Numerical Example Suppose that

$$\begin{array}{lll} \delta = 0.5 & n = 3 & k_p = k_r = 0.01 \\ s_1 = 1 & s_2 = 2 & s_3 = 4 \end{array}$$

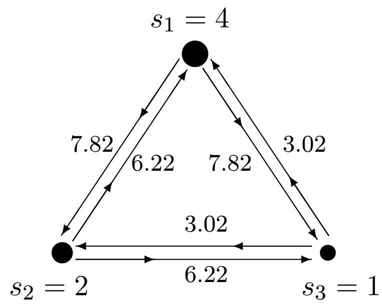
Then

$$\begin{array}{ll} r_{21} = r_{31} = 4.54 & p_{21} = p_{31} = 9.08 \\ r_{12} = r_{32} = 9.34 & p_{12} = p_{32} = 18.68 \\ r_{13} = r_{23} = 23.48 & p_{13} = p_{23} = 11.74 \end{array}$$

and

$$
\begin{aligned}
\frac{r_{21}p_{21}}{r_{21+}p_{21}} &= \frac{r_{31}p_{31}}{r_{31+}p_{31}} = \frac{4.542 \times 9.084}{4.542 + 9.084} = 3.02 \\
\frac{r_{12}p_{12}}{r_{12} + p_{12}} &= \frac{r_{32}p_{32}}{r_{32} + p_{32}} = \frac{9.342 \times 18.685}{9.342 + 18.685} = 6.22 \\
\frac{r_{13}p_{13}}{r_{13} + p_{13}} &= \frac{r_{23}p_{23}}{r_{23} + p_{23}} = \frac{23.484 \times 11.742}{23.484 + 11.742} = 7.82
\end{aligned}
$$

The share of expenditure going to active communication is the same in all three cases and it's $\frac{1}{3}$



5 Properties of Equilibrium Communication

We now use Theorem 6 to study the properties of the communication networks that are formed in equilibrium.

5.1 What Determines Overall Communication?

To begin, what determines the overall intensity of communication within a set of agents? Let us consider change on three possible dimensions: We say that:

- Overall interaction increases if matrix $\boldsymbol{D}'$ is such that $d'_{ij} \geq d_{ij}$ for all i and all j and $d'_{ij} > d_{ij}$ for at least one pair (i, j) (which may be the same);
- Communication cost decreases if $k'_r \leq k_r$ and $k'_p \leq k_p$ with a strict inequality for at least one of the two cost parameters;
- Overall uncertainty increases if $s'_i \leq s_i$ for all i with a strict inequality for at least one i.

Proposition 11 *Communication intensities increase when the communication cost decreases and the overall uncertainty increases. Communication intensitites increase when the overall interaction increases under some particular transformations in the interaction matrix. Formally, suppose that the game with $(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ has an interior solution, then:*

(i) the game with $(\boldsymbol{D}', \boldsymbol{k}, \boldsymbol{s})$, where $\boldsymbol{D}'$ is obtained by multiplying row i of $\boldsymbol{D}$ by $\lambda_i > 1$, also has an interior solution and satisfies that $b'_{ji} > b_{ji}$, $b'_{ij} > b_{ij}$ for all j, and $r'_{ji} > r_{ji}$, $r'_{ij} > r_{ij}$, $p'_{ji} > p_{ji}$ and $p'_{ij} > p_{ij}$ for all $j \neq i$.

(ii) the game with $(\boldsymbol{D}, \boldsymbol{k}', \boldsymbol{s})$ also has an interior solution and satisfies that, for every i and every j, $b'_{ij} > b_{ij}$, $r'_{ij} > r_{ij}$, and $p'_{ij} > p_{ij}$.

(iii) the game with $(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s}')$ also has an interior solution and satisfies that, $b'_{ji} > b_{ji}$, $r'_{ji} > r_{ji}$, and $p'_{ji} > p_{ji}$.

This proposition determines the sign of natural comparative statics with respect to the primitives of our model

The first part of the proposition ensures that if we increase proportionally all interaction coefficients of a single agent, everybody is going to pay more attention to the information he transmits about his signal, that he is also going to pay more attention to the information communicated by all the rest of agents, and that active and passive communication precisions in his bilateral communications increase as well.

The second part of the proposition presents a natural conclusion: if communication costs decrease then all decision parameters move up, i.e. every agent is going to increase the intensity of all bilateral communications and is going to pay more attention to the communication reports received; hence, all communication precisions increase accordingly,

Finally, the third part states that if an agent receives local information with an smaller amount of uncertainty, everybody is going to pay more attention to the information this agent is going to transmit. Agents might find beneficial to adopt actions closer to the information with less intrinsic uncertainty as a focal point to alleviate miscoordination.

5.2 Who Communicates with Whom?

A more subtle question is: What determines the relative communication intensities? We have already provided a number of comparative statics results with respect to the primitives of the model. Here we are interested in the effect of a transfer of weights in the interaction coefficients of a memeber of the organization.

Proposition 12 *Starting from a symmetric situation, an increase in d_{ij} compensated by an equal decrease in d_{ik} (where i, k, and j are distinct integers) leads to an increase in b_{ji}, r_{ji}, p_{ji} and a decrease in b_{ki}, r_{ki}, p_{ki}, while other other variables are unchanged.*

When we slightly perturb a symmetric situation, in which all pairwise interaction coefficients coincide, and slightly increase the coordination motive of agent i with respect to agent j, counterbalanced by a decrease of same magnitude in the coordination motive of agent i with respect to agent k, the comparative statics are clear: agent j, resp. k, cares more (resp.less) about the information received from agent i in the action he undertakes, he pays more (resp. less) attention to this information, and agent i is going to put more (resp. less) effort in communicating accurately his local information.

5.2.1 Example: Matrix Form (continued)

In the matrix form example, we obtain sharp predictions about which agents communicate more intensely.

Proposition 13 *Suppose that k_r, k_p, and L are sufficiently small. Then:*

(i) If there are the same number of countries and functions ($n_F = n_C$), communication within functions is more intense than communication within countries if and only if $F > C$.

(ii) If the interaction within countries and the interaction across functions are equally strong ($F = C$), communication within functions is stronger than communication within units if and only if there are less functions than countries ($n_F < n_C$)

This proposition ties intensity of communication to the characteristics of each one of the layers that determine the matrix structure of the organization. When the number of possible labels in each of the layers (functions or countries) coincides the intensity of communication depends is strongly linked to the interaction pattern: if coordination among different local functions is more important than international coordination, we naturally expect more communication at the functional level. If the cordination motives at the functional and international level are the same, we conclude that communication is going to be focused on the layer that shows an smaller number of elements: if there are less functions than countries, communication is going to be mostly executed among the different local functions again.

5.3 Which Way Does Information Flow?

Is the amount of information communicated by agent i to j greater than the amount communicated by j to i?

The precision of communication from i to j is given by

$$q_{ij} = \frac{1}{\rho_{ij}^2 + \pi_{ij}^2} = \frac{r_{ij} p_{ij}}{r_{ij} + p_{ij}}$$

Proposition 14 *Suppose that, for two agents i and j, $w_{jj} = w_{ii}$ and $s_j = s_i$. Then,*

$$\frac{q_{ij}}{q_{ji}} = \frac{b_{ii}}{b_{jj}}.$$

The proposition compares two agents who face the same amount of local uncertainty and who place the same relative importance of adapting to the local state and coordinating with other agents. The proposition shows that the ratios between the precisions of communication from i to j is the same as the ratio of the own local state decision coefficients of the same two agents. An agent who, in equilibrium, pays more attention to his own state is an agent who communicates more information to others.

5.4 Who Wields Influence?

While the previous section dealt with relative influence in pairwise comparison, one can ask which agents are most influential overall.

It turns out that the answer to this question links Theorem 6 with a family of measures of centrality developed in sociology (Bonacich 1987) and applied to economics by Ballester et al (2006).

We begin by introducing basic notions of network centrality. Consider a network described by an n-square adjacency matrix G, where $g_{ij} \in [0,1]$ measures the strength of the path from i to j. For any positive scalar a (sufficiently low), define the matrix

$$\boldsymbol{M}(\boldsymbol{G}, a) = [\boldsymbol{I} - a\boldsymbol{G}]^{-1}.$$

Each element m_{ij} of the matrix $\boldsymbol{M}$ can be interpreted as a weighted sum of the paths – direct or indirect – leading from node i to node j. The parameter a is a decay parameter that may put a lower weight on less direct effects. Let $m_{ij}(\boldsymbol{G}, a)$ be the ij element of $\boldsymbol{M}$.

The Bonacich centrality measure of node i is defined as

$$\beta_i(\boldsymbol{G}, a) = \sum_{j=1}^{n} m_{ij}(\boldsymbol{G}, a).$$

The centrality of node i is determined by the weighted sum of paths to all nodes that begin in i.

In our context, we let $B_i = \sum_{j=1}^{n} b_{ji}$ be the *global influence* of agent i. Note that this corresponds to the sum of the expected effects of a change on the agent's local state on all actions (including the agent's own action).[13]

We begin by characterizing global influence:

Proposition 15 *The global influence of agent i can be expressed as a weighted sum of all the agents' Bonacich centrality measures, computed on Ω' with decay factor one, where the weights are given by the $h_{\cdot i}$*

$$B_i = \sum_{j=1}^{n} \beta_j\left(\boldsymbol{\Omega}', 1\right) h_{ji}.$$

The global influence of agent i is a sum of weighted Bonacich measures, where the weights on the agent's own measure is positive (because $h_{ii} = \omega_{ii}$) while all the other weights are negative. Hence, an agent's global influence depends positively on the centrality of that agent and negatively on the centrality of all other agents.

The following is an immediate consequence of the Proposition:

Corollary 16 *When $k_p, k_r \to 0$, the global influence of agent i corresponds to the product of his Bonacich centrality index, computed on Ω' with decay parameter one, and the agent's sensitivity to his own action.*

$$\sum_{j=1}^{n} b_{ji} = \beta_i\left(\boldsymbol{\Omega}', 1\right) \omega_{ii}$$

When communication costs vanish, local states of the world become common knowledge. The influence of agent i's local states on other agents depends on how connected agent i is to the rest of the network. The local state of an agent with stronger links is more influential. The right way to compute connectedness in this case is the Bonacich centrality measure

[13] While the notions of network centrality that we employ are already present in the literature, to the definition of global influence is, to the best of our knowledge, novel.

based on the transpose of $\mathbf{\Omega}$ with decay factor 1. The use of the transpose depends on the fact that the influence of i on j is measured by ω_{ji} (had we swapped indices we would not use the transpose).

The global influence of agent i also depends on his sensitivity to his own action ω_{ii}. The other agents' payoffs do not depend directly on i's local state. They are affected by it indirectly through the interaction factors of the form $d_{ji}\left(a_j - a_i\right)^2$. Hence, other agents will be more eager to adapt to i's local state if i's action is strongly influenced by i's local state, which happens when ω_{ii} is high. In the extreme case where $\omega_{ii} \to 0$, the agent's action is independent of his local state and other agents have no reason to adapt their actions to i's state.[14]

Our characterization of global influence produces sharp predictions. First, define an increase in agent i´s importance as follows. For one agent i, let $d'_{ij} = \lambda d_{ij}$ for all j. Consider an infinitesimal increase of λ starting from $\lambda = 1$.

Proposition 17 *If i becomes more important, all global influences increase. The global influence of i increases more than the others if and only if the cost of passive communication is sufficiently low compared to the cost of active communication:*

$$\frac{k_p}{k_r} \leq \sum_{k \neq i} \frac{\beta_k\left(\mathbf{\Omega}', 1\right)}{\beta_i\left(\mathbf{\Omega}', 1\right)} \frac{\sqrt{D_i}}{\sqrt{d_{ik}}}.$$

We know from *(i)* in Proposition 11 that when an agent becomes more important everybody is going to increase the weight of her communication reports in others' actions. This explains why her global influence increases. But the increase on the level of externalities is also partially internalized by the rest of agents in the organization: through the now more important agent, all other agents also care more for coordination with the rest of the organization. In other words, if a first agent wants to coordinate with the now more important second agent, and hence this second agent aims to coordinate more with a third one, the first agent needs also to coordinate better with the third one. That's the reason behind the increase of all global influences. Furthermore, when passive communication is less costly than active communication, the best way to achieve coordination is by paying attention to the messages send by the rest of agents inside the organization. In particular, people is going to pay more attention to the information transmitted by the now more important agent (that translates into first-order level of externalities) than on paying attention to rest (that would come from the second, and higher, order of externality spreading). This ensures that the global influence of the now more important agent increases more than the rest.

Second, consider the effect of a change in the uncertainty faced by agent i.

Proposition 18 *If s_i increases, i's global influence decreases and the global influence for the rest of agents does not change.*

This result provides an straightforward relation between local information uncertainty and global influence. Ceteris paribus, an increase in local uncertainty implies a decrease in

[14] Another extremely important measure of network centrality is the invariant method (Palacios-Huerta and Volij 2004). The relation between our results and the invariant method is unclear.

global influence of the local agent. The reason why this only affects the local agent and does not spread into a decrease for the rest of agents as well is that here there are no second-order externalities. An increase of uncertainty does not change the indirect coordination motives of the rest of the population.

5.4.1 Example: Network of Professionals (continued)

The two effects described in Propositions 17 and 18 are easy to observe in our second example.

Proposition 19 *In the Network of Professionals example, the global influence of agent i is*

$$B_i = 1 - s_i k_p \sqrt{1 + (n-1)\,\delta} \sum_{j \neq i} \frac{1}{\sqrt{W_j}} - (n-1)\, s_i k_r \frac{1 + (n-1)\,\delta}{\sqrt{\delta W_i}}$$

The global influence of agent i increases whenever W_i or W_j, with $j \neq i$, increase. When s_i increases B_i decreases, and the expression of B_i does not depend on s_j for $j \neq i$.

In the special case where $W_i = 1$ for all agents,

$$B_i = 1 - (n-1)\, s_i k_p \sqrt{1 + (n-1)\,\delta} - (n-1)\, s_i k_r \frac{1 + (n-1)\,\delta}{\sqrt{\delta}}$$

Numerical example Suppose that $\delta = 0.5$, $n = 3$, $k_p = k_r = 0.01$. Compare the case where $s_1 = s_2 = s_3 = 2$ with the case we saw before $s_1 = 1$, $s_2 = 2$, $s_3 = 4$.

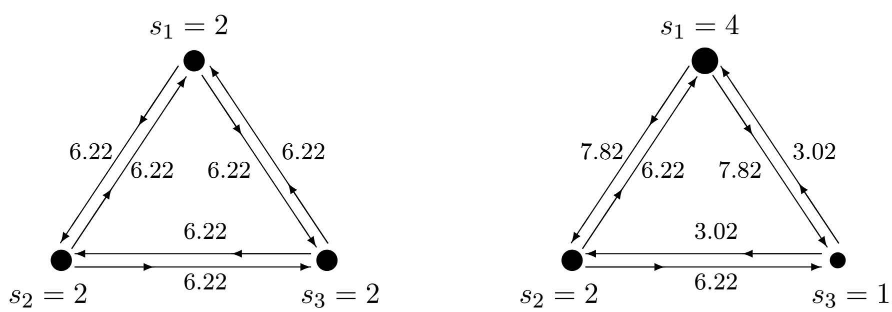
In the first case, the solution is symmetric across agents, and we have

$$r_{ij} = 9.34 \quad p_{ij} = 18.68 \ ,$$

with communication precision

$$\frac{r_{ij} p_{ij}}{r_{ij} + p_{ij}} = 6.22.$$

The two equilibrium communication networks are depicted in the figure below



In the first case, all agents have the same global influence: for $i = 1, 2, 3$,

$$B_i = 0.83$$

In the second case, as one would expect, our influence measure is lower for the first agent and higher for the third.

$$B_1 = 0.66 \quad B_2 = 0.83 \quad B_3 = 0.91$$

5.5 Adaptation vs. Coordination

Agents' payoffs reflect both an adaptation and a coordination concern. On the one hand, agents want to adapt to their local state. On the other one, they want to coordinate with the rest of members of the organization. In this section we show that communication intensities show an inverse U shape form. In particular, we prove that communication activity by agent i vanishes whenever the adaptation concern is too intense or too weak compared to coordination motives.

To formalize this result, define $d'_{ii} = \lambda d_{ii}$ and $d'_{ij} = (2-\lambda)\, d_{ij}$ for all $j \neq i$. Observe that:

- If $\lambda \to 0$ then $d'_{ii} \to 0$ and $d'_{ij} \to 2d_{ij} \geq 0$, and $\frac{d'_{ii}}{d'_{ij}} \to 0$. Coordination outweighs adaptation.
- if $\lambda = 1$ we have the initial vector of i's interaction terms d.
- If $\lambda \to 2$ we obtain $d'_{ii} = 2d_{ii} > 0$ and $d'_{ij} \to 0$, and $\frac{d'_{ii}}{d'_{ij}} \to +\infty$. Adaptation outweighs coordination.

Proposition 20 *If $\lambda \to 0$ or $\lambda \to 2$ agent i does not engage in active communication, and no agent passively communicates with him.*

The reasons why communication vanishes when we approach the two extreme situations is different in each case. When coordination motives outweigh the adaptation motive, communication engagement is null because there is a natural focal point that resolves coordination problems: agent, according to prior information, fix their actions to be 0. This trivially resolves coordination and does not affect the decision problem that right now is of negligible magnitude. Local information is unnecesary.

On the other hand, when adaptation outweighs coordination, agents primarly want to resolve their respective local decision problems. The obvious way is to fix their action close to the local information they possess. In this case, prior information is redundant.

5.6 Active vs. Passive Communication

Do agents invest more in speaking or listening? For any directed link between two agents, we can compare the intensity of active communication with the intensity of passive communication: r_{ij}/p_{ij}.

From Theorem 6, we immediately see that

Proposition 21 *The ratio between active and passive communication is*

$$\frac{r_{ij}}{p_{ij}} = \frac{k_p}{k_r}\sqrt{\frac{d_{ji}}{D_i}} \qquad \textit{for all } i \neq j$$

Clearly, the ratio depends on the ratio between the cost parameters for active and passive communication. More interestingly, the balance between active and passive communication depends on the ratio d_{ji}/D_i.

In general, we should expect $D_i > d_{ji}$. Only in the case that agent i is particularly prominent and the interaction coefficient d_{ji} for another agent j is larger than the sum of interaction coefficients D_i that affect agent i, active communication can be relatively more intensive than passive communication.

If the problem is symmetric ($d_{ij} = \bar{d}Q$ for all $i \neq j$ and $d_{ii} = \left(1 - (n-1)\,\bar{d}\right) Q$), we have

$$\gamma_{ij} = \bar{d} \qquad \text{for all } i \neq j.$$

Consider the case in which active and passive communication are equally costly, i.e. $k_p = k_r$. As $\bar{d} < \frac{1}{n-1}$, this means that the ratio between active and passive communication is bounded above by $\frac{1}{n-1}$, implying that: (i) it is smaller than 1; (ii) it becomes lower as n increases.

As we mentioned after Proposition **??**, there is a strategic asymmetry between active and passive communication, which favors passive communication. In our set-up, spending resources to inform another agent has a diluted effect: the signal I send you makes you change your behavior and that affects my payoff as well as the payoff of other agents (but I only internalize my part). Hence, the positive effect is a linear function of d_{ji}. Instead, if I spend more resources to understand your state of the world, I can use this information directly to change my decision, and that affects the interaction of my decision with my own state and with the decisions of all the other agents. Hence the effect is a linear function of D_i.

The only case where passive communication does not have an intrinsic advantage is when there are only two agents. Conversely, as the number of agents increases, the ratio tends to zero.

5.7 Is Equilibrium Communication Efficient?

One can ask whether the communication network that emerges in the equilibrium of the non-cooperative game that we have studied is efficient. This question can be asked in two ways, depending on which benchmark is used. In particular one can compare the equilibrium outcome to the outcome that would arise if communication intensities were chosen by a planner, while decision functions were still be delegated to agents. Or one can use as a benchmark the case where the planner is also responsible for choosing decision functions. Here, we analyse the first case, while we leave the second for the team theory section.

Reconsider the symmetric-information static game $\tilde{\Gamma}\left(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s}\right)$. Keep the same payoff functions u_i defined in (1), but now assume that each agent i solves

$$\max_{\{b_{ij}\}_{j=1}^{n}} E\left[u_i\right]$$

while a planner solves

$$\max_{\left\{\rho_{ji}^2\right\}_{j=1}^{n}, \left\{\gamma_{ij}^2\right\}_{j=1}^{n}} \sum_{i=1}^{n} E\left[u_i\right]$$

The planner and the agents make their decisions simultaneously. Call this new game $\Gamma^*\left(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s}\right)$.

By an argument similar to Theorem 2, a pure-strategy Nash equilibrium of $\Gamma^*\left(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s}\right)$ corresponds to a perfect-Bayesian equilibrium of a two-stage game where in the first stage the planner chooses communication intensities but the intensities are not observed by agents, while in the second stage agents receive signals and select actions.

We can offer an equilibrium characterization that mirrors the one of Theorem 6:

Proposition 22 *The decision network and communication network that arise in equilibrium are given by:*

$$\begin{aligned}
\boldsymbol{b}_{\cdot i} &= (\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\cdot\tilde{\boldsymbol{h}}_{\cdot i} \quad \text{for all } i\\
r_{ij} &= \sqrt{\sum_{k=1}^{n} d_{ik}+\sum_{k\neq i}^{n} d_{ki}}\,\frac{b_{ij}}{k_r} \qquad \text{for all } i\neq j\\
p_{ij} &= \sqrt{\sum_{k=1}^{n} d_{ik}+\sum_{k\neq i}^{n} d_{ki}}\,\frac{b_{ij}}{k_p} \qquad \text{for all } i\neq j
\end{aligned}$$

with

$$h_{ji}=\begin{cases}\omega_{ii} & \text{if } i=j\\ -s_i\frac{k_p+k_r}{\sqrt{\sum_{k=1}^{n}d_{ik}+\sum_{k\neq i}^{n}d_{ki}}} & \text{otherwise}\end{cases}$$

Note that this implies

Corollary 23 *In the efficient communication network, $k_r r_{ij}^2 = k_p p_{ij}^2$ for all i and j.*

The asymmetry between active and passive communication that we discussed earlier is not present with a planner. For any level of precision of communication between i and j, the planner minimizes the total cost of achieving it by equalizing the marginal costs of active and passive communication.

By an argument similar to the one in Proposition, we can show:

Proposition 24 *In the equilibrium of $\Gamma^*(\boldsymbol{D},\boldsymbol{k},\boldsymbol{s})$ all the decision coefficients and communication intensities are larger than in the equilibrium of $\tilde{\Gamma}(\boldsymbol{D},\boldsymbol{k},\boldsymbol{s})$.*

In a game of coordination, communication creates positive externalities that players do not internalize in the non-cooperative game.

6 Team Problem

In this section we extend the previous analysis by allowing agents to be grouped in teams. All agents in the same team share the same utility function, that is obtained by adding up the previously defined utility functions of all team members. Two extreme cases are encompassed in this framework. When each team is formed by only one agent we come back to the initial game-theoretical formulation of the problem. When all agents are in the same team, we are in the classical formulation of a team problem. In particular, in this second case, we model how internal communication intensities in a team are endogenously determined by the agents that form it.

Now the n agents are divided into different disjoint teams. For a given agent i, we denote by $T(i)$ the team to which he belongs. Each agent shares the same utility function with the rest of members of his team. Define $t_{ij} = d_{ij} + d_{ji}$. Note that $t_{ij} = t_{ji}$.The utility function of a team T is

$$u_T = -\sum_{i\in T} d_{ii}(a_i-\theta_i)^2 - \sum_{i,j\in T, i\neq j} \frac{t_{ij}}{2}(a_i-a_j)^2 - \sum_{i\in T, j\notin T} d_{ij}(a_i-a_j)^2 - \sum_{i\in T}\sum_{j\neq i} c_r(r_{ji}) - \sum_{i\in T}\sum_{j\neq i} c_p(p_{ij})$$

Some notation is necessary. Let

$$\phi_{ij} = \begin{cases} \frac{d_{ii}}{d_{ii}+\sum_{j\in T(i), j\neq i} t_{ij} + \sum_{j\notin T(i)} d_{ij}} & \text{if } i=j \\ \frac{t_{ij}}{d_{ii}+\sum_{j\in T(i), j\neq i} t_{ij} + \sum_{j\notin T(i)} d_{ij}} & \text{if } i\neq j, T(i)=T(j) \\ \frac{d_{ij}}{d_{ii}+\sum_{j\in T(i), j\neq i} t_{ij} + \sum_{j\notin T(i)} d_{ij}} & \text{if } i\neq j, T(i)\neq T(j) \end{cases}$$

Let $\mathbf{\Phi}$ be the $n\times n$ matrix with zeros in the diagonal and outside diagonal entries equal to Φ_{ij}. As before, $\boldsymbol{b}_{\cdot i}$ is the column vector with entries b_{ji}. $\boldsymbol{z}_{\cdot i}$ is the column vector with entries

$$z_{ki} = \begin{cases} \frac{d_{ii}}{d_{ii}+\sum_{j\in T(i), j\neq i} t_{ij} + \sum_{j\notin T(i)} d_{ij}} & \text{if } i=k \\ -s_i \frac{k_p+k_r}{\sqrt{d_{kk}+\sum_{j\in T(k), j\neq k} t_{kj} + \sum_{j\notin T(k)} d_{kj}}} & \text{if } T(i)=T(k) \\ -s_k\left(\frac{k_p}{\sqrt{d_{kk}+\sum_{j\in T(k), j\neq k} t_{kj} + \sum_{j\notin T(k)} d_{kj}}} + \frac{k_r}{\sqrt{\sum_{j\in T(i)} d_{jk}}}\right) & \text{if } T(i)\neq T(k) \end{cases}$$

Proposition 25 *The solution to the team-theory problem is given by:*
(i) Optimal decisions coefficients are

$$\boldsymbol{b}_{\cdot i} = (\boldsymbol{I}-\mathbf{\Phi})^{-1}\cdot \boldsymbol{z}_{\cdot i}$$

(ii) Active communication is

$$r_{ik} = \begin{cases} \frac{\sqrt{d_{kk}+\sum_{j\in T(i), j\neq k} t_{jk} + \sum_{j\notin T(i)} d_{kj}}}{k_r} b_{ik} & \textit{if } k\in T(i) \\ \frac{\sqrt{\sum_{j\in T(i)} d_{jk}}}{k_r} b_{ik} & \textit{if } k\notin T(i) \end{cases} \tag{6}$$

(iii) Passive communication is

$$p_{ik} = \frac{\sqrt{d_{ii}+\sum_{j\in T(i), j\neq i} t_{ij} + \sum_{j\notin T(i)} d_{ij}}}{k_p} b_{ik} \quad \textit{for all } i\neq k$$

Observe that the matrix $\mathbf{\Phi}$ in the characterization of the team theory solution is equivalent to a matrix $\mathbf{\Omega}$ in the game theory solution associated to a different interaction matrix: when agents i and j are in the same team, the interaction coefficients d_{ij} and d_{ji} rise up to $t_{ij} = d_{ij}+d_{ji}$, while the rest remain the same. Then D_i is now equal to $d_{ii}+\sum_{j\in T(i), j\neq i} t_{ij} + \sum_{j\notin T(i)} d_{ij}$. But the vector $\boldsymbol{z}_{\cdot i}$ in the team theory solution is not equivalent to the $\boldsymbol{h}_{\cdot i}$ in the game theory solution with the same transformation in interaction terms. Hence, the team theory solution is in general not equivalent to the solution of an alternative game theory problem with different interaction structures.

6.0.1 Example: Networks of Professionals (continued)

Re-consider the case with three players and $s_1 = s_2 = s_3 = 2$. It is interesting to focus on a symmetric example, to illustrate the asymmetry that arises when some agents are grouped in teams.

In the firgure below, the first graph depicts a situation where there are no teams. In the second graph, agents 1 and 2 belong to the same team. Using the Theorem above, we compute the equilibrium communication network as follows:

$$\begin{array}{ll} r_{12} = r_{21} = 33.07 & p_{12} = p_{21} = 33.07 \\ r_{13} = r_{23} = 11.81 & p_{13} = p_{23} = 18.68 \\ r_{31} = r_{32} = 10.35 & p_{31} = p_{32} = 20.70 \end{array}$$

which yields a high within-team communication intensity

$$\frac{r_{12}p_{12}}{r_{12} + p_{12}} = \frac{r_{21}p_{21}}{r_{21+}p_{21}} = 16.53$$

and a slightly higher across-team communication intensity

$$\begin{aligned} \frac{r_{13}p_{13}}{r_{13} + p_{13}} &= \frac{r_{23}p_{23}}{r_{23} + p_{23}} = 7.23 \\ \frac{r_{31}p_{31}}{r_{31} + p_{31}} &= \frac{r_{32}p_{32}}{r_{32} + p_{32}} = 6.90 \end{aligned}$$

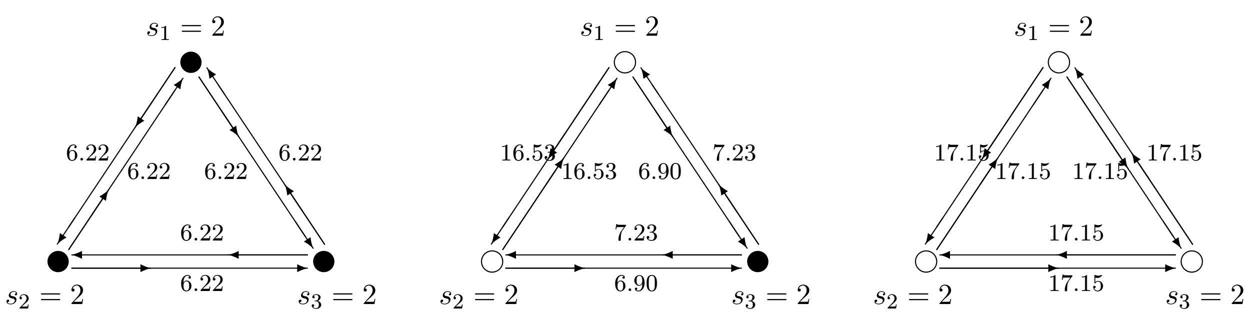
Finally, in the the third graph we examine the case where all agents belong to the same team. Symmetry is restored and communication intensity continues to increase:

$$r_{ij} = 34.30 \quad p_{ij} = 34.30$$

and

$$\frac{r_{ij}p_{ij}}{r_{ij} + p_{ij}} = 17.15$$

Note also that within teams active and passive communication intensities are balanced.



6.0.2 Example: Matrix-form Organizations (continued)

Consider again the numerical example developed before, in which there are five countries ($n_C = 5$) and three functions ($n_F = 3$); interaction terms are $G = 10$, $C = 2$, $F = 5$ and $L = 1$; communication cost parameters are $k_r = k_p = 0.1$; and $s_i = 0.05$.

We analyze here the case in which the diferent units in each country with same function are grouped to form a team. Hence, there are three different teams, one for each function.

In this numerical example we obtain that the weights in equilibrium actions are

$$\begin{aligned} b_G &= 0.1714 \\ b_F &= 5.5937 \times 10^{-2} \\ b_C &= 1.5058 \times 10^{-2} \\ b_L &= 9.9441 \times 10^{-3} \end{aligned}$$

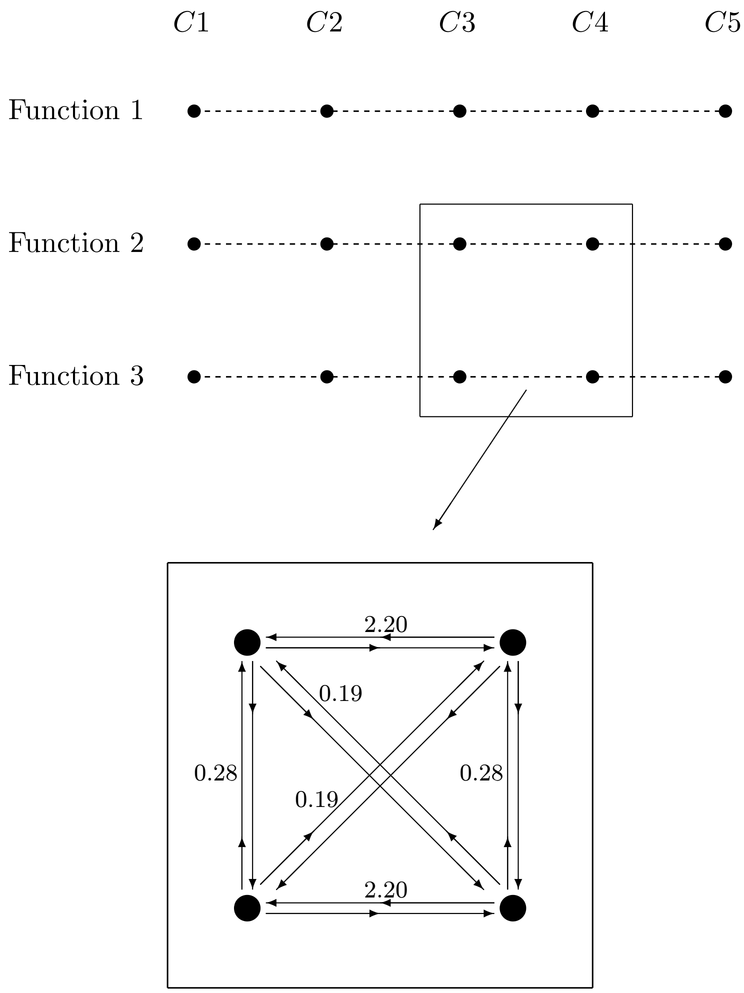
and the equilibrium precisions of active and passive communication are

$$\begin{aligned} r_F &= 4.4045;\ r_C = 0.36884;\ r_L = 0.24358 \\ p_F &= 4.4045;\ p_C = 1.1857;\ p_L = 0.78300 \end{aligned}$$

The overall equilibrium communication precisions are

$$\begin{aligned} \frac{r_F p_F}{r_F + p_F} &= 2.2023 \\ \frac{r_C p_C}{r_C + p_C} &= 0.28133 \\ \frac{r_L p_L}{r_L + p_L} &= 0.18578 \end{aligned}$$

The following figure provides the equilibrium precisions of pairwise communication.



As expected, there is more intrateam communication than in the game theory case. Because of the complementarity between equilibrium action decisions and equilibrium communication decisions, also units in the same team put more weight on the communication reports received from the rest of units with same function than in the game theory case, and it decreases the weight on its own local information. Interestingly, communication among units of different teams decreases, both in the case that they are in the same country or in different ones. Similarly, the complementarity between actions and communication decisions is reflected in that the weight of the communication reports of units in different countries is smaller than in the game theory case.

6.1 Negative Inter-Team Effects

As the numerical matrix-form example illustrates, the creation of teams may reduce communication between units that now belong to different teams. This effect can also be seen analytically, at least in a special case:

Proposition 26 *Consider a matrix-form organization with two countries and two functions. Suppose $C = F$ and consider the limit case as $L \to 0$, $k_r \to 0$, and $k_p \to 0$. Starting from a situation where all units are independent, group all units in a function into a single team. Then:*

(i) Communication and influence within a function go up.

(ii) The within-country decision-coefficent b_C is reduced. Within-country active communication r_C is reduced. The sign of the change in passiev communication p_C is ambiguous.

The intuition behind this result has to do with the fact that units that now belong to the same team internalize the damage that they impose on other team members when they vary their action in response to signals they receive from non-team members.

7 Conclusion

The present paper is a first step towards modeling equilibrium information flows in network games with asymmetric information. We arrived at a close-form characterization of the matrix of communication intensities that agents select in equilibrium. The characterization was then used to obtain a number of comparative statics results.

Does our model have the potential to be used for empirical work? What kind of data could identify the model? Suppose that we observe the information flows among nodes of a network (e.g what amount of resources each firm spends for liaising with other firms) but not the underlying interaction matrix, communication cost parameters, local state uncertainty, or decision functions. Can back out the primitives of the model?

The potential for identification is there. If the number of agents is at least four, Theorem 6 supplies a number restrictions that is at least as large as the number of primitive variables to be estimated.[15]

A similar identification potential exists in the other formulation of the problem, which is explored in the Appendix. This observation, although preliminary, appears to indicate that data on information flows could be a fruitful avenue for investigating organizations empirically, if combined with a model – not necessarily the present one – of endogenous communication in network games.

The model can be applied to a variety of setups, including matrix-form organizations and networks of professionals, but also to other ones. For example, as Kranton and Minehart (2000) highlight, buyer-seller relationships determine a network structure of vertical contacts. When the formation of this network occurs in a decentralized manner, coordination problems may emerge. Communication could be introduced in a similar model to ensure that miscoordination in the network formation process is minimized.

Finally, we have considered and static setup an, although this seems a natural starting point for our incquire, it would be interesting to analyze dynamic communication protocols in a similar environment. Information would then come from direct communication and from learning of the past activity of some, or all, agents in the organization. This relates to

[15] The Theorem consists of system of $n\,(3n-2)$ equations in $2n^2+n+2$ variables (b's, d's, s's, and k's). There are at least as many equations as variables if $n \geq 4$.

a recent literature on social learning in networks (see for example, Acemoglu et al., 2008, and Bala and Goyal, 2008) and we plan to pursue this analysis in future research.

Appendix A. Proofs

Proof of Theorem 2:

The theorem is proven through the three lemmas in text. Proofs of the lemmas follow. ■

Proof of Lemma 3:

For analytical tractability, in proofs we use variances instead of precisions. We denote by $\sigma_i = 1/s_i$ the variance of θ_i, $\rho_{ij} = 1/r_{ij}$ the variance of ε_{ij}, and $\pi_{ij} = 1/p_{ij}$ the variance of η_{ij} (the omission of the square sign is intentional: σ, ρ and π are variances not standard deviations). The expected payoff of agent i is then

$$\begin{aligned}-E\left[u_i\right] &= d_{ii}\left(\left(b_{ii}-1\right)^2\sigma_i + \sum_{k\neq i} b_{ik}^2\left(\sigma_k+\rho_{ik}+\pi_{ik}\right)\right) \\ &\quad + \sum_{j\neq i} d_{ij}\left(\sum_k \left(b_{ik}-b_{jk}\right)^2\sigma_k + \sum_{k\neq i} b_{ik}^2\left(\rho_{ik}+\pi_{ik}\right) + \sum_{k\neq j} b_{jk}^2\left(\rho_{jk}+\pi_{jk}\right)\right) \\ &\quad + k_r^2\sum_{j\neq i}\frac{1}{\rho_{ji}} + k_p^2\sum_{j\neq i}\frac{1}{\pi_{ij}}.\end{aligned} \tag{7}$$

The necessary and sufficient first-order conditions are:

$$\begin{aligned}-\frac{1}{2}\frac{\partial E\left[u_i\right]}{\partial b_{ii}} &= d_{ii}\left(b_{ii}-1\right)\sigma_i + \sum_{k\neq i} d_{ij}\left(b_{ii}-b_{ji}\right)\sigma_i = 0 \\ -\frac{1}{2}\frac{\partial E\left[u_i\right]}{\partial b_{ij}} &= d_{ii}b_{ij}\left(\sigma_j+\rho_{ij}+\pi_{ij}\right) + \sum_{k\neq i} d_{ik}\left(\left(b_{ij}-b_{kj}\right)\sigma_j + b_{ij}\rho_{ij} + b_{ij}\pi_{ij}\right) = 0 \\ -\frac{\partial E\left[u_i\right]}{\partial \rho_{ji}} &= d_{ij}b_{ji}^2 + k_r^2\frac{1}{\rho_{ji}} = 0 \\ -\frac{\partial E\left[u_i\right]}{\partial \pi_{ij}} &= D_i b_{ij}^2 + k_p^2\frac{1}{\pi_{ij}} = 0.\end{aligned}$$

Move now to the other version of the one-shot game. The payoff for agent i in $\tilde{\Gamma}_\theta\left(\boldsymbol{D},\boldsymbol{k},\boldsymbol{s}\right)$ is given by

$$u_i = -\left(d_{ii}\left(a_i-\theta_i\right)^2 + \sum_{j\neq i} d_{ij}\left(a_i-a_j\right)^2 + \sum_{j\neq i} c_r\left(\frac{1}{\rho_{ji}}\right) + \sum_{j\neq i} c_p\left(\frac{1}{\pi_{ij}}\right)\right), \tag{8}$$

or

$$
\begin{aligned}
-u_i &= d_{ii}\left((b_{ii}-1)\,\theta_i+\sum_{k\neq i} b_{ik}\left(\theta_k+\varepsilon_{ik}+\eta_{ik}\right)\right)^2 \\
&\quad+\sum_{j\neq i} d_{ij}\left(\sum_{k}(b_{ik}-b_{jk})\,\theta_k+\sum_{k\neq i} b_{ik}\varepsilon_{ik}+\sum_{k\neq i} b_{ik}\eta_{ik}-\sum_{k\neq j} b_{jk}\varepsilon_{jk}-\sum_{k\neq j} b_{jk}\eta_{jk}\right)^2 \\
&\quad+\sum_{j\neq i} c_r\left(\frac{1}{\rho_{ji}}\right)+\sum_{j\neq i} c_p\left(\frac{1}{\pi_{ij}}\right)
\end{aligned}
$$

which can be re-written as

$$
\begin{aligned}
-u_i &= d_{ii}\left(((b_{ii}-1)\,\theta_i)^2+2\left((b_{ii}-1)\,\theta_i\right)\left(\sum_{k\neq i} b_{ik}\left(\theta_k+\varepsilon_{ik}+\eta_{ik}\right)\right)+\left(\sum_{k\neq i} b_{ik}\left(\theta_k+\varepsilon_{ik}+\eta_{ik}\right)\right)^2\right) \\
&\quad+\sum_{j\neq i} d_{ij}\left((b_{ii}-b_{ji})\,\theta_i+\sum_{k\neq i}(b_{ik}-b_{jk})\,\theta_k+\sum_{k\neq i} b_{ik}\varepsilon_{ik}+\sum_{k\neq i} b_{ik}\eta_{ik}-\sum_{k\neq j} b_{jk}\varepsilon_{jk}-\sum_{k\neq j} b_{jk}\eta_{jk}\right)^2 \\
&\quad+\sum_{j\neq i} c_r\left(\frac{1}{\rho_{ji}}\right)+\sum_{j\neq i} c_p\left(\frac{1}{\pi_{ij}}\right)
\end{aligned}
$$

The expected payoff for agent i is

$$
\begin{aligned}
-E[u_i] &= d_{ii}\left((b_{ii}-1)^2\,\theta_i^2+\sum_{k\neq i} b_{ik}^2\left(\sigma_k+\rho_{ik}+\pi_{ik}\right)\right) \\
&\quad+\sum_{j\neq i} d_{ij}\left((b_{ii}-b_{ji})^2\,\theta_i^2+\sum_{k\neq i}(b_{ik}-b_{jk})^2\,\sigma_k+\sum_{k\neq i} b_{ik}^2\left(\rho_{ik}+\pi_{ik}\right)+\sum_{k\neq j} b_{jk}^2\left(\rho_{jk}+\pi_{jk}\right)\right) \\
&\quad+\sum_{j\neq i} c_r\left(\frac{1}{\rho_{ji}}\right)+\sum_{j\neq i} c_p\left(\frac{1}{\pi_{ij}}\right)
\end{aligned}
$$

First-order conditions are

$$
\begin{aligned}
-\frac{1}{2}\frac{\partial E\left[u_i\right]}{\partial b_{ii}} &= d_{ii}\left(b_{ii}-1\right)\theta_i^2+\sum_{k\neq i} d_{ij}\left(b_{ii}-b_{ji}\right)\theta_i^2=0 \\
-\frac{1}{2}\frac{\partial E\left[u_i\right]}{\partial b_{ij}} &= d_{ii}b_{ij}\left(\sigma_j+\rho_{ij}+\pi_{ij}\right)+\sum_{k\neq i} d_{ik}\left(\left(b_{ij}-b_{kj}\right)\sigma_j+b_{ij}\rho_{ij}+b_{ij}\pi_{ij}\right)=0 \\
-\frac{\partial E\left[u_i\right]}{\partial \rho_{ji}} &= d_{ij}b_{ji}^2+c_r'\left(\frac{1}{\rho_{ji}}\right)=0 \\
-\frac{\partial E\left[u_i\right]}{\partial \pi_{ij}} &= D_i b_{ij}^2+c_p'\left(\frac{1}{\pi_{ij}}\right)=0.
\end{aligned}
$$

Observe that except for the first of these conditions, the rest of conditions coincide with those for $\tilde{\Gamma}(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$. But, indeed, the first condition is also equivalent to the first one for $\tilde{\Gamma}(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$, since

$$\begin{aligned} d_{ii}(b_{ii}-1)\theta_i^2 + \sum_{k\neq i} d_{ij}(b_{ii}-b_{ji})\theta_i^2 &= 0 \iff d_{ii}(b_{ii}-1) + \sum_{k\neq i} d_{ij}(b_{ii}-b_{ji}) = 0 \\ &\iff d_{ii}(b_{ii}-1)\sigma_i^2 + \sum_{k\neq i} d_{ij}(b_{ii}-b_{ji})\sigma_i^2 = 0 \end{aligned}$$

Hence the first-order conditions for $\tilde{\Gamma}_\theta(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ are equivalent to the ones for $\tilde{\Gamma}(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$.

The statement of the lemma is obtained by applying the notation of total sensitivities.■

Proof of Lemma 4:

For the first part, begin by considering the second stage of $\Gamma(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$. Consider player i. The other players have chosen $(\hat{\boldsymbol{r}}_{-i}, \hat{\boldsymbol{p}}_{-i})$ in the previous stage and are going to play linear strategies denoted with $\hat{\boldsymbol{b}}_{-i}$.

As player i may have deviated from equilibrium play in stage 1, denote his precision vectors with $(\boldsymbol{r}_i, \boldsymbol{p}_i)$. He now must choose his decision function $a_i(\theta_i, \boldsymbol{y}_i)$.

In the perfect Bayesian equilibrium, player i forms beliefs on $(\boldsymbol{r}_{-i}, \boldsymbol{p}_{-i})$. Given that the signal $\boldsymbol{y}_i$ he receives have full support under all possible $(\boldsymbol{r}_{-i}, \boldsymbol{p}_{-i})$, there are no out-of-equilibrium beliefs. In the putative equilibrium, player i's beliefs are constant and equal to $(\hat{\boldsymbol{r}}_{-i}, \hat{\boldsymbol{p}}_{-i})$.

Given i's beliefs, his expected payoff function can be represented in a familiar quadratic form (see the proof of Lemma 3). The best response is unique, linear and the coefficients must satisfy:

$$\begin{aligned} D_i b_{ii} &= d_{ii} + \sum_{j\neq i} d_{ij}\hat{b}_{ji} \\ D_i b_{ij} &= \frac{\hat{r}_{ij}p_{ij}}{s_j\hat{r}_{ij} + s_j p_{ij} + \hat{r}_{ij}p_{ij}} \sum_{k\neq i} d_{ik}\hat{b}_{kj} \qquad \text{for all } j \neq i \end{aligned}$$

In the first stage of the game, player i chooses $(\boldsymbol{r}_i, \boldsymbol{p}_i)$ knowing what his best response will be in the second stage. His decision does not affect the beliefs of other players which are constant and equal to $(\hat{\boldsymbol{r}}_i, \hat{\boldsymbol{p}}_i)$. Hence, it is easy to see that the optimal decision satisfies the two remaining first-order conditions of Lemma 3, namely (4) and (5).

For the second part of the lemma, suppose that there exists a linear perfect Bayesian equilibrium of $\Gamma(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ that does not satisfy the first-order conditions of Lemma 3. From the first part ofthis proof, it is easy to see that the first-order conditions for this equilibrium would be the same as in Lemma 3, and hence it would correspond to an equilibrium of $\tilde{\Gamma}(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$. ■

Proof of Lemma 5:

The first part is similar to the first part of Lemma 4 and can be omitted. The second part is instead somewhat more complex.

In the game where the local state is observed before making communication investment decisions, a first-stage strategy profile of the form $(\hat{\boldsymbol{r}}, \hat{\boldsymbol{p}})$ is highly restrictive because players do not condition their investments on available information. In the case of the one-shot game $\tilde{\Gamma}_\theta(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$, we already know that all equilibria are of that form, but this point needs to be checked for the two-stage game $\Gamma_\theta(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$.

Suppose that $\Gamma_\theta(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ has a linear perfect Bayesian equilibrium where, for at least one player i at least one communication precision depends on the θ_i he observes. Such a strategy profile would not be an equilibrium of $\tilde{\Gamma}_\theta(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$.

However, note that by assumption the decision function of all players can be expressed as a linear function with certain coefficients $\tilde{\boldsymbol{b}}$, which may not be a solution of the system formed by (2) and (3). Still, the expected payoff of player i can be written as a quadratic function of the variances

$$\begin{aligned}
-E[u_i] \quad = \quad & d_{ii}\left(\left(\tilde{b}_{ii}-1\right)^2\theta_i^2+\sum_{k\neq i}\tilde{b}_{ik}^2\left(\sigma_k+\rho_{ik}+\pi_{ik}\right)\right) \\
& +\sum_{j\neq i} d_{ij}\left(\sum_{k}\left(\tilde{b}_{ik}-\tilde{b}_{jk}\right)^2\theta_i^2+\sum_{k\neq i}^{2}\tilde{b}_{ik}^2\left(\rho_{ik}+\pi_{ik}\right)+\sum_{k\neq j}\tilde{b}_{jk}^2\left(\rho_{jk}+\pi_{jk}\right)\right) \\
& +k_r^2\sum_{j\neq i}\frac{1}{\rho_{ji}}+k_p^2\sum_{j\neq i}\frac{1}{\pi_{ij}}.
\end{aligned}$$

But the problem of choosing variances to maximize $E[u_i]$ has clearly a unique (strict) solution for any possible matrix $\tilde{\boldsymbol{b}}$ and this solution does not depend on θ_i, because the payoff function is additively separable in θ_i and all the variance terms. Hence, player i must choose the same vectors $(\boldsymbol{r}_i, \boldsymbol{p}_i)$ for all realizations of θ_i, which means that there exists no linear equilibrium where communication investments depend on local states. ■

Proof of Theorem 6:

The equivalent condition for (3) for the pair ji is

$$D_j b_{ji} = \frac{p_{ji}}{p_{ji}+q_i}\sum_{k\neq j} d_{jk} b_{ki}$$

Plugging the expression (4) in this last equation, we get

$$D_j b_{ji} = \frac{r_{ji}p_{ji}}{s_i r_{ji}+s_i p_{ji}+r_{ji}p_{ji}}\sum_{k\neq j} d_{jk} b_{ki}$$

$$b_{ji}-\sum_{k\neq j}\frac{d_{jk}}{D_j}b_{ki} = -s_i\left(\frac{k_p}{\sqrt{D_j}}+\frac{k_r}{\sqrt{d_{ij}}}\right)$$

that with (2) form the following system of equations

$$\begin{aligned} b_{ii} - \sum_{j \neq i} \frac{d_{ij}}{D_i} b_{ji} &= \frac{d_{ii}}{D_i} \\ b_{ji} - \sum_{k \neq j} \frac{d_{jk}}{D_j} b_{ki} &= -s_i \left(\frac{k_p}{\sqrt{D_j}} + \frac{k_r}{\sqrt{d_{ij}}} \right) \qquad \text{if } i \neq j \end{aligned}$$

and

$$\begin{aligned} r_{ji} &= \frac{\sqrt{d_{ij}} b_{ji}}{k_r} \\ p_{ij} &= \frac{\sqrt{D_i} b_{ij}}{k_p} \end{aligned}$$

■

7.0.1 Proof of Corollary 7:

When k_r and k_p are equal to 0,

$$\sum_j b_{ij} = \sum_j (\boldsymbol{I} - \boldsymbol{\Omega})^{-1}_{ij} \, \omega_{jj}.$$

We want to check that in this case

$$(\boldsymbol{I} - \boldsymbol{\Omega})^{-1} \cdot \begin{pmatrix} \omega_{11} \\ \vdots \\ \omega_{nn} \end{pmatrix} = \mathbf{1}.$$

But this is trivial, since

$$\begin{pmatrix} \omega_{11} \\ \vdots \\ \omega_{nn} \end{pmatrix} = (\boldsymbol{I} - \boldsymbol{\Omega}) \cdot \mathbf{1}.$$

by definition.

When k_r and/or k_p are greater than 0, all vectors $\boldsymbol{h}_{\cdot j}$ have strictly negative entries except for $\boldsymbol{h}_{jj}$. This immediately implies that $\sum_j b_{ij} < 1$. ■

Proof of Proposition 8:

Let

$$W = G + (n_f - 1) F + (n_c - 1) C + ((n_f - 1)(n_c - 1) - 1) L.$$

Then,

$$I-\Omega = \frac{1}{W}\begin{bmatrix} \begin{bmatrix} W & -F & \cdots & -F \\ -F & W & \cdots & -F \\ \vdots & \vdots & \ddots & \vdots \\ -F & -F & \cdots & W \end{bmatrix} & \begin{bmatrix} -C & -L & \cdots & -L \\ -L & -C & \cdots & -L \\ \vdots & \vdots & \ddots & \vdots \\ -L & -L & \cdots & -C \end{bmatrix} & \cdots & \begin{bmatrix} -C & -L & \cdots & -L \\ -L & -C & \cdots & -L \\ \vdots & \vdots & \ddots & \vdots \\ -L & -L & \cdots & -C \end{bmatrix} \\ \begin{bmatrix} -C & -L & \cdots & -L \\ -L & -C & \cdots & -L \\ \vdots & \vdots & \ddots & \vdots \\ -L & -L & \cdots & -C \end{bmatrix} & \begin{bmatrix} W & -F & \cdots & -F \\ -F & W & \cdots & -F \\ \vdots & \vdots & \ddots & \vdots \\ -F & -F & \cdots & W \end{bmatrix} & \cdots & \begin{bmatrix} -C & -L & \cdots & -L \\ -L & -C & \cdots & -L \\ \vdots & \vdots & \ddots & \vdots \\ -L & -L & \cdots & -C \end{bmatrix} \\ \vdots & \vdots & \ddots & \vdots \\ \begin{bmatrix} -C & -L & \cdots & -L \\ -L & -C & \cdots & -L \\ \vdots & \vdots & \ddots & \vdots \\ -L & -L & \cdots & -C \end{bmatrix} & \begin{bmatrix} -C & -L & \cdots & -L \\ -L & -C & \cdots & -L \\ \vdots & \vdots & \ddots & \vdots \\ -L & -L & \cdots & -C \end{bmatrix} & \cdots & \begin{bmatrix} W & -F & \cdots & -F \\ -F & W & \cdots & -F \\ \vdots & \vdots & \ddots & \vdots \\ -F & -F & \cdots & W \end{bmatrix} \end{bmatrix}$$

and it is easy to check that the inverse must take the following form

$$(I-\Omega)^{-1} = \begin{bmatrix} \begin{bmatrix} H_G & H_F & \cdots & H_F \\ H_F & H_G & \cdots & H_F \\ \vdots & \vdots & \ddots & \vdots \\ H_F & H_F & \cdots & H_G \end{bmatrix} & \begin{bmatrix} H_C & H_L & \cdots & H_L \\ H_L & H_C & \cdots & H_L \\ \vdots & \vdots & \ddots & \vdots \\ H_L & H_L & \cdots & H_C \end{bmatrix} & \cdots & \begin{bmatrix} H_C & H_L & \cdots & H_L \\ H_L & H_C & \cdots & H_L \\ \vdots & \vdots & \ddots & \vdots \\ H_L & H_L & \cdots & H_C \end{bmatrix} \\ \begin{bmatrix} H_C & H_L & \cdots & H_L \\ H_L & H_C & \cdots & H_L \\ \vdots & \vdots & \ddots & \vdots \\ H_L & H_L & \cdots & H_C \end{bmatrix} & \begin{bmatrix} H_G & H_F & \cdots & H_F \\ H_F & H_G & \cdots & H_F \\ \vdots & \vdots & \ddots & \vdots \\ H_F & H_F & \cdots & H_G \end{bmatrix} & \cdots & \begin{bmatrix} H_C & H_L & \cdots & H_L \\ H_L & H_C & \cdots & H_L \\ \vdots & \vdots & \ddots & \vdots \\ H_L & H_L & \cdots & H_C \end{bmatrix} \\ \vdots & \vdots & \ddots & \vdots \\ \begin{bmatrix} H_C & H_L & \cdots & H_L \\ H_L & H_C & \cdots & H_L \\ \vdots & \vdots & \ddots & \vdots \\ H_L & H_L & \cdots & H_C \end{bmatrix} & \begin{bmatrix} H_C & H_L & \cdots & H_L \\ H_L & H_C & \cdots & H_L \\ \vdots & \vdots & \ddots & \vdots \\ H_L & H_L & \cdots & H_C \end{bmatrix} & \cdots & \begin{bmatrix} H_G & H_F & \cdots & H_F \\ H_F & H_G & \cdots & H_F \\ \vdots & \vdots & \ddots & \vdots \\ H_F & H_F & \cdots & H_G \end{bmatrix} \end{bmatrix}$$

Then

$$h_{ji} = \begin{bmatrix} \frac{G}{W} \\ \begin{bmatrix} -s\left(\frac{k_p}{\sqrt{W}} + \frac{k_r}{\sqrt{F}}\right) \\ -s\left(\frac{k_p}{\sqrt{W}} + \frac{k_r}{\sqrt{F}}\right) \\ \vdots \\ -s\left(\frac{k_p}{\sqrt{W}} + \frac{k_r}{\sqrt{F}}\right) \end{bmatrix} \\ \begin{bmatrix} -s\left(\frac{k_p}{\sqrt{W}} + \frac{k_r}{\sqrt{C}}\right) \\ -s\left(\frac{k_p}{\sqrt{W}} + \frac{k_r}{\sqrt{L}}\right) \\ \vdots \\ -s\left(\frac{k_p}{\sqrt{W}} + \frac{k_r}{\sqrt{L}}\right) \end{bmatrix} \\ \vdots \\ \begin{bmatrix} -s\left(\frac{k_p}{\sqrt{W}} + \frac{k_r}{\sqrt{C}}\right) \\ -s\left(\frac{k_p}{\sqrt{W}} + \frac{k_r}{\sqrt{L}}\right) \\ \vdots \\ -s\left(\frac{k_p}{\sqrt{W}} + \frac{k_r}{\sqrt{L}}\right) \end{bmatrix} \end{bmatrix} \equiv \begin{bmatrix} h_G \\ \begin{bmatrix} h_F \\ h_F \\ \vdots \\ h_F \end{bmatrix} \\ \begin{bmatrix} h_C \\ h_L \\ \vdots \\ h_L \end{bmatrix} \\ \vdots \\ \begin{bmatrix} h_C \\ h_L \\ \vdots \\ h_L \end{bmatrix} \end{bmatrix}$$

Then, for $i = 1$,

$$b_{\cdot 1} = \frac{G}{W} \begin{bmatrix} H_G h_G + (n_F - 1) H_F h_F + (n_C - 1) H_C h_C + (n_F - 1)(n_C - 1) H_L h_L \\ \begin{bmatrix} H_F h_G + H_G h_F + (n_F - 2) H_F h_F + (n_C - 1) H_L h_C + (n_C - 1) H_C h_L + (n_F - 2)(n_C - 1) H_L h_L \\ H_F h_G + H_G h_F + (n_F - 2) H_F h_F + (n_C - 1) H_L h_C + (n_C - 1) H_C h_L + (n_F - 2)(n_C - 1) H_L h_L \\ \vdots \\ H_F h_G + H_G h_F + (n_F - 2) H_F h_F + (n_C - 1) H_L h_C + (n_C - 1) H_C h_L + (n_F - 2)(n_C - 1) H_L h_L \end{bmatrix} \\ \begin{bmatrix} H_F h_G + H_G h_C + (n_C - 2) H_C h_C + (n_F - 1) H_L h_F + (n_F - 1) H_F h_L + (n_C - 2)(n_F - 1) H_L h_L \\ H_L h_G + (n_F - 2) H_L h_F + H_C h_F + (n_C - 2) H_L h_C + (n_C - 2) H_C h_L + H_F h_C + (n_F - 2) H_F h_L + H_G h_L + (n_C - 2)(n_F - 2) H_L h_L \\ \vdots \\ H_L h_G + (n_F - 2) H_L h_F + H_C h_F + (n_C - 2) H_L h_C + (n_C - 2) H_C h_L + H_F h_C + (n_F - 2) H_F h_L + H_G h_L + (n_C - 2)(n_F - 2) H_L h_L \end{bmatrix} \\ \vdots \\ \begin{bmatrix} H_F h_G + H_G h_C + (n_C - 2) H_C h_C + (n_F - 1) H_L h_F + (n_F - 1) H_F h_L + (n_C - 2)(n_F - 1) H_L h_L \\ H_L h_G + (n_F - 2) H_L h_F + H_C h_F + (n_C - 2) H_L h_C + (n_C - 2) H_C h_L + H_F h_C + (n_F - 2) H_F h_L + H_G h_L + (n_C - 2)(n_F - 2) H_L h_L \\ \vdots \\ H_L h_G + (n_F - 2) H_L h_F + H_C h_F + (n_C - 2) H_L h_C + (n_C - 2) H_C h_L + H_F h_C + (n_F - 2) H_F h_L + H_G h_L + (n_C - 2)(n_F - 2) H_L h_L \end{bmatrix} \end{bmatrix}$$

In general

$$b_{ij} = \begin{cases} H_G h_G + (n_F - 1) H_F h_F + (n_C - 1) H_C h_C + (n_F - 1)(n_C - 1) H_L h_L & \text{if } f_i = f_j \text{ and } c_i = c_j \\ H_F h_G + H_G h_F + (n_F - 2) H_F h_F + (n_C - 1) H_L h_C + (n_C - 1) H_C h_L + (n_F - 2)(n_C - 1) H_L h_L & \text{if } f_i = f_j \text{ and } c_i \neq c_j \\ H_F h_G + H_G h_C + (n_C - 2) H_C h_C + (n_F - 1) H_L h_F + (n_F - 1) H_F h_L + (n_C - 2)(n_F - 1) H_L h_L & \text{if } f_i \neq f_j \text{ and } c_i = c_j \\ H_L h_G + (n_F - 2) H_L h_F + H_C h_F + (n_C - 2) H_L h_C + (n_C - 2) H_C h_L + H_F h_C & \text{otherwise} \\ \quad + (n_F - 2) H_F h_L + H_G h_L + (n_C - 2)(n_F - 2) H_L h_L & \end{cases}$$

which can be re-written as

$$b_{ij} = \begin{cases} H_G h_G + (n_F - 1) H_F h_F + (n_C - 1) H_C h_C + \Delta & \text{if } f_i = f_j \text{ and } c_i = c_j \\ H_F h_G + (H_G + (n_F - 2) H_F) h_F + (n_C - 1) H_L h_C + (n_C - 1)(H_C - H_L) h_L + \Delta & \text{if } f_i = f_j \text{ and } c_i \neq c_j \\ H_F h_G + (H_G + (n_C - 2) H_C) h_C + (n_F - 1) H_L h_F + (n_F - 1)(H_F - H_L) h_L + \Delta & \text{if } f_i \neq f_j \text{ and } c_i = c_j \\ H_L h_G + ((n_F - 2) H_L + H_C) h_F + ((n_C - 2) H_L + H_F) h_C & \text{otherwise} \\ \quad + ((n_C - 2)(H_C - H_L) + (n_F - 2)(H_F - H_L) + (H_G - H_L)) h_L + \Delta & \end{cases}$$

■

Proof of Proposition 9:

The interaction matrix is now

$$\boldsymbol{D} = \begin{bmatrix} W_1 & \delta W_1 & \cdots & \delta W_1 \\ \delta W_2 & W_2 & \cdots & \delta W_2 \\ \vdots & \vdots & \ddots & \vdots \\ \delta W_n & \delta W_n & \cdots & W_n \end{bmatrix}$$

and

$$\boldsymbol{\Omega} = \begin{bmatrix} 0 & \frac{\delta}{1+(n-1)\delta} & \cdots & \frac{\delta}{1+(n-1)\delta} \\ \frac{\delta}{1+(n-1)\delta} & 0 & \cdots & \frac{\delta}{1+(n-1)\delta} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\delta}{1+(n-1)\delta} & \frac{\delta}{1+(n-1)\delta} & \cdots & 0 \end{bmatrix}$$

$$(\boldsymbol{I}-\boldsymbol{\Omega})^{-1} = \begin{bmatrix} 1 & -\frac{\delta}{1+(n-1)\delta} & \cdots & -\frac{\delta}{1+(n-1)\delta} \\ -\frac{\delta}{1+(n-1)\delta} & 1 & \cdots & -\frac{\delta}{1+(n-1)\delta} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{\delta}{1+(n-1)\delta} & -\frac{\delta}{1+(n-1)\delta} & \cdots & 1 \end{bmatrix}^{-1} = \frac{1+(n-1)\,\delta}{1+n\delta} \begin{bmatrix} 1+\delta & \delta & \cdots & \delta \\ \delta & 1+\delta & \cdots & \delta \\ \vdots & \vdots & \ddots & \vdots \\ \delta & \delta & \cdots & 1+\delta \end{bmatrix}$$

and

$$h_{ij} = \begin{cases} \frac{1}{1+(n-1)\delta} & \text{if } i=j \\ -s_j\left(\frac{k_p}{\sqrt{(1+(n-1)\delta)W_j}} + \frac{k_r}{\sqrt{\delta W_i}}\right) & \text{otherwise} \end{cases}$$

Then, for $j=1$,

$$\begin{aligned}
\boldsymbol{b}_{\cdot 1} &= \frac{1+(n-1)\,\delta}{1+n\delta} \begin{bmatrix} 1+\delta & \delta & \cdots & \delta \\ \delta & 1+\delta & \cdots & \delta \\ \vdots & \vdots & \ddots & \vdots \\ \delta & \delta & \cdots & 1+\delta \end{bmatrix} \begin{bmatrix} \frac{1}{1+(n-1)\delta} \\ -s_1\left(\frac{k_p}{\sqrt{(1+(n-1)\delta)W_1}} + \frac{k_r}{\sqrt{\delta W_2}}\right) \\ \vdots \\ -s_1\left(\frac{k_p}{\sqrt{(1+(n-1)\delta)W_1}} + \frac{k_r}{\sqrt{\delta W_n}}\right) \end{bmatrix} \\
&= \frac{1+(n-1)\,\delta}{1+n\delta} \begin{bmatrix} \frac{1+\delta}{1+(n-1)\delta} - s_1\left(\frac{k_p(n-1)\delta}{\sqrt{(1+(n-1)\delta)W_1}} + \sum_{i\neq 1}\frac{k_r\delta}{\sqrt{\delta W_i}}\right) \\ \frac{\delta}{1+(n-1)\delta} - s_1\left(\frac{k_p(1+(n-1)\delta)}{\sqrt{(1+(n-1)\delta)W_1}} + \frac{k_r(1+\delta)}{\sqrt{\delta W_2}} + \sum_{i\neq\{1,2\}}\frac{k_r\delta}{\sqrt{\delta W_i}}\right) \\ \vdots \\ \frac{\delta}{1+(n-1)\delta} - s_1\left(\frac{k_p(1+(n-1)\delta)}{\sqrt{(1+(n-1)\delta)W_n}} + \frac{k_r(1+\delta)}{\sqrt{\delta W_n}} + \sum_{i\neq\{1,n\}}\frac{k_r\delta}{\sqrt{\delta W_i}}\right) \end{bmatrix} \\
&= \begin{bmatrix} \frac{1+\delta}{1+n\delta} - s_1\frac{1+(n-1)\delta}{1+n\delta}\left(\frac{k_p(n-1)\delta}{\sqrt{(1+(n-1)\delta)W_1}} + \sum_{i\neq 1}\frac{k_r\delta}{\sqrt{\delta W_i}}\right) \\ \frac{\delta}{1+n\delta} - s_1\frac{1+(n-1)\delta}{1+n\delta}\left(\frac{k_p(1+(n-1)\delta)}{\sqrt{(1+(n-1)\delta)W_1}} + \frac{k_r(1+\delta)}{\sqrt{\delta W_2}} + \sum_{i\neq\{1,2\}}\frac{k_r\delta}{\sqrt{\delta W_i}}\right) \\ \vdots \\ \frac{\delta}{1+n\delta} - s_1\frac{1+(n-1)\delta}{1+n\delta}\left(\frac{k_p(1+(n-1)\delta)}{\sqrt{(1+(n-1)\delta)W_n}} + \frac{k_r(1+\delta)}{\sqrt{\delta W_n}} + \sum_{i\neq\{1,n\}}\frac{k_r\delta}{\sqrt{\delta W_i}}\right) \end{bmatrix}
\end{aligned}$$

In general

$$b_{ij}=\begin{cases}\frac{1+\delta}{1+n\delta}-s_i\left(\frac{k_p(n-1)\delta}{\sqrt{(1+(n-1)\delta)W_i}}+\sum_{k\neq i}\frac{k_r\delta}{\sqrt{\delta W_k}}\right) & \text{if } i=j\\ \frac{\delta}{1+n\delta}-s_j\left(\frac{k_p(1+(n-1)\delta)}{\sqrt{(1+(n-1)\delta)W_i}}+\frac{k_r(1+\delta)}{\sqrt{\delta W_j}}+\sum_{k\neq\{i,j\}}\frac{k_r\delta}{\sqrt{\delta W_k}}\right) & \text{otherwise}\end{cases}$$

Active communication is

$$r_{ij}=\frac{\sqrt{\delta W_j}b_{ij}}{k_r}\qquad\text{for all } i\neq j;$$

Passive communication is

$$p_{ij}=\frac{\sqrt{W_i}b_{ij}}{k_p}\qquad\text{for all } i\neq j$$

■

Proof of Proposition 11:

(i) The multiplication of all entries in row i by $\lambda_i>1$ does not affect matrix $\mathbf{\Omega}$, since

$$\omega'_{ij}=\frac{d'_{ij}}{d'_{i1}+\cdots d'_{in}}=\frac{\lambda d_{ij}}{\lambda\left(d_{i1}+\cdots d_{in}\right)}=\frac{d_{ij}}{d_{i1}+\cdots d_{in}}=\omega_{ij}$$

Also, ω_{ii} is unchanged. The effect of this transformation concentrates on the elements h_{ij} and h_{ji} for all $j\neq i$. In particular, if $j\neq i$, we have that

$$\begin{aligned} h'_{ij} &= -s_j\left(\frac{k_p}{\sqrt{D'_i}}+\frac{k_r}{\sqrt{d'_{ji}}}\right)=-s_j\left(\frac{k_p}{\sqrt{\lambda_i D_i}}+\frac{k_r}{\sqrt{d_{ji}}}\right)>h_{ij}\\ h'_{ji} &= -s_i\left(\frac{k_p}{\sqrt{D'_j}}+\frac{k_r}{\sqrt{d'_{ij}}}\right)=-s_i\left(\frac{k_p}{\sqrt{D_j}}+\frac{k_r}{\sqrt{\lambda_i d_{ij}}}\right)>h_{ji}\end{aligned}$$

because $\lambda_i>1$. Therefore, condition (i) of Theorem 6 implies that this change in $\boldsymbol{D}$ causes an increase in all b_{ji}'s and all b_{ij}'s, for all $\cancel{j}$. By conditions (ii) and (iii), communication intensities r_{ji},r_{ij},p_{ji} and p_{ij} increase as well, for all $j\neq i$.

(ii) For communication costs, note that all the elements in the matrix $(\boldsymbol{I}-\mathbf{\Omega})^{-1}$ are positive (because it can be represented as a series of of matrix with non-negative elements: $\boldsymbol{I}+\mathbf{\Omega}+\mathbf{\Omega}^2+...$). Then, condition (i) of Theorem 6 implies that an increase in either k_p or k_r causes a decrease in all the decision coefficient b's (including b_{ii}). By conditions (ii) and (iii), communication intensities decrease as well.

(iii) A decrease in s_i only affects the vector $\boldsymbol{h}_{\cdot i}$, in which all negative entries increase and the unique non-negative entry, ω_{ii}, is unchanged. Condition (i) of Theorem 6 implies that this decrease in s_i causes an increase in all b_{ji}'s, for all j. By conditions (ii) and (iii), communication intensities r_{ji} and p_{ji} increase as well, for all $j\neq i$. ■

Proof of Proposition 12:

Let $\boldsymbol{M} = (\boldsymbol{I} - \boldsymbol{\Omega})^{-1}$, and let a denote the increase (resp. decrease) in d_{ij} (resp. d_{ik}). We have

$$\frac{d}{da}\boldsymbol{b}_{\cdot i} = \frac{d}{da}\boldsymbol{M} \cdot \boldsymbol{h}_{\cdot i} + \boldsymbol{M} \cdot \frac{d}{da}\boldsymbol{h}_{\cdot i}$$

Note that

$$\begin{aligned}
\frac{d}{da}\boldsymbol{M} &= \frac{d}{da}\left((\boldsymbol{I} - \boldsymbol{\Omega})^{-1}\right) \\
&= -\left((\boldsymbol{I} - \boldsymbol{\Omega})^{-1}\right)' \frac{d}{da}(\boldsymbol{I} - \boldsymbol{\Omega})(\boldsymbol{I} - \boldsymbol{\Omega})^{-1} \\
&= \left((\boldsymbol{I} - \boldsymbol{\Omega})^{-1}\right)' \frac{d}{da}\boldsymbol{\Omega}(\boldsymbol{I} - \boldsymbol{\Omega})^{-1}
\end{aligned}$$

We will evaluate the effect of a change of d_{ij} and d_{ik} starting form a symmetric situation, namely $d_{ij} = d$ for all i and j and $s_i = s$ for all i. Then we have

$$\boldsymbol{I} - \boldsymbol{\Omega} = \begin{bmatrix} 1 & \cdots & -\omega \\ \vdots & & \vdots \\ -\omega & \cdots & 1 \end{bmatrix}$$

with inverse

$$\boldsymbol{M} = K\begin{bmatrix} 1-(n-1)\,\omega & \cdots & \omega \\ \vdots & & \vdots \\ \omega & \cdots & 1-(n-1)\,\omega \end{bmatrix}$$

where K is a constant that depend on w. We also have

$$h_{ji} = \begin{cases} 1-(n-1)\,\omega & \text{if } i = j \\ -s\left(\frac{k_p}{\sqrt{nd}} + \frac{k_r}{\sqrt{d}}\right) & \text{otherwise} \end{cases}$$

We have

$$\frac{d}{da}\boldsymbol{M} = \begin{bmatrix} 1-(n-1)\,\omega & \cdots & \omega \\ \vdots & & \vdots \\ \omega & \cdots & 1-(n-1)\,\omega \end{bmatrix} \frac{d}{da}\boldsymbol{\Omega} \begin{bmatrix} 1-(n-1)\,\omega & \cdots & \omega \\ \vdots & & \vdots \\ \omega & \cdots & 1-(n-1)\,\omega \end{bmatrix}$$

But note that $\frac{d}{da}\boldsymbol{\Omega}$ contains zero everywhere excepts a 1 in ij and -1 in ik. Hence it is easy to see that $\frac{d}{da}\boldsymbol{M} = 0$. Thus $\frac{d}{da}\boldsymbol{b}_{\cdot i} = \boldsymbol{M} \cdot \frac{d}{da}\boldsymbol{h}_{\cdot i}$. Now note that $\frac{d}{da}\boldsymbol{h} = 0$ everywhere except for

$$\begin{aligned}
\tfrac{d}{da}h_{ji} &= -s\tfrac{d}{da}\left(\tfrac{k_p}{\sqrt{D}} + \tfrac{k_r}{\sqrt{d_{ji}}}\right) = dh > 0 \\
\tfrac{d}{da}h_{ki} &= -s\tfrac{d}{da}\left(\tfrac{k_p}{\sqrt{D}} + \tfrac{k_r}{\sqrt{d_{ki}}}\right) = -dh
\end{aligned}$$

Then note that

$$
\begin{aligned}
\frac{d}{da}\boldsymbol{b}_{\cdot i} &= \begin{bmatrix} 1-(n-1)\,\omega & \cdots & \omega \\ \vdots & & \vdots \\ \omega & \cdots & 1-(n-1)\,\omega \end{bmatrix} \cdot \frac{d}{da}\boldsymbol{h}_{\cdot i} \\
&= \begin{cases} 0 & \text{if} \\ H\,(1-nw) & \text{for } b_{ji} \\ -H\,(1-nw) & \text{for } b_{ki} \end{cases}
\end{aligned}
$$

So the only b's that change are b_{ji} (up) and b_{ki} (down). All the others are unaffected.

The only communication intensities to be affected are

$$
\begin{aligned}
r_{ji} &= \frac{\sqrt{d_{ij}}b_{ji}}{k_r} \\
r_{ki} &= \frac{\sqrt{d_{ik}}b_{ki}}{k_r} \\
p_{ji} &= \frac{\sqrt{D}b_{ji}}{k_p} \\
p_{ki} &= \frac{\sqrt{D}b_{ki}}{k_p}
\end{aligned}
$$

■

Proof of Proposition 13:

For the first part, we begin by proving that $H_F > H_C$ if and only if $F > C$. To see this, note that because $(\boldsymbol{I}-\boldsymbol{\Omega})^{-1}(\boldsymbol{I}-\boldsymbol{\Omega}) = \boldsymbol{I}$, we have that

$$
\begin{aligned}
\left((\boldsymbol{I}-\boldsymbol{\Omega})^{-1}(\boldsymbol{I}-\boldsymbol{\Omega})\right)_{f_i=f_j,c_i\neq c_j} &= \boldsymbol{0} \\
\left((\boldsymbol{I}-\boldsymbol{\Omega})^{-1}(\boldsymbol{I}-\boldsymbol{\Omega})\right)_{f_i\neq f_j,c_i=c_j} &= \boldsymbol{0}
\end{aligned}
$$

and hence

$$
\left((\boldsymbol{I}-\boldsymbol{\Omega})^{-1}(\boldsymbol{I}-\boldsymbol{\Omega})\right)_{f_i=f_j,c_i\neq c_j} - \left((\boldsymbol{I}-\boldsymbol{\Omega})^{-1}(\boldsymbol{I}-\boldsymbol{\Omega})\right)_{f_i\neq f_j,c_i=c_j} = \boldsymbol{0}
$$

If $n_C = n_F$,

$$
\begin{aligned}
& \left((\boldsymbol{I}-\boldsymbol{\Omega})^{-1}(\boldsymbol{I}-\boldsymbol{\Omega})\right)_{f_i=f_j,c_i\neq c_j} - \left((\boldsymbol{I}-\boldsymbol{\Omega})^{-1}(\boldsymbol{I}-\boldsymbol{\Omega})\right)_{f_i\neq f_j,c_i=c_j} \\
= \;& H_F\,(W-(n_F-2)\,F+(n_F-1)\,L) - H_C\,(W-(n_F-2)\,C+(n_F-1)\,L) \\
& -(F-C)\,((H_G-(n_F-1)\,H_L)) \\
= \;& 0
\end{aligned}
$$

If L is sufficiently small, then $H_G > (n_F-1)\,H_L$ and

$$
sign\,(H_F\,(W-(n_F-2)\,F+(n_F-1)\,L) - H_C\,(W-(n_F-2)\,C+(n_F-1)\,L)) = sign\,(F-C)
$$

Suppose $F > C$. Then,

$$W - (n_F - 2)\, F + (n_F - 1)\, L < W - (n_F - 2)\, C + (n_F - 1)\, L$$

and a necessary conditions for the equation above to be satisfied is that $H_F > H_C$. The opposite is true when $F < C$.

Note that, when communication costs go to zero, all h's go to zero but h_G. Hence[16]

$$b_{ij} = \begin{cases} H_F h_G & \text{if } f_i = f_j \text{ and } c_i \neq c_j \\ H_C h_G & \text{if } f_i \neq f_j \text{ and } c_i = c_j \end{cases}$$

For the second part, if $C = F$, it is easy to see that $H_C = H_F$ and $h_C = h_F$, and

$$b_{ij} = \begin{cases} H_F h_G + H_G h_F + (n_F - 2)\, h_F H_F + (n_C - 1)\, h_L H_F + (n_C - 1)\, h_F H_L + ((n_F - 1)\, (n_C - 2) - 1)\, h_L H_L \\ H_F h_G + (n_F - 1)\, h_F H_L + H_G h_F + (n_F - 1)\, h_L H_F + (n_C - 2)\, h_F H_F + ((n_F - 2)\, (n_C - 1) - 1)\, h_L H_L \end{cases}$$

The difference between b_{ij} within a function and b_{ij} within a country is

$$(n_C - n_F)\, (H_F h_L - H_F h_F + H_L h_F + H_L h_L)\,.$$

If L is sufficiently small, h_L vanishes and the sign of the second term is determined by

$$-\, (H_F - H_L)\, h_F$$

which is positive. Hence, b_{ij} between units with the same function is greater than b_{ij} between units in the same country if and only if $n_C > n_F$. ■

Proof of Proposition 14:

From Theorem 6,

$$q_{ij} = \frac{\frac{\sqrt{d_{ji}} b_{ij}}{k_r} \frac{\sqrt{D_i} b_{ij}}{k_p}}{\frac{\sqrt{d_{ji}} b_{ij}}{k_r} + \frac{\sqrt{D_i} b_{ij}}{k_p}} = \frac{\sqrt{d_{ji} D_i}}{k_p \sqrt{d_{ji}} + k_r \sqrt{D_i}} b_{ij}$$

and analogously

$$q_{ji} = \frac{\sqrt{d_{ij} D_j}}{k_p \sqrt{d_{ij}} + k_r \sqrt{D_j}} b_{ji}$$

Note that, for any i and j,

$$\frac{b_{\cdot j}}{b_{\cdot i}} = \frac{h_{\cdot j}}{h_{\cdot i}}$$

In particular

$$\frac{b_{ij}}{b_{ii}} = \frac{h_{ij}}{h_{ii}} \qquad \frac{b_{jj}}{b_{ji}} = \frac{h_{jj}}{h_{ji}},$$

[16] As k_r and k_p do not enter Ω, we can first find an L that guarantees that $H_F > H_C$ (assuming $F > C$) and then take k_r and k_p small enough to: (i) have an interior solution; (ii) guarantee that all h's but h_G are sufficiently small.

so that

$$\begin{aligned}
\frac{b_{ij}}{b_{ji}} &= \frac{b_{ii}}{b_{jj}}\frac{h_{jj}}{h_{ii}}\frac{h_{ij}}{h_{ji}} = \frac{b_{ii}}{b_{jj}}\frac{w_{jj}}{w_{ii}}\frac{s_j\left(\frac{k_p}{\sqrt{D_i}}+\frac{k_r}{\sqrt{d_{ji}}}\right)}{s_i\left(\frac{k_p}{\sqrt{D_j}}+\frac{k_r}{\sqrt{d_{ij}}}\right)} \\
&= \frac{b_{ii}}{b_{jj}}\frac{w_{jj}}{w_{ii}}\frac{s_j\frac{k_p\sqrt{d_{ji}}+k_r\sqrt{D_i}}{\sqrt{D_i d_{ji}}}}{s_i\frac{k_p\sqrt{d_{ij}}+k_r\sqrt{D_j}}{\sqrt{D_j d_{ij}}}}
\end{aligned}$$

We have

$$\frac{q_{ij}}{q_{ji}} = \frac{\frac{\sqrt{d_{ji}D_i}}{k_p\sqrt{d_{ji}}+k_r\sqrt{D_i}}b_{ij}}{\frac{\sqrt{d_{ij}D_j}}{k_p\sqrt{d_{ij}}+k_r\sqrt{D_j}}b_{ji}} = \frac{b_{ii}}{b_{jj}}\frac{w_{jj}}{w_{ii}}\frac{s_j}{s_i}$$

If we assume $w_{jj} = w_{ii}$ and $s_j = s_i$, we have the statement of the proposition. ■

Proof of Proposition 15:

From Theorem 6, we know that, for all i,

$$\boldsymbol{b}_{\cdot i} = (\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\cdot \boldsymbol{h}_{\cdot i}.$$

We can write

$$\begin{aligned}
\begin{bmatrix} b_{1i} \\ \vdots \\ b_{ni} \end{bmatrix} &= \begin{bmatrix} \left((\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\right)_{11} & \cdots & \left((\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\right)_{1n} \\ \vdots & & \vdots \\ \left((\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\right)_{n1} & \cdots & \left((\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\right)_{nn} \end{bmatrix} \cdot \begin{bmatrix} h_{1i} \\ \vdots \\ h_{ni} \end{bmatrix} \\
&= \sum_{j=1}^{n} h_{ji} \begin{bmatrix} \left((\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\right)_{1j} \\ \vdots \\ \left((\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\right)_{nj} \end{bmatrix} = \sum_{j=1}^{n} h_{ji} \begin{bmatrix} \left((\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\right)'_{j1} \\ \vdots \\ \left((\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\right)'_{jn} \end{bmatrix}
\end{aligned}$$

so that

$$B_i = \sum_{k=1}^{n} b_{ki} = \sum_{k=1}^{n}\sum_{j=1}^{n} h_{ji}\left((I-\Omega)^{-1}\right)'_{jk} = \sum_{j=1}^{n}\left(\sum_{k=1}^{n}\left((\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\right)'_{jk}\right)h_{ji}.$$

If we define the $\boldsymbol{G}$ matrix in the Bonacich measure to be the transpose of the $\boldsymbol{\Omega}$ matrix used in Theorem 6 and we let $a = 1$, we have:

$$M(G,a) = M\left(\boldsymbol{\Omega}',1\right) = \left[\boldsymbol{I}-\boldsymbol{\Omega}'\right]^{-1} = \left(\left[\boldsymbol{I}-\boldsymbol{\Omega}\right]^{-1}\right)'$$

and hence

$$\sum_{k=1}^{n}\left((\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\right)'_{jk} = \beta_j\left(\boldsymbol{\Omega}',1\right),$$

so that

$$B_i = \sum_{j=1}^{n} \beta_j \left(\mathbf{\Omega}', 1\right) h_{ji}$$

■

Proof of Proposition 17:

Let $\boldsymbol{d}'_{i\cdot} = \lambda \boldsymbol{d}_{i\cdot}$. In this case $\mathbf{\Omega}$ does not change, and the effect is in h_{jk} as follows:

$$\frac{dh_{jk}}{d\lambda} = \begin{cases} 0 & \text{if } j \neq i \text{ and } k \neq i \\ s_i \frac{k_p}{2\sqrt{D_i}} & \text{if } j = i \text{ and } k \neq i \\ s_i \frac{k_r}{2\sqrt{d_{ij}}} & \text{if } j \neq i \text{ and } k = i \\ 0 & \text{if } j = k = i \end{cases}$$

We have

$$\frac{dB_j}{d\lambda} = \sum_{k=1}^{n} \beta_k \left(\mathbf{\Omega}', 1\right) \frac{dh_{kj}}{d\lambda} = \begin{cases} \beta_i \left(\mathbf{\Omega}', 1\right) s_i \frac{k_p}{2\sqrt{D_i}} & \text{if } j \neq i \\ \sum_{k \neq i} \beta_k \left(\mathbf{\Omega}', 1\right) s_i \frac{k_r}{2\sqrt{d_{ik}}} & \text{if } j = i \end{cases}$$

Note that $\frac{dB_j}{d\lambda}$ is the same for all $j \neq i$ and that

$$\frac{dB_j}{d\lambda} \leq \frac{dB_i}{d\lambda}$$

If and only if

$$\beta_i \left(\mathbf{\Omega}', 1\right) s_i \frac{k_p}{2\sqrt{D_i}} \leq \sum_{k \neq i} \beta_k \left(\mathbf{\Omega}', 1\right) s_i \frac{k_r}{2\sqrt{d_{ik}}}$$

correponding to

$$\frac{k_p}{k_r} \leq \sum_{k \neq i} \frac{\beta_k \left(\mathbf{\Omega}', 1\right)}{\beta_i \left(\mathbf{\Omega}', 1\right)} \frac{\sqrt{D_i}}{\sqrt{d_{ik}}}$$

■

Proof of Proposition 18:

The precision s_i only affects the influence elements h_{ji} , for all $j \neq i$. Hence, it only affects i's global influence. When s_i increases, all negative entries in the vector $\boldsymbol{h}_{\cdot i}$ decrease while the positive entry, w_{ii}, does not change, and agent i's global influence decreases. An increase in s_i does not change $\mathbf{\Omega}$ and $\boldsymbol{h}_{\cdot j}$ for $j \neq i$. Therefore, j's global influence does not change.
■

Proof of Proposition 19:

The Bonacich centrality measure of agent i is defined as

$$\beta_j\left(\boldsymbol{\Omega}', 1\right) = \sum_{k=1}^{n}\left(\left(\boldsymbol{I}-\boldsymbol{\Omega}\right)^{-1}\right)'_{jk}$$

Note that

$$\left(\left(\boldsymbol{I}-\boldsymbol{\Omega}\right)^{-1}\right)' = \frac{1+(n-1)\,\delta}{1+n\delta}\begin{bmatrix} 1+\delta & \delta & \cdots & \delta \\ \delta & 1+\delta & \cdots & \delta \\ \vdots & \vdots & \ddots & \vdots \\ \delta & \delta & \cdots & 1+\delta \end{bmatrix}$$

Hence

$$\beta_j\left(\boldsymbol{\Omega}', 1\right) = \frac{1+(n-1)\,\delta}{1+n\delta}\left(1+n\delta\right) = 1+(n-1)\,\delta$$

The global influence of agent i is simply

$$B_i = \sum_{j=1}^{n}\beta_j\left(\boldsymbol{\Omega}', 1\right)h_{ji} = \left(1+(n-1)\,\delta\right)\sum_{j=1}^{n}h_{ji}$$

Recall that

$$h_{ji} = \begin{cases} \frac{1}{1+(n-1)\delta} & \text{if } i=j \\ -s_i\left(\frac{k_p}{\sqrt{(1+(n-1)\delta)W_j}}+\frac{k_r}{\sqrt{\delta W_i}}\right) & \text{otherwise} \end{cases}$$

Hence

$$\begin{aligned} B_i &= \left(1+(n-1)\,\delta\right)\left(\frac{1}{1+(n-1)\,\delta}-\sum_{j\neq i}s_i\left(\frac{k_p}{\sqrt{\left(1+(n-1)\,\delta\right)W_j}}+\frac{k_r}{\sqrt{\delta W_i}}\right)\right) \\ &= 1-s_ik_p\sqrt{1+(n-1)\,\delta}\sum_{j\neq i}\frac{1}{\sqrt{W_j}}-(n-1)\,s_ik_r\frac{1+(n-1)\,\delta}{\sqrt{\delta W_i}} \end{aligned}$$

■

Proof of Proposition 20:

(i) If $\lambda = 0$ then $\omega_{ii} = 0$, and this immediately implies that we hit a boundary equilibrium in which $b_{ji} = 0$ for all j. This implies that agent i is not going to put effort in actively communicating with agent j, and that agent j is not going to exert any kind of effort in passive communication to learn about agent i's state of the world.

(ii) If $\lambda \to 2$ the matrix $\boldsymbol{\Omega}$ tends to $\boldsymbol{\Omega}'$, where $\boldsymbol{\Omega}'$ is equal to $\boldsymbol{\Omega}$ except that row i's entries in $\boldsymbol{\Omega}'$ are equal to 0. Also

$$h'_{ji} = \begin{cases} w'_{ii} \to 1 & \text{if } i=j \\ -s_i\left(\frac{k_p}{\sqrt{D_j}}+\frac{k_r}{\sqrt{d'_{ij}}}\right) \to -\infty & \text{otherwise} \end{cases}$$

It is easy to see that the non-negative matrix $(\boldsymbol{I}-\boldsymbol{\Omega}')^{-1}$ satisfies that all entries in row i are also equal to 0, except for $(\boldsymbol{I}-\boldsymbol{\Omega}')^{-1}_{ii}=1$. Hence, following our equilibrium characterization, the elements b'_{ji} in equilibrium actions would satisfy that, when $\lambda\to 2$, $b'_{ii}\to 1$ and $b'_{ji}\to-\infty$ if $j\neq i$. But this implies that we hit an equilibrium in the boundary that satisfies $b'_{ji}=0$ for all $j\neq i$. Therefore, again there is neither passive communication by agent j nor active communication by agent i. ■

Proof of Proposition 22

The agents' actions first-order conditions are the same as above. Instead the condition on communication intensities are now

$$
\begin{aligned}
-\frac{\partial}{\partial\rho_{ji}}\sum_{k=1}^{n}E\left[u_k\right] &= \left(\sum_{k=1}^{n}d_{ik}+\sum_{k\neq i}^{n}d_{ki}\right)b_{ij}^2+c_r'\left(\frac{1}{\rho_{ji}}\right)=0\\
-\frac{\partial}{\partial\pi_{ij}}\sum_{k=1}^{n}E\left[u_k\right] &= \left(\sum_{k=1}^{n}d_{ik}+\sum_{k\neq i}^{n}d_{ki}\right)b_{ij}^2+c_p'\left(\frac{1}{\pi_{ij}}\right)=0
\end{aligned}
$$

which can be re-written as

$$
\begin{aligned}
r_{ij} &= \sqrt{\sum_{k=1}^{n}d_{ik}+\sum_{k\neq i}^{n}d_{ki}}\,\frac{b_{ij}}{k_r} \qquad \text{for all } i\neq j\\
p_{ij} &= \sqrt{\sum_{k=1}^{n}d_{ik}+\sum_{k\neq i}^{n}d_{ki}}\,\frac{b_{ij}}{k_p} \qquad \text{for all } i\neq j
\end{aligned}
$$

■

Proof of Proposition 25:

The expected utility of team T is given by

$$
\begin{aligned}
-E\left[u_T\right] &= \sum_{i\in T}d_{ii}\left((b_{ii}-1)^2\sigma_i+\sum_{k\neq i}b_{ik}^2\left(\sigma_k+\rho_{ik}+\pi_{ik}\right)\right)\\
&\quad+\sum_{i,j\in T,i\neq j}\frac{t_{ij}}{2}\left(\sum_k\left(b_{ik}-b_{jk}\right)^2\sigma_k+\sum_{k\neq i}b_{ik}^2\left(\rho_{ik}+\pi_{ik}\right)+\sum_{k\neq j}b_{jk}^2\left(\rho_{jk}+\pi_{jk}\right)\right)\\
&\quad+\sum_{i\in T,j\notin T}d_{ij}\left(\sum_k\left(b_{ik}-b_{jk}\right)^2\sigma_k+\sum_{k\neq i}b_{ik}^2\left(\rho_{ik}+\pi_{ik}\right)+\sum_{k\neq j}b_{jk}^2\left(\rho_{jk}+\pi_{jk}\right)\right)\\
&\quad+\sum_{i\in T}\sum_{j\neq i}c_r\left(\frac{1}{\rho_{ji}}\right)+\sum_{i\in T}\sum_{j\neq i}c_p\left(\frac{1}{\pi_{ij}}\right)
\end{aligned}
$$

The FOC's for optimal decisions for an agent $i \in T(i)$ are

$$
\begin{aligned}
-\frac{1}{2}\frac{\partial E\left[u_{T(i)}\right]}{\partial b_{ii}} &= d_{ii}\left(1-b_{ii}\right)\sigma_i + \sum_{j\in T(i),j\neq i} t_{ij}\left(b_{ii}-b_{ji}\right)\sigma_i + \sum_{j\notin T(i)} d_{ij}\left(b_{ii}-b_{ji}\right)\sigma_i = 0 \\
-\frac{1}{2}\frac{\partial E\left[u_{T(i)}\right]}{\partial b_{ik}} &= d_{ii}b_{ik}\left(\sigma_k+\rho_{ik}+\pi_{ik}\right) + \sum_{j\in T(i),j\neq i} t_{ij}\left[b_{ik}\left(\sigma_k+\rho_{ik}+\pi_{ik}\right)-b_{jk}\sigma_k\right] \\
&\quad + \sum_{j\notin T(i)} d_{ij}\left[b_{ik}\left(\sigma_k+\rho_{ik}+\pi_{ik}\right)-b_{jk}\sigma_k\right] = 0 \qquad \text{,if } k\neq i \\
-\frac{\partial E\left[u_{T(i)}\right]}{\partial \pi_{ik}} &= \left(d_{ii} + \sum_{j\in T(i),j\neq i} t_{ij} + \sum_{j\notin T(i)} d_{ij}\right) b_{ik}^2 + c_p'\left(\frac{1}{\pi_{ik}}\right) = 0 \\
-\frac{\partial E\left[u_{T(i)}\right]}{\partial \rho_{ki}} &= \begin{cases} \left(d_{kk}+\sum_{j\in T(i),j\neq k} t_{jk}+\sum_{j\notin T(i)} d_{kj}\right) b_{ki}^2 + c_r'\left(\frac{1}{\rho_{ki}}\right) = 0 & \text{if } k\in T(i) \\ \left(\sum_{j\in T(i)} d_{jk}\right) b_{ki}^2 + c_r'\left(\frac{1}{\rho_{ki}}\right) = 0 & \text{if } k\notin T(i) \end{cases}
\end{aligned}
$$

These conditions are rewriten as

$$
\left(d_{ii} + \sum_{j\in T(i),j\neq i} t_{ij} + \sum_{j\notin T(i)} d_{ij}\right) b_{ii} = d_{ii} + \sum_{j\in T(i),j\neq i} t_{ij}b_{ji} + \sum_{j\notin T(i)} d_{ij}b_{ji} \tag{9}
$$

$$
\left(d_{ii} + \sum_{j\in T(i),j\neq i} t_{ij} + \sum_{j\notin T(i)} d_{ij}\right) b_{ik} = \frac{r_{ik}p_{ik}}{s_k r_{ik} + s_k p_{ik} + r_{ik}p_{ik}} \left(\sum_{j\in T(i),j\neq i} t_{ij}b_{jk} + \sum_{j\notin T(i)} d_{ij}b_{jk}\right) \tag{10}
$$

$$
\frac{\sqrt{d_{ii}+\sum_{j\in T(i),j\neq i} t_{ij}+\sum_{j\notin T(i)} d_{ij}}}{k_p} b_{ik} = p_{ik} \tag{11}
$$

$$
\frac{\sqrt{d_{kk}+\sum_{j\in T(i),j\neq k} t_{jk}+\sum_{j\notin T(i)} d_{kj}}}{k_r} b_{ki} = r_{ki} \qquad \text{if } k\in T(i) \tag{12}
$$

$$
\frac{\sqrt{\sum_{j\in T(i)} d_{jk}}}{k_r} b_{ki} = r_{ki} \qquad \text{if } k\notin T(i) \tag{13}
$$

If $i \in T(k)$, plugging the expressions of (11) and (12) in (10), we obtain:

$$
\begin{aligned}
& b_{ik} - \sum_{j\in T(i),j\neq i} \frac{t_{ij}}{d_{ii}+\sum_{j\in T(i),j\neq i} t_{ij}+\sum_{j\notin T(i)} d_{ij}} b_{jk} - \sum_{j\notin T(i)} \frac{d_{ij}}{d_{ii}+\sum_{j\in T(i),j\neq i} t_{ij}+\sum_{j\notin T(i)} d_{ij}} b_{jk} \\
= \; & -\frac{s_k k_p + s_k k_r}{\sqrt{d_{ii}+\sum_{j\in T(i),j\neq i} t_{ji}+\sum_{j\notin T(i)} d_{ij}}}
\end{aligned}
$$

While, if $k \notin T(i)$, plugging the expressions of (11) and (13) in (10), we obtain:

$$b_{ik} - \sum_{j\in T(i), j\neq i} \frac{t_{ij}}{d_{ii} + \sum_{j\in T(i), j\neq i} t_{ij} + \sum_{j\notin T(i)} d_{ij}} b_{jk} - \sum_{j\notin T(i)} \frac{d_{ij}}{d_{ii} + \sum_{j\in T(i), j\neq i} t_{ij} + \sum_{j\notin T(i)} d_{ij}} b_{jk}$$

$$= \quad -\left(\frac{s_k k_p}{\sqrt{d_{ii} + \sum_{j\in T(i), j\neq i} t_{ij} + \sum_{j\notin T(i)} d_{ij}}} + \frac{s_k k_r}{\sqrt{\sum_{j\in T(k)} d_{ji}}} \right)$$

These two sets of equations can be subsumed in matricial form as

$$(\boldsymbol{I} - \boldsymbol{\Phi}) \cdot \boldsymbol{b}_{\cdot k} = \boldsymbol{z}_{\cdot k}$$

with matrix $\boldsymbol{\Phi}$ and vector $\boldsymbol{z}_{\cdot k}$ as previously defined. ■

Proof of Proposition 26

Suppose $C = F$ and $L \to 0$. In the four-agent case, the limit of Ω is

$$\frac{1}{W}\begin{bmatrix} W & -F & -F & 0 \\ -F & W & 0 & -F \\ -F & 0 & W & -F \\ 0 & -F & -F & W \end{bmatrix}$$

with inverse

$$\frac{1}{W^2 - 4F^2}\begin{bmatrix} W^2 - 2F^2 & FW & FW & 2F^2 \\ FW & W^2 - 2F^2 & 2F^2 & FW \\ FW & 2F^2 & W^2 - 2F^2 & FW \\ 2F^2 & FW & FW & W^2 - 2F^2 \end{bmatrix}$$

As communication costs tend to zero, the decision coefficients tend to

$$\frac{1}{W^2 - 4F^2}\begin{bmatrix} W^2 - 2F^2 & FW & FW & 2F^2 \\ FW & W^2 - 2F^2 & 2F^2 & FW \\ FW & 2F^2 & W^2 - 2F^2 & FW \\ 2F^2 & FW & FW & W^2 - 2F^2 \end{bmatrix}\begin{bmatrix} \frac{W-2F}{W} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1}{W}\frac{W^2-2F^2}{W^2-4F^2}(W-2F) \\ \frac{F}{W^2-4F^2}(W-2F) \\ \frac{F}{W^2-4F^2}(W-2F) \\ 2\frac{F^2}{W(W^2-4F^2)}(W-2F) \end{bmatrix}$$

If we group agents by function, we have

$$\Phi = \frac{1}{W+F}\begin{bmatrix} W+F & -2F & -F & 0 \\ -2F & W+F & 0 & -F \\ -F & 0 & W+F & -2F \\ 0 & -F & -2F & W+F \end{bmatrix}$$

with inverse

$$\left[\frac{W+F}{-16F^3W-4F^2W^2+4FW^3+W^4}\begin{matrix} -4F^3-2F^2W+3FW^2+W^3 & -4F^3+4F^2W+2FW^2 & 4 \\ -4F^3+4F^2W+2FW^2 & -4F^3-2F^2W+3FW^2+W^3 & \\ 4F^3+2F^2W+FW^2 & 4F^3+4WF^2 & -4F^3 \\ 4F^3+4WF^2 & 4F^3+2F^2W+FW^2 & -4 \end{matrix}\right.$$

As communication costs tend to zero, the decision coefficients tend to

$$\begin{bmatrix} \frac{W-2F}{-16F^3W-4F^2W^2+4FW^3+W^4}\left(-4F^3-2F^2W+3FW^2+W^3\right) \\ (W-2F)\frac{-4F^3+4F^2W+2FW^2}{-16F^3W-4F^2W^2+4FW^3+W^4} \\ (W-2F)\frac{4F^3+2F^2W+FW^2}{-16F^3W-4F^2W^2+4FW^3+W^4} \\ (W-2F)\frac{4F^3+4WF^2}{-16F^3W-4F^2W^2+4FW^3+W^4} \end{bmatrix}$$

Compare b_C with an without teams

$$2\frac{F^2}{W\left(W^2-4F^2\right)}-\frac{4F^3+2F^2W+FW^2}{-16F^3W-4F^2W^2+4FW^3+W^4}=-\frac{F}{W\left(4F+W\right)}$$

which shows that b_C is always lower when units are grouped by function.

It is also immediate to see that, as $L \to 0$, active communication within countries is given by

$$\frac{\sqrt{F}}{k_r}b_C$$

Hence, also active communication goes down when units are grouped by function.

Instead, the sign of the change in passive communication precision is ambiguous. Hence, the chang ein overall communication precision is ambiguous too. ■

Appendix B. Uniqueness

Consider the following variation of our game:

- payoffs are the same as before
- local information is bounded: $\theta_i \in \left[-\bar{\theta},\bar{\theta}\right]$, with $\bar{\theta} \in \mathbb{R}$, follows a *truncated* normal distribution with mean 0 and precision s.
- the set of possible actions is bounded. In particular, $a_i \in [-\bar{a},\bar{a}]$ for all i, where $\bar{a} = c\bar{\theta}$, for some $c \geq 1$. Note that this implies that $\left[-\bar{\theta},\bar{\theta}\right] \subseteq [-\bar{a},\bar{a}]$.
- communication reports are defined as in text and, thus, are unbounded: $y_{ij} = \theta_i + \varepsilon_{ij} + \eta_{ij}$ with

$$\begin{aligned} \varepsilon_{ij} &\sim N\left(0, r_{ij}\right) \\ \eta_{ij} &\sim N\left(0, p_{ij}\right) \end{aligned}$$

Observe that as $\bar{\theta} \to +\infty$ we converge to our initial specification of the model.

We define the following expectation operators: $E_i[\cdot] = E\left[\cdot \mid \theta_i, \{y_{ij}\}_{j \neq i}\right]$ for every $i \in \{1, \ldots, n\}$.

Lemma 27 *For any action profile $(a_1, \ldots, a_n)$ we have that $\omega_{ii}\theta_i + \sum_{j \neq i} \omega_{ij} E_i[a_j] \in [-\bar{a}, \bar{a}]$ for all i.*

Proof. Just note that $E_i[a_j] \in [-\bar{a}, \bar{a}]$ for all i, j. Since $\theta_i \in \left[-\bar{\theta}, \bar{\theta}\right] \subset [-\bar{a}, \bar{a}]$, the linear combination $\omega_{ii}\theta_i + \sum_{j \neq i} \omega_{ij} E_i[a_j]$ must be in $[-\bar{a}, \bar{a}]$. ■

Lemma 28 *The matrix $\boldsymbol{\Omega}$ with off-diagonal entries equal to ω_{ij} and diagonal entries equal to 0 is a contraction.*

Proof. Gerschgorin theorem says that all eigenvalues of a matrix $\boldsymbol{M}$ are in the union of the following sets

$$F_i = \left\{ \lambda \mid |\lambda - m_{ii}| \leq \sum_{j \neq i} |m_{ij}| \right\}.$$

In our case, $\omega_{ii} = 0$ and $\sum_{j \neq i} |\omega_{ij}| = 1 - \frac{d_{ii}}{D_i}$, and hence all eigenvalues have absolute value smaller than 1. This is the necessary and sufficient condition for $\boldsymbol{\Omega}$ being a contraction. ■

Proposition 29 *Given $\bar{\theta}, \bar{a}$, and $(r_{ij}, p_{ij})_{i,j}$ the game in which agents choose actions $\{a_i\}_i$ has a unique equilibrium.*

Proof. Expected payoffs are

$$-E_i[u_i] = d_{ii}(a_i - \theta_i)^2 + \sum_{j \neq i} d_{ij}\left(a_i^2 - 2a_i E[a_j] + E\left[a_j^2\right]\right) - k_r^2 \sum_{j \neq i} r_{ji} - k_p^2 \sum_{j \neq i} p_{ij}.$$

Therefore, first order conditions with respect to actions are

$$-\frac{\partial E_i[u_i]}{\partial a_i} = 2d_{ii}(a_i - \theta_i) + 2\sum_{j \neq i} d_{ij}(a_i - E_i[a_j]) = 0.$$

Given information sets $\{y_i\}_i$ individual actions satisfy Kuhn-Tucker's conditions. Thus, for each $i \in \{1, \ldots, n\}$ either

$$a_i = \omega_{ii}\theta_i + \sum_{j \neq i} \omega_{ij} E_i[a_j]$$

or

$$a_i \in \{-\bar{a}, \bar{a}\}.$$

More precisely:

$$BR_i(a_{-i}) = \begin{cases} \omega_{ii}\theta_i + \sum_{j \neq i} \omega_{ij} E_i[a_j] & \text{if } \omega_{ii}\theta_1 + \sum_{j \neq i} \omega_{ij} E_i[a_j] \in [-\bar{a}, \bar{a}] \\ \bar{a} & \text{if } \omega_{ii}\theta_1 + \sum_{j \neq i} \omega_{ij} E_i[a_j] > \bar{a} \\ -\bar{a} & \text{if } \omega_{ii}\theta_1 + \sum_{j \neq i} \omega_{ij} E_i[a_j] < -\bar{a} \end{cases}$$

We can make use of Lemma 27 to show that, indeed,

$$BR_i\left(a_{-i}\right) = \omega_{ii}\theta_i + \sum_{j\neq i}\omega_{ij}E_i\left[a_j\right] \qquad \text{for all } i.$$

Hence, equilibrium conditions become

$$a_i^* = \omega_{ii}\theta_i + \sum_{j\neq i}\omega_{ij}E_i\left[a_j^*\right] \quad i = 1,\ldots,n. \tag{14}$$

Nesting these conditions we get

$$a_i^* = \omega_{ii}\theta_i + \sum_{j\neq i}\omega_{ij}E_i\left[\omega_{jj}\theta_j + \sum_{k\neq j}\omega_{jk}E_j\left[a_k^*\right]\right] = \underbrace{\omega_{ii}\theta_i + \sum_{j\neq i}\omega_{ij}\omega_{jj}E_i\left[\theta_j\right]}_{\text{expectations on primitives}} + \underbrace{\sum_{j\neq i}\sum_{k\neq j}\omega_{ij}\omega_{jk}E_iE_j\left[a_k^*\right]}_{\text{strategic interdependence}} \tag{15}$$

The last term in this expression allows for a new level of nestedness that we obtain plugging (14) in (15):

$$a_i^* = \underbrace{\omega_{ii}\theta_i + \sum_{j\neq i}\omega_{ij}\omega_{jj}E_i\left[\theta_j\right] + \sum_{j\neq i}\sum_{k\neq j}\omega_{ij}\omega_{jk}\omega_{kk}E_iE_j\left[\theta_k\right]}_{\text{expectations on primitives}} + \underbrace{\sum_{j\neq i}\sum_{k\neq j}\sum_{s\neq k}\omega_{ij}\omega_{jk}\omega_{ks}E_iE_jE_k\left[a_s^*\right]}_{\text{strategic interdependence}}$$

Observe that, again, this last interdependence term allows for adding another level of nestedness, and that we can keep repeating this nestedness procedure up to any level. In particular, if we repeat this l times we obtain the following expression

$$a_i^* = \underbrace{\omega_{ii}\theta_i + \sum_{k\neq i}\omega_{ik}\omega_{kk}E_i\left[\theta_k\right] + \cdots + \sum_{i_1}\cdots\sum_{i_l}\sum_{k\neq i_l}\omega_{i,i_1}\cdots\omega_{i_l,k}\omega_{kk}E_iE_{i_1}\cdots E_{i_l}\left[\theta_k\right]}_{\text{expectations on primitives}} \tag{16}$$

$$+\underbrace{\sum_{i_1}\cdots\sum_{i_l}\sum_{k\neq i_l}\sum_{s\neq k}\omega_{i,i_1}\cdots\omega_{i_l,k}\omega_{ks}E_iE_{i_1}\cdots E_{i_l}E_k\left[a_s^*\right]}_{\text{strategic interdependence}} \tag{17}$$

where, $i_1,\ldots,i_l$ are indices that run from 1 to n.

We want to show that as $l \to +\infty$ this expression converges and, therefore, that the equilibrium is unique. We are going to show this in two steps:

(i) first, we are going to show that the limit when $l \to +\infty$ of *expectations on primitives* is bounded above and below; this ensures that the expression of *expectations on primitives* is well-defined at the limit;

(ii) second, we are going to show that the expression of *strategic interdependencies* vanishes when $l \to +\infty$.

The proofs of both steps rely on Lemma 28.

To prove (i), first note that all expectations $E_i[\theta_k], E_iE_j[\theta_k], \ldots, E_iE_{i_1}\cdots E_{i_l}[\theta_k]$ are bounded above by $\bar{\theta}$ and bounded below by $-\bar{\theta}$. Then, the expression

$$\sum_{k\neq i}\omega_{ik}\omega_{kk}E_i[\theta_k]+\sum_{j\neq i}\sum_{k\neq j}\omega_{ij}\omega_{jk}\omega_{kk}E_iE_j[\theta_k]+\cdots+\sum_{i_1}\cdots\sum_{i_l}\sum_{k\neq i_l}\omega_{i,i_1}\cdots\omega_{i_l,k}\omega_{kk}E_iE_{i_1}\cdots E_{i_l}[\theta_k]$$

is bounded above by

$$\bar{\theta}\left(\sum_{k\neq i}\omega_{ik}\omega_{kk}+\sum_{j\neq i}\sum_{k\neq j}\omega_{ij}\omega_{jk}\omega_{kk}+\cdots+\sum_{i_1}\cdots\sum_{i_l}\sum_{k\neq i_l}\omega_{i,i_1}\cdots\omega_{i_l,k}\omega_{kk}\right)$$

and bounded below by

$$-\bar{\theta}\left(\sum_{k\neq i}\omega_{ik}\omega_{kk}+\sum_{j\neq i}\sum_{k\neq j}\omega_{ij}\omega_{jk}\omega_{kk}+\cdots+\sum_{i_1}\cdots\sum_{i_l}\sum_{k\neq i_l}\omega_{i,i_1}\cdots\omega_{i_l,k}\omega_{kk}\right).$$

We can apply now the following result: the entry (i,j) of $\mathbf{\Omega}^l$, that we denote $\omega_{ij}^{[l]}$, is equal to $\sum_{i_1}\cdots\sum_{i_{l-1}}\omega_{i,i_1}\omega_{i_1,i_2}\cdots\omega_{i_{l-2},i_{l-1}}\omega_{i_{l-1},j}$. Hence

$$\sum_{k\neq i}\omega_{ik}\omega_{kk}+\sum_{j\neq i}\sum_{k\neq j}\omega_{ij}\omega_{jk}\omega_{kk}+\cdots+\sum_{i_1}\cdots\sum_{i_l}\sum_{k\neq i_l}\omega_{i,i_1}\cdots\omega_{i_l,k}\omega_{kk}=\omega_{kk}\sum_{j=1}^{l}\omega_{ik}^{[j]}$$

The element $\sum_{j=1}^{l}\omega_{ik}^{[j]}$ is the (i,k) entry of the matrix $\sum_{1\leq j\leq l}\mathbf{\Omega}^j$. A sufficient condition for the infinite sum $\sum_{j\geq 1}\mathbf{\Omega}^j$ to converge is that $\mathbf{\Omega}$ is a contraction. Thus by Lemma 28 $\omega_{kk}\sum_{j=1}^{l}\omega_{ik}^{[j]}$ is bounded when $l\to+\infty$ and hence the expression of *expectations on primitives* is bounded too. This proves (i).

To prove (ii), first note that, trivially, $E_iE_{i_1}\cdots E_{i_l}E_k[a_s^*]$ is bounded above by $\bar{a}$ and below by $-\bar{a}$. Hence the expression

$$\sum_{i_1}\cdots\sum_{i_l}\sum_{k\neq i_l}\sum_{s\neq k}\omega_{i,i_1}\cdots\omega_{i_l,k}\omega_{ks}E_iE_{i_1}\cdots E_{i_l}E_k[a_s^*]$$

is bounded above by

$$\bar{a}\sum_{i_1}\cdots\sum_{i_l}\sum_{k\neq i_l}\sum_{s\neq k}\omega_{i,i_1}\cdots\omega_{i_l,k}\omega_{ks}$$

and below by

$$-\bar{a}\sum_{i_1}\cdots\sum_{i_l}\sum_{k\neq i_l}\sum_{s\neq k}\omega_{i,i_1}\cdots\omega_{i_l,k}\omega_{ks}.$$

Then, since $\sum_{i_1}\cdots\sum_{i_l}\sum_{k\neq i_l}\sum_{s\neq k}\omega_{i,i_1}\cdots\omega_{i_l,k}\omega_{ks}=\sum_{s\neq k}\omega_{is}^{[l+1]}$ and $\omega_{is}^{[l+1]}\to 0$ when $l\to\infty$ for all $s=1,\ldots,n,$[17] we can ensure that $\sum_s\omega_{is}^{[l+1]}\to 0$ when $l\to\infty$. Therefore,

[17] This is, precisely, because $\sum_{l\geq 1}\mathbf{\Omega}^l$ converges.

the upper and lower bounds of the strategic intrerdependecies term tend to 0 when $l \to \infty$. This proves (ii). ■

Observation: This proof does not require our structure of communication reports. Any other information structure would do not change the unicity result. Of course, it would change the shape of this equilibrium.

Proposition 30 *The unique equilibrium of the game when $\bar{\theta} \to +\infty$ (and, therefore, $\bar{a} \to +\infty$ too) is linear.*

Proof. The previous proposition states that the equilibrium for any given $\bar{\theta}$ and $\bar{a}$ is given by

$$a_i^* = \lim_{l \to +\infty} \left\{ \omega_{ii}\theta_i + \sum_{k \neq i} \omega_{ik}\omega_{kk} E_i [\theta_k] + \cdots + \sum_{i_1} \cdots \sum_{i_l} \sum_{k \neq i_l} \omega_{i,i_1} \cdots \omega_{i_l,k}\omega_{kk} E_i E_{i_1} \cdots E_{i_l} [\theta_k] \right\}. \tag{18}$$

We have to compute explicitly the expectations in the previous expression when $\bar{\theta} \to +\infty$. Observe that when $\bar{\theta} \to +\infty$ all θ_is probability distributions tend to the normal distribution with mean 0 and precision s. Bayesian updating with normal distributions takes a simple linear form. To be more precise, in our setup, since the mean of all prior distributions is equal to 0, we have that

$$\begin{aligned} E_i [\theta_j] &= \alpha_{ij} y_{ij} \qquad \text{for all } i \neq j \\ E_i [y_{jk}] &= \beta_{ijk} y_{ik} \qquad \text{for all } k \neq i \neq j \neq k \end{aligned}$$

with $\alpha_{ij} \in [0,1]$ and $\beta_{ijk} \in [0,1]$ being constants that depend on the precisions $(r_{ij}, p_{ij})_{i,j}$ that are chosen at the first stage of the game. Observe that this immediately implies that also higher-order expectations $E_i E_{i_1} \cdots E_{i_l} [\theta_k]$ are linear in $\{y_{ij}\}_{j \neq i}$. In particular, $E_i E_{i_1} \cdots E_{i_l} [\theta_k] = \varphi_{ik}^{[l]} y_{ik}$ where $\varphi_{ik}^{[l]}$ is a product of one α (in particular, of $\alpha_{i_l,k}$) and $l-1$ different βs. Note that $\varphi_{ik}^{[l]} \in [0,1]$ for all i, k, l. Therefore

$$a_i^* = \omega_{ii}\theta_i + \sum_k \omega_{kk} \sum_{l=1}^{+\infty} \varphi_{ik}^{[l]} \omega_{ik}^{[l]} y_{ik} \qquad \text{for all } i. \tag{19}$$

To show that this expression is well-defined we proceed in a similar way than in the proof of Proposition 29. The expression $\sum_{l=1}^{+\infty} \varphi_{ik}^{[l]} \omega_{ik}^{[l]}$ is weighted below by 0 and above by $\sum_{l=1}^{+\infty} \omega_{ik}^{[l]}$. This last infinite sum is the entry (i,k) of the matrix $\sum_{l \geq 1} \mathbf{\Omega}^l$ that is well-defined because $\mathbf{\Omega}$ is a contraction. Thus, we conclude that the expression in 19 is well-defined for all players and linear in $\left(\theta_i, \{y_{ij}\}_{j \neq i}\right)$ for each $i \in \{1, \ldots, n\}$. ■

Appendix C. Precluding Communication and Tranfers

In this appendix we want to show up a possible strategic effect that we have not considered in text. Besides direct communication, another possible tool an agent could use for his own purpose is trying to exclude some possible ways of communication. For example, an agent can inhibit a communication channel by paying some monetary transfer to the agents that would be involved in it. With the use of a simple three-agent numerical example we show that this incentive exists.

Consider an organization formed by three agents with interaction matrix

$$\boldsymbol{D} = \begin{pmatrix} 0.3 & 0.3 & 1 \\ 0.3 & 0.3 & 1 \\ 1 & 100 & 1 \end{pmatrix}$$

and such that $s_i = 0.1$ for all i, $k_r = k_r = 0.01$. Agents 1 and 2 occupy an equivalent position inside the organization, and they want primarily to coordinate with agent 3. Instead, agent 3 shows a severe coordination motive with agent 2, compared with any other payoff externality.

When considering unrestricted communication, as we do in text, the final utilities of each agent are

$$u_1 = -7.\,034\,6, u_2 = -3.\,593\,2, u_3 = -17.\,789.$$

If, instead, we consider a setup with inhibited communication in which agents 1 and 2 can not communicate with each other, some algebra shows that agents' utilities under this communication restriction are

$$u_1 = -11.\,446, u_2 = -6.\,129\,7, u_3 = -16.\,267.$$

Comparing utilities in both cases, one immediately observes that agent 3 benefits from inhibited communication in the communication lines among agents 1 and 2, while this two agents end up in a worst situation. This suggests that there is room in this model to analyze monetary transfers among agents to limit information transmission. Of courses, this would rise up other strategic considerations, such as the enforcement of the possible agreements reached, something that would possibly displace the focus and aim of our analysis.

Appendix D. Broadcasting

We analyze here a variation of the game theoretical version of our model. In particular, we consider the situation in which each agent chooses a unique r_i, a common precision for active communication with the rest of individuals. This can be understood as an approximation to the analysis of broadcasting. Agents exert the same effort in actively communicating with every else but can freely choose to which messages they want to pay more atention. This could be the case of e-mail lists, where the sender is allowed to send a unique message tot he organization as a whole, and it is at the discretion of each one of the receivers to attend to it. In our model, when the agent chooses the precision r_i, he determines the possible ambiguity in the message: if the signal is very precise, everybody is going to receive essentially the same common signal; if the signal is very noisy, the receiver needs to exert a high effort to decode this message.

Before proceeding to present and prove the chracterization of the equilibrium in the broadcasting case we need to introduce a bit of notation. Given a vector $\boldsymbol{\lambda}_{\cdot i}$ Let

$$g_{ji}\left(\boldsymbol{\lambda}_{\cdot i}\right) = \begin{cases} \omega_{ii} & \text{if } i = j \\ -s_i\left(\frac{k_p}{\sqrt{D_j}} + \frac{k_r}{\lambda_{ij}}\right) & \text{otherwise} \end{cases}$$

Observe that this is a variation of the previously defined

Proposition 31 *For any* $(\boldsymbol{D}, \boldsymbol{s})$*, if* k_r *and* k_p *are sufficiently low, the game* $\tilde{\Gamma}(\boldsymbol{D}, \boldsymbol{k}, \boldsymbol{s})$ *has a unique pure-strategy equilibrium:*

(i) Decisions are given by

$$\boldsymbol{b}_{\cdot j} = (\boldsymbol{I} - \boldsymbol{\Omega})^{-1} \cdot \boldsymbol{g}_{\cdot j}(\boldsymbol{\lambda}_{\cdot j}) \quad \textit{for all } j;$$

where $\boldsymbol{\lambda}_{\cdot j}$ *is an endogenously determined vector with positives entries that satisfy* $\sum_{k \neq j} d_{jk} b_{kj}^2 = \lambda_{ij}^2 b_{ij}^2$

(ii) Active communication is

$$r_i = \frac{\lambda_{ij} b_{ij}}{k_r} \qquad \textit{for all } j;$$

(iii) Passive communication is

$$p_{ij} = \frac{\sqrt{D_i} b_{ij}}{k_p} \qquad \textit{for all } i \neq j$$

Proof. If agent i chooses a unique ρ_i, the set of first-order conditions is equal to

$$\begin{aligned}
-\frac{1}{2}\frac{\partial E[u_i]}{\partial b_{ii}} &= d_{ii}(b_{ii} - 1)\sigma_i + \sum_{k \neq i} d_{ij}(b_{ii} - b_{ji})\sigma_i = 0 \\
-\frac{1}{2}\frac{\partial E[u_i]}{\partial b_{ij}} &= d_{ii} b_{ij}\left(\sigma_j + \rho_j + \pi_{ij}\right) + \sum_{k \neq i} d_{ik}\left((b_{ij} - b_{kj})\sigma_j + b_{ij}\rho_j + b_{ij}\pi_{ij}\right) = 0 \\
-\frac{\partial E[u_i]}{\partial \rho_i} &= \sum_{j \neq i} d_{ij} b_{ji}^2 + c_r'\left(\frac{1}{\rho_i}\right) = 0 \\
-\frac{\partial E[u_i]}{\partial \pi_{ij}} &= D_i b_{ij}^2 + c_p'\left(\frac{1}{\pi_{ij}}\right) = 0.
\end{aligned}$$

This set of first-order conditions is equivalent to

$$D_i b_{ii} = d_{ii} + \sum_{k \neq i} d_{ij} b_{ji} \tag{20}$$

$$D_i b_{ij} = \frac{\sigma_j}{\sigma_j + \rho_j + \pi_{ij}} \sum_{k \neq i} d_{ik} b_{kj} \tag{21}$$

$$\frac{\sqrt{\sum_{k \neq i} d_{ik} b_{ki}^2}}{k_r} = r_i \tag{22}$$

$$\frac{\sqrt{D_i} b_{ij}}{k_p} = p_{ij} \tag{23}$$

Since

$$\frac{r_j p_{ij}}{s_j r_j + s_j p_{ij} + r_j p_{ij}} = \frac{\sigma_j}{\sigma_j + \rho_j + \pi_{ij}}$$

condition (21) becomes

$$D_i b_{ij} = \frac{r_j p_{ij}}{s_j r_j + s_j p_{ij} + r_j p_{ij}} \sum_{k \neq i} d_{ik} b_{kj}$$

By permuting i and j in this last expression, we get

$$D_j b_{ji} = \frac{r_i p_{ji}}{s_i r_i + s_i p_{ji} + r_i p_{ji}} \sum_{k \neq i} d_{jk} b_{ki} \tag{24}$$

Since

$$\frac{\sqrt{\sum_{k \neq i} d_{ik} b_{ki}^2}}{k_r} = r_i$$

we can define an endogenous value λ_{ji} such that $\sqrt{\sum_{k \neq i} d_{ik} b_{ki}^2} = \lambda_{ji} b_{ji}$ for each $j \neq i$. In particular, it is the unique positive number such that $\sum_{k \neq i} d_{ik} b_{ki}^2 = \lambda_{ji}^2 b_{ji}^2$. Then, the first order condition associated to ρ_i can be rewriten as

$$\frac{\lambda_{ji} b_{ji}}{k_r} = r_i$$

for any $j \neq i$. Plugging this expression and (23) in (24), we get that

$$b_{ji} - \sum_{k \neq i} w_{jk} b_{ki} = -s_i \left(\frac{k_p}{\sqrt{D_j}} + \frac{k_r}{\lambda_{ji}} \right) \qquad \text{for all } i$$

or, equivalently, in matrix form

$$\boldsymbol{b}_{\cdot i} = (\boldsymbol{I} - \boldsymbol{\Omega})^{-1} \cdot \boldsymbol{g}(\boldsymbol{\lambda}_{\cdot i})$$

■

Observe that the main difference of the equilibrium action in the broadcasting case compared with the one in Theorem 6 is the change from the vector $\boldsymbol{h}$ to the vector $\boldsymbol{g}$. The matrix that relates these vectors with the equilibrium actions $\boldsymbol{b}$ remains the same in both cases.

A natural question that arises with the analysis of this new communication protocol is whether we should expect that agents engage in more active communication than before or not. The following result gives us an answer in terms of the ratio of passive versus active communication already considered in a previous section.

Proposition 32 *In the symmetric case, in which $d_{ij} = \bar{d}Q$ for all $i \neq j$ and $d_{ii} = \left(1 - (n-1)\,\bar{d}\right) Q$, for some $Q > 0$, the ratio of passive versus active communication is*

$$\frac{k_p}{k_r} \sqrt{(n-1)\,\gamma_{ij}}.$$

Proof. Because of symmetry, for all pairwise different i, j, k we have that $b_{ji} = b_{ki} = b^*$. Therefore,

$$\lambda_{ji} = \sqrt{\sum_{k \neq i} d_{ik}} = \sqrt{(n-1)\, \bar{d}Q}$$

This implies that

$$\begin{aligned} \frac{\sqrt{(n-1)\, \bar{d}Q}b^*}{k_r} &= r_i \\ \frac{\sqrt{Q}b^*}{k_p} &= p_{ij} \end{aligned}$$

The ratio between active and passive communication in this case is

$$\frac{r_i}{p_{ij}} = \frac{k_p}{k_r}\sqrt{\frac{(n-1)\, \bar{d}Q}{Q}} = \frac{k_p}{k_r}\sqrt{(n-1)\, \bar{d}} = \frac{k_p}{k_r}\sqrt{(n-1)\, \gamma_{ij}}$$

Again, when active and passive communication are equally costly, i.e $k_p = k_r$, the upper bound for this ratio is 1. Observe also, that the ratio in the case of broadcasting does not necessarily decreases when n increases. When $\bar{d} = \frac{1}{n}$, we obtain that the ratio of active versus passive communication is $\sqrt{(n-1)/n}$, that increases and tends to 1 when n increases. In clear contrast with the case of pairwise communication, active and passive communication are almost equal when there are enough agents. ■

Appendix E. Robustness to Alternative Communication Protocols

In this appendix we show that linearity of equilibrium is preserved under an alternative communication protocol. This ensures that similar comparative statics results can be obtained under different communication restrictions.

Assume that externalities, in the form of coordination requirements, only arise between pairs of agents that are linked. This is natural in social environments where conformity with social connections is a determinant of individual behavior.[18] Furthermore, assume that communication is retricted to acquitances. While in our model of organizations, where the formal hierarchy and the formal and informal chart of communication capabilities don't necessarily coincide, this assumption makes sense, for example, in the analysis of communication in social networks that link friends and acquitances. Keeping the same notation as in text, this means that pairwise communication between agent i and agent j can only take place when $d_{ij} \neq 0$.[19] Under this specification, differing weights in coordination problems in the utility function can be interpreted as heterogeneous social preferences, in which an agent seeks to coordination more precisely with individuals for whom she cares the most.

[18] See for example, Akerlof (1997) and Bernheim (1994).

[19] We assume here, in accordance with a vast literature on social networks, that relations are bidirectional and, hence, that $d_{ij} \neq 0$ whenever $d_{ji} \neq 0$, though d_{ij} and d_{ji} do not need necesarily to coincide.

Altogether, matrix $\mathbf{D}$ determines both the shape of social preferences and all communication restrictions.

Communication is modelled in the same terms as in text: two agents i and j that are linked (d_{ij} and d_{ji} are different than 0) choose the precisions for pairwise active and passive communication. Of course, first-order conditions with respect to actions do not change and lead to the following system of equations, that determines equilibrium conditions:

$$a_i^* = \omega_{ii}\theta_i + \sum_{j \neq i} \omega_{ij} E_i \left[a_j^*\right] \quad i = 1, \ldots, n$$

Hence, the equalities obtained through nestedness (15) and (16) in Appendix B are still valid under this new specification. To apply a similar argument to the proof of Proposition 30, we want to show that all higher order expectations

$$E_{i_1} E_{i_2} \cdots E_{i_l} \left[\theta_k\right]$$

are linear in communication reports for any sequence $i_1, \ldots, i_l$. The following observations are useful:

$$\begin{aligned} E_i\left[\theta_j\right] &= \begin{cases} 0 & \text{if } d_{ij} = 0 \\ \alpha_{ij} y_{ij} & \text{if } d_{ij} \neq 0 \end{cases} \\ E_i\left[y_{jk}\right] &= \begin{cases} 0 & \text{if } d_{ik} = 0 \\ \beta_{ijk} y_{ik} & \text{if } d_{ik} \neq 0 \end{cases} \end{aligned}$$

Applying these equalities one immediately obtaines the following result: either the value of $E_{i_1} E_{i_2} \cdots E_{i_l} \left[\theta_k\right]$ is going to take the same form as in the proof of Proposition 30 and, hence, it is linear in y_{ik}, or otherwise it is equal to 0. The latter case occurs when either in the sequence $i_1, \ldots, i_l$ there exists an $s \leq l - 1$ such that i_s and i_{s+1} do not communicate (i.e. $d_{i_s i_{s+1}} = 0$) or there exists and $s \leq l - 2$ such that i_s and i_{s+2} do not communicate (i.e. $d_{i_s i_{s+2}} = 0$).

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