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Renegotiating Moral Hazard Contracts under Limited Liability and Monotonicity

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Abstract

A moral hazard model with renegotiation is studied in which contracts must satisfy two natural restrictions, limited liability and monotonicity of payments. After he has chosen his effort, and before its consequence is realized, a risk averse entrepreneur (agent) may renegotiate his contract with a risk neutral investor (principal). Assuming the agent has the renegotiation bargaining power, a debt contract is the optimal initial contract.

Keywords: *moral hazard, renegotiation, limited liability, debt*

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1. Introduction

Current models of moral hazard contracts subject to renegotiation do not restrict the nature of sharing rules.[1] This paper considers the effects of two realistic restrictions, limited liability and monotonicity of returns to investors.

For concreteness the model is put in terms of financing a new firm. The agent is a risk averse entrepreneur who wishes to start up a firm. The principal is a risk neutral investor who provides the start-up funds in return for a payment contingent on the realized profit of the firm. After the agent takes an unobservable effort, but before the profit is realized, the parties may renegotiate the contract – they may want to do so because its incentive provisions are then irrelevant.

The first contract restriction is limited liability, a constraint that prohibits the agent from paying back more than the enterprise earns. It is a consequence of legal provisions, or of an inability to verify outside income. The second restriction is monotonicity, a constraint that requires the payment to the principal to weakly increase with the realized profit of the project. Monotonicity is a consequence of ex post moral hazard considerations, as, e.g., Innes (1990) observes. (This is discussed in Section 2.)

We focus on a renegotiation process in which all bargaining power rests with the agent: he makes an ultimatum offer to the principal. This is in keeping with the owner of a start-up approaching competitive financial markets with a new security offering, rather than *vice versa*. When the agent has the bargaining power and there are no constraints on the form of a contract, Matthews (1995) shows that an equilibrium initial contract is a "sales contract" in which the payment to the principal is not contingent. A central task of this paper is to determine

[1] Renegotiation of moral hazard contracts is the subject of Edlin and Hermalin (1997), Fudenberg and Tirole (1990), Hermalin and Katz (1991), Ma (1991,4), Matthews (1995), and Segal and Tadelis (1994).

how that result is altered by the presence of limited liability and monotonicity constraints.

We find that an optimal initial contract now takes the form of debt – both parties are better off renegotiating a debt contract than they are renegotiating any other feasible contract. The optimal debt contract is not renegotiation-proof, as it is typically renegotiated in order to give the agent better insurance. This is consistent with the stylized fact that new ventures are initially financed by debt-like securities which impose a large amount of risk on entrepreneurs, but are later refinanced by the sale of common stock. The model gives an explanation for the pervasiveness of debt that is quite distinct from others found in the literature.[2]

A precursor to this paper is Innes (1990), which shows that debt is an efficient incentive contract if contracts must satisfy limited liability and monotonicity, if the stochastic technology satisfies the monotone likelihood ratio property, and if both parties are risk neutral. The agent's risk neutrality is crucial for this result. If he is instead risk averse, debt is generally not an efficient incentive contract. In addition, risk aversion is what makes renegotiation an issue – only if the agent is risk averse will the parties want to adopt a better risk-sharing contract after he takes his effort. This paper shows that even when the agent is risk averse, so long as the initial contract can be renegotiated, it should still be debt.

As in most moral hazard models, a contract in this paper is a single sharing rule. This is in contrast to contracts in, e.g., Fudenberg and Tirole (1990) and Ma (1994). Those contracts are revelation mechanisms (menus of sharing rules) that determine the sharing rule, after renegotiation has ceased, as a function of the agent's reported effort. In this paper greater welfare would sometimes be achieved by a menu contract.[3] We have two main reasons for nonetheless

[2] Other rationales for debt rely on costly state verification, or on control rights and managerial profit appropriability. See, e.g., the surveys by Allen (1990) and by Harris and Raviv (1992).

[3] However, menus have no benefit if a particular "sales" or "riskless debt" contract is feasible.

restricting attention to contracts that are single sharing rules. First, the rarity of menu contracts in reality suggests that they may often have low benefit or high cost. For example, a menu may be difficult to describe; there may not be time for the agent to select from a menu after renegotiation ceases and before he knows the consequences of his action; or there may not even be a date prior to the consequences of his actions at which renegotiation must cease. Second, sharp answers to questions about how renegotiation proceeds requires a model in which renegotiation must occur. This rules out menu contracts, since any equilibrium is equivalent to one in which a menu contract is adopted and not renegotiated (the renegotiation-proof principle).

The remainder of this paper begins with a description of the environment in Section 2. A key property of debt is discussed in Section 3. The renegotiation game is presented in Section 4. The main results regarding the emergence of debt as an optimal and equilibrium initial contract are derived in Sections 5 and 6. Examples are considered in Section 7. The possible value of random contracts is considered in Section 8, and two extensions of the model are sketched. The Appendix details one of these extensions. Section 9 concludes.

2. Environment

A risk averse *agent* (agent) has a project requiring an investment K. This investment can be provided by a risk neutral *principal* (principal) who embodies a competitive market of potential principals. The agent provides an unobservable effort. The principal provides start-up funds and absorbs risk.

The (*gross*) *profit* of the project is verifiable and has $n \geq 2$ possible realizations: $\pi_1 < \pi_1 < \cdots < \pi_n$. The agent can take any effort in a nonempty, compact subset E of the real line, with $\underline{e} \equiv \min(E)$, and $\bar{e} \equiv \max(E)$. If he chooses

This is shown in one setting in Matthews (1995), and in another here.

effort e, the realized profit is π_i with probability $p_i(e)$. Each p_i is positive and twice continuously differentiable on $[\underline{e}, \bar{e}]$. Letting $p(e) \equiv (p_1(e), \ldots, p_n(e))$ and $\pi \equiv (\pi_1, \ldots, \pi_n)$, the expected gross profit generated by effort e is $\pi \cdot p(e)$. The distributions satisfy the strict monotone likelihood ratio property:

$$\text{(MLRP)} \quad \frac{p_i'(e)}{p_i(e)} \text{ increases in } i \text{ for all } e \in E.$$

Consequently, $p(e)$ stochastically dominates $p(d)$ if $e > d$.

An *allocation* is a pair (ψ, e), where $\psi = (\psi_1, \ldots, \psi_n)$ consists of contingent payments from the agent to the principal. The principal's expected return from (ψ, e), net of the start-up funds, is $\psi \cdot p(e) - K$.

The agent's utility function for money y and effort e is $u(y) - c(e)$. Define $u(y_1, \ldots, y_n) \equiv (u(y_1), \ldots, u(y_n))$. The agent's expected utility is

$$U(\psi, e) \equiv u(\pi - \psi) \cdot p(e) - c(e).$$

The maintained assumptions are $u' > 0$, $u'' < 0$, and $c' > 0$.

A *contract* is an agreement specifying that the principal provide the funds K in return for a payment contingent on the realized profit. A contract is represented by its vector of contingent payments, ψ.

Contracts are restricted in two ways. The first is *limited liability* for the agent, which prohibits him from paying back more than the project earns:

$$\text{(LL)} \quad \psi_i \leq \pi_i \text{ for all } i \leq n.$$

It would also be natural to require limited liability for the principal, $\psi_i \geq 0$. But this would not alter the results, and so for simplicity only (LL) is imposed.

The second restriction on contracts is *monotonicity*:

$$\text{(MON)} \quad \psi_i \leq \psi_{i+1} \text{ for all } i < n.$$

Monotonicity requires the principal's earnings to weakly increase with the project's observed profit. This is true of virtually all real contracts. It can be derived as a consequence of ex post moral hazard, as Innes (1990) observes. For example, the principal may be able to sabotage the firm, thereby distorting the observed π_i downwards. Or the agent may be able to borrow secretly from a lender after a contract has been signed, thereby distorting the observed π_i upwards. Given such abilities to distort apparent profit, (MON) must be satisfied by any implementable contract.[4]

The set of feasible contracts is

$$\Psi \equiv \{\psi \in \Re^n \,|\, \psi \text{ satisfies (LL) and (MON)}\}.$$

3. Debt Contracts

A *debt contract,* $\delta = (\delta_1, \ldots \delta_n)$, takes the form

$$\delta_i = \min(\pi_i, D) \text{ for all } i \leq n, \tag{1}$$

where D is its *face value*. Debt contracts satisfy limited liability and monotonicity.

Given any feasible contract $\psi \in \Psi$ and effort level $e \in [\underline{e}, \bar{e}]$, a debt contract δ exists that has the same expected value as ψ given e, i.e., it satisfies

$$\delta \cdot p(e) = \psi \cdot p(e). \tag{2}$$

We can view the expected values of the two contracts, $\delta \cdot p(\cdot)$ and $\psi \cdot p(\cdot)$, as functions of effort. Given (2), their graphs cross at e. Furthermore, single-crossing is satisfied: $\delta \cdot p(d)$ is greater (lower) than $\psi \cdot p(d)$ at efforts d that are lower (higher) than e. This is because the payments specified by a debt contract are the maximum possible at low profit levels given limited liability, and they are the minimum possible at high profit levels given monotonicity. By (MLRP), lowering

[4]Other ex post moral hazards, such as the entrepreneur destroying realized profit, or the investor injecting cash so as to inflate the observed profit, lead to another monotonicity constraint, $\psi_{i+1} - \psi_i \leqslant \pi_{i+1} - \pi_i$. Adding it would not change the main results.

effort shifts probability evenly from high to low profits, and so decreases the debt's expected value relatively little.

The essence of this observation was observed by Innes (1990). A somewhat different statement and proof are given here.[5]

Lemma 1. *For any* $\psi \in \Psi$ *and debt contract* $\delta \neq \psi$*, suppose* $\psi \cdot p(e) = \delta \cdot p(e)$ *for some* $e \in [\underline{e}, \bar{e}]$*. Then*

$$\psi \cdot p(d) \lesseqgtr \delta \cdot p(d) \text{ for } d \lesseqgtr e.$$

Proof. Let $v = \psi - \delta$. A routine calculus argument proves the result if for all $d \in [\underline{e}, \bar{e}]$, $v \cdot p'(d) > 0$ whenever $v \cdot p(d) = 0$. So assume $v \cdot p(d) = 0$. Since $v \neq 0$, and $p(d)$ has full support, v has negative and positive components. Let k be the largest i such that $v_i < 0$. Because δ is debt and ψ satisfies (LL) and (MON), $v_i \leq 0$ for $i \leq k$, and $v_i \geq 0$ for $i > k$. Thus, since v has a positive component, (MLRP) and $v \cdot p(d) = 0$ imply

$$\begin{aligned} v \cdot p'(d) &= \sum_{i=1}^{n} \left(\frac{p_i'(d)}{p_i(d)} \right) p_i(d) v_i \\ &> \sum_{i=1}^{k} \left(\frac{p_k'(d)}{p_k(d)} \right) p_i(d) v_i + \sum_{i=k+1}^{n} \left(\frac{p_k'(d)}{p_k(d)} \right) p_i(d) v_i \\ &= \left(\frac{p_k'(d)}{p_k(d)} \right) [v \cdot p(d)] = 0. \quad \blacksquare \end{aligned}$$

Lemma 1 immediately yields the following result of Innes (1990): If the agent is risk neutral, then debt solves the standard principal-agent problem, which in

[5] Lemma 1 states that $(\psi - \delta) \cdot p(d)$ satisfies single-crossing in d. By (LL) and (MON), $\psi - \delta$ satisfies single-crossing in π_i (see the proof of the lemma). Thus, Lemma 1 is a special case of Theorem 3 in Athey (1998), which shows more generally that (MLRP) implies this inheritability of the single-crossing property.

this case is

$$\underset{\psi\in\Psi, e\in E}{\text{Max}} \ (\pi-\psi)\cdot p(e)-c(e) \ \text{ subject to}$$

$$e\in\arg\max_{d\in E} \ (\pi-\psi)\cdot p(d)-c(d),$$

$$\psi\cdot p(e)\geq K.$$

To see why, note that a risk neutral agent only cares about his expected payment to the principal. Suppose a non-debt contract $\psi\in\Psi$ induces effort e, and consider a lower effort $d<e$. The expected payment $\psi\cdot p(d)$ must be too high for the agent to want to choose d instead of e. Now, let δ be the debt contract that, together with ψ and e, satisfies (2). As he always has the option of choosing e, the (risk neutral) agent must (weakly) prefer δ to ψ. Lemma 1 implies that the principal agrees. For, if $d<e$, the expected payment $\delta\cdot p(d)$ is greater than $\psi\cdot p(d)$. Since the latter was already high enough to deter the agent from choosing d, he will also not choose it when δ is the contract. So δ induces only efforts greater than or equal to e. Because the expected value of δ exceeds that of ψ at efforts greater than e, the principal must also (weakly) prefer δ to ψ.

If the agent is risk averse, the principal-agent problem is generally not solved by debt, nor by a first-best risk-sharing contract. The latter fact makes renegotiation an issue. If the parties were to agree to an optimal incentive contract for a risk averse agent, they would renegotiate it to a better risk-sharing contract after he chooses effort. In equilibrium the agent would foresee this, and so disregard the initial contract's incentives.

4. Renegotiation Game

We turn now to the game played after the adoption of an initial contract. In this game the agent chooses an unobservable effort, and the parties bargain over a new contract. It will be referred to as the *renegotiation game*. (This is a slight misnomer, since a non-bargaining action, the effort, is taken in this game.)

The initial contract is denoted ψ^0. It is renegotiated via ultimatum bargaining, with the agent proposing a new contract on a take-it-or-leave-it basis. The game starts with the agent choosing an effort $e \in E$ and a proposal $\psi \in \Psi$. (It is irrelevant here whether e and ψ are chosen simultaneously or in either order.) The principal does not observe e. Upon receiving the proposal, she agrees to renegotiate by selecting it, or she refuses to renegotiate by selecting ψ^0 to be the final contract. Denote this game as $\Gamma(\psi^0)$.

The agent's strategy is a proposal and an effort, (ψ, e). The principal's strategy is a selection rule: she selects $s(\psi) \in \{\psi^0, \psi\}$ when ψ is proposed. Her beliefs about the agent's effort when he proposes ψ are given by a probability distribution $\beta(\cdot|\psi)$ on E. Attention is restricted for now to pure strategy equilibria. Accordingly, an *assessment* is a profile of strategies and beliefs, (ψ, e, s, β). A *(perfect Bayesian) equilibrium* is an assessment satisfying sequential rationality, and Bayes' rule on the equilibrium path.

Suppose (ψ, e, s, β) is an equilibrium in which the principal selects ψ. The outcome is then (ψ, e). Because e is the agent's optimal effort given ψ, this outcome satisfies incentive compatibility:

$$\text{(IC)} \quad U(\psi, e) \geq U(\psi, d) \text{ for all } d \in E.$$

Because the principal must prefer ψ to ψ^0 when ψ is proposed, (ψ, e) also satisfies an individual rationality condition:

$$(\text{IR-}\psi^0) \quad \psi \cdot p(e) \geq \psi^0 \cdot p(e).$$

An allocation is a equilibrium outcome if it maximizes the agent's payoff subject to these constraints. This program is

$$(\text{P-}\psi^0) \quad V(\psi^0) \equiv \underset{\psi\in\Psi, e\in E}{\text{Max}}\ U(\psi, e)$$

$$\text{subject to (IC) and (IR-}\psi^0).$$

Let $A(\psi^0)$ be the set of allocations that solve (P-ψ^0).

Lemma 2. *For any* $\psi^0 \in \Psi$, *(i)* $A(\psi^0) \neq \emptyset$, *and (ii) every* $(\psi, e) \in A(\psi^0)$ *is an equilibrium outcome of* $\Gamma(\psi^0)$.

Proof. The proof of (i) is well-known.[6] To prove (ii), let $(\bar{\psi}, \bar{e}) \in A(\psi^0)$. Define a belief function $\bar{\beta}$ as follows. If $\bar{\psi}$ is proposed, the principal believes $\bar{e}$ was chosen: $\bar{\beta}(\bar{e} \,|\, \bar{\psi}) = 1$. For any $\psi \neq \bar{\psi}$, let

$$E(\psi) \equiv \{e \in E \,|\, \psi \cdot p(e) \leq \psi^0 \cdot p(e)\}.$$

If $E(\psi) \neq \emptyset$, let $\bar{\beta}(\cdot|\psi)$ be any distribution with support in $E(\psi)$. If $E(\psi) = \emptyset$, let $\bar{\beta}(\cdot|\psi)$ be any distribution on E. Define a selection rule $\bar{s}$ by $\bar{s}(\bar{\psi}) = \bar{\psi}$, and for $\psi \neq \bar{\psi}$,

$$\bar{s}(\psi) = \begin{cases} \psi^0 & \text{if } E(\psi) \neq \emptyset \\ \psi & \text{if } E(\psi) = \emptyset. \end{cases}$$

The assessment $(\bar{\psi}, \bar{e}, \bar{s}, \bar{\beta})$ has outcome $(\bar{\psi}, \bar{e})$, and it satisfies Bayes rule on its path. The pair $(\bar{s}, \bar{\beta})$ is sequentially rational at proposals $\psi \neq \bar{\psi}$ by construction, and at $\bar{\psi}$ because $(\bar{\psi}, \bar{e})$ satisfies (IR-ψ^0). When this assessment is played, the agent's payoff is $V(\psi^0) = U(\bar{\psi}, \bar{e})$. He cannot get a greater payoff by deviating to some $(\bar{\psi}, d)$, since $(\bar{\psi}, \bar{e})$ satisfies (IC). Suppose he instead deviates to (ψ, d), where $\psi \neq \bar{\psi}$. Suppose $E(\psi) \neq \emptyset$. Then the principal rejects ψ, and the resulting

[6] Because $p > 0$ on the compact set E, the contract set can be bounded. See Grossman and Hart (1983), or Lemma A1 in the Appendix of this paper.

allocation is (ψ^0, e). Letting e^0 be an optimal effort for the agent when ψ^0 is the contract, (ψ^0, e^0) is feasible for (P-ψ^0). Hence, $U(\psi^0, d) \leq U(\psi^0, e^0) \leq V(\psi^0)$. Now suppose $E(\psi) = \emptyset$. Then the principal accepts ψ, and the resulting allocation is (ψ, d). Let e be an optimal effort for the agent when ψ is the contract. Since $E(\psi) = \emptyset$, (ψ, e) satisfies (IR-ψ^0). So (ψ, e) is feasible for (P-ψ^0), and again $U(\psi, d) \leq U(\psi, e) \leq V(\psi^0)$. This shows that $(\bar{\psi}, \bar{e})$ is a best reply to $\bar{s}$, and so $(\bar{\psi}, \bar{e}, \bar{s}, \bar{\beta})$ is an equilibrium. ■

Conservative beliefs are used in the proof of Lemma 2 to show that the solutions of (P-ψ^0) are equilibrium outcomes. According to these beliefs, if the agent makes a non-equilibrium proposal, the principal believes (if possible) that the chosen effort is such that she should refuse to renegotiate. This suspicious attitude on the part of the principal when faced with an unexpected proposal may or may not seem plausible. However, other beliefs can also be used. For example, the principal can be required to believe, when faced with a non-equilibrium proposal, that the agent chose an effort that is optimal under the assumption that his proposal will be accepted. That is, the $(\bar{s}, \bar{\beta})$ in the proof can be replaced by a $(\hat{s}, \hat{\beta})$ defined as follows for non-equilibrium proposals $\psi : \hat{\beta}(e_\psi \,|\, \psi) = 1$, where $e_\psi \in \arg\max_{e \in E} U(\psi, e)$, and

$$\hat{s}(\psi) = \psi \iff \psi \cdot p(e_\psi) \geq \psi^0 \cdot p(e_\psi).$$

The principal holds these beliefs if she thinks the agent would make a non-equilibrium proposal only if he thought it would be accepted.

Some equilibria may have outcomes that do not solve (P-ψ^0).[7] We do not consider them, but instead adopt the usual practice of focusing on the proposer's

[7] The beliefs in such equilibria cause an out-equilibrium proposal ψ to be rejected even though an effort e exists such that the principal prefers (ψ, e) to (ψ^0, e), and the agent prefers (ψ, e) to both (ψ, d) and (ψ^0, d) for all $d \in E$.

best equilibrium outcomes, $A(\psi^0)$. This is consistent with the supposition that the agent has all the bargaining power.

Remark 1. *A standard argument shows that individual rationality constraints in problems like (P-ψ^0) bind. Suppose (ψ, e) satisfies (IC) and, with slack, (IR-ψ^0). Then each payment in ψ can be lowered to obtain a contract ψ' such that (ψ', e) also satisfies (IC) and (IR-ψ^0). Since $U(\psi', e) > U(\psi, e)$, this shows that (ψ, e) does not solve (P-ψ^0). The problem with this argument here is that if $\psi_j = \psi_{j+1}$ for some j, then ψ' violates (MON).*[8] *Thus, $(\psi, e) \in A(\psi^0)$ necessarily satisfies (IR-ψ^0) with equality only if ψ is strictly monotonic.*

5. Optimal Initial Contracts

In this section we show that, given any possible initial contract, a debt contract exists that both parties would prefer to be the initial contract. The argument uses two lemmas. The first is a revealed preference proposition about the agent's preferences over initial contracts, as embodied in the utility function $V(\cdot)$.

Lemma 3. *Let ψ^0 be any contract, and let $(\psi^*, e^*) \in A(\psi^0)$. Suppose ψ^1 is a contract for which $\psi^1 \cdot p(e^*) \leq \psi^* \cdot p(e^*)$. Then $V(\psi^1) \geq V(\psi^0)$.*

Proof. By hypothesis, (ψ^*, e^*) satisfies (IR-ψ^1). It also satisfies (IC), since it solves (P-ψ^0). So (ψ^*, e^*) is feasible for (P-ψ^1), and the result follows. ■

A simple application is to contracts ψ^0 and $\psi^1 \leq \psi^0$, so that each contingent payment is no less in ψ^0 than in ψ^1. By Lemma 3, the agent weakly prefers the

[8] Contract ψ' is defined, for some $v > 0$ and all $i \leqslant n$, by $u(\pi_i - \psi'_i) = u(\pi_i - \psi_i) + v$. Assume $\psi_j = \psi_{j+1} = \bar{\psi}$. Define $g(\pi)$ by $u(\pi - g(\pi)) = u(\pi_i - \bar{\psi}) + v$. Then $g'(\pi) = 1 - u'(\pi - \bar{\psi})/u'(\pi - g(\pi)) < 0$, since $u'' < 0$. Hence, $\psi'_j = g(\pi_j) > g(\pi_{j+1}) = \psi'_{j+1}$, violating (MON).

outcomes in $A(\psi^1)$ to those in $A(\psi^0)$. For example, he prefers to renegotiate a debt contract with a low face value than one with a high face value.

It is now useful to view program (P-ψ^0) as being solved in two stages. In the first a contract is chosen given a fixed effort:[9]

$$(\text{P-}\psi^0, e) \quad V(e \,|\, \psi^0) \equiv \max_{\psi \in \Psi} U(\psi, e)$$
$$\text{subject to (IC) and (IR-}\psi^0\text{)}.$$

In the second stage the effort is chosen:

$$V(\psi^0) = \max_{e \in E} V(e \,|\, \psi^0). \tag{3}$$

The function $V(\cdot\,|\,\cdot)$ satisfies a single-crossing property. Suppose a non-debt contract $\psi^0 \in \Psi$ and a debt contract δ have the same expected value given effort e^*. Then programs (P-δ, e^*) and (P-ψ^0, e^*) are the same, since the right sides of their respective individual rationality constraints are the same. Thus, $V(e^*|\,\delta) = V(e^*|\,\psi^0)$. Now suppose $d > e^*$. Then, by Lemma 1, $\psi^0 \cdot p(d) > \delta \cdot p(d)$, so that the principal prefers ψ^0 to δ if she thinks the agent took effort d. This implies that there are more contracts that she will accept in lieu of δ than she will accept in lieu of ψ^0, given effort d. Accordingly, the agent is better off renegotiating δ than ψ^0 if he wants to take efforts higher than $e^* : V(d\,|\,\delta) \geq V(d\,|\,\psi^0)$ for $d > e^*$. A similar argument shows that $V(d\,|\,\delta) \leq V(d\,|\,\psi^0)$ for $d < e^*$.

This single-crossing property is not quite what we need. This is because it is about a debt contract that gives the principal the same expected payment as the initial contract ψ^0 given e^*, which is less than her payoff from a solution of (P-ψ^0, e^*) when (IR-ψ^0) does not bind. Thus, the principal is not necessarily better off when this debt contract rather than ψ^0 is renegotiated. The actual single-crossing property that we need is established in the following lemma.

[9] If the constraint set of (P-ψ^0, e) is empty, let $V(e\,|\,\psi^0) = -\infty$.

Lemma 4. *For any non-debt contract* $\psi^0 \in \Psi$, *effort* $e^* \in [\underline{e}, \bar{e}]$, *and debt contract* δ*: if* $\delta \cdot p(e^*) \geq \psi^0 \cdot p(e^*)$, *then*

(*i*) $V(d \,|\, \delta) \leq V(d \,|\, \psi^0)$ *for* $d \leq e^*$;

(*ii*) $V(d \,|\, \delta) < V(d \,|\, \psi^0)$ *for* $d < e^*$ *at which* (IR-ψ^0) *binds in* (P-ψ^0, d).

Proof. For $d \leq e^*$, Lemma 1 implies $\delta \cdot p(d) \geq \psi^0 \cdot p(d)$. So if (ψ, d) satisfies (IR-δ), it also satisfies (IR-ψ^0). This proves (*i*). If $d < e^*$, Lemma 1 implies $\delta \cdot p(d) > \psi^0 \cdot p(d)$. So if (ψ, d) satisfies (IC) and (IR-δ), it is feasible for (P-ψ^0, d), but it does not satisfy (IR-ψ^0) with equality. Thus, $U(\psi, d) < V(d \,|\, \psi^0)$ if (IR-ψ^0) binds in (P-ψ^0, d). This proves (*ii*). ■

We now show that both parties prefer to renegotiate debt. Let $\psi^0 \in \Psi$, and consider the case in which $A(\psi^0)$ contains a single outcome, say (ψ^*, e^*). The desired debt contract δ has the same expected value as ψ^* given e^*:[10]

$$\delta \cdot p(e^*) = \psi^* \cdot p(e^*). \tag{4}$$

Lemma 3 immediately implies that the agent weakly prefers to renegotiate the debt contract: $V(\delta) \geq V(\psi^0)$. We show that the principal agrees.

The interesting case is that in which the agent has a strict preference, so that $V(\delta) > V(\psi^0)$. Adopting δ as the initial contract instead of ψ^0 then results in the agent taking a greater effort. To see this, note that

$$\delta \cdot p(e^*) \geq \psi^0 \cdot p(e^*), \tag{5}$$

since (ψ^*, e^*) satisfies (IR-ψ^0) and δ satisfies (4). Lemma 4 (*i*) thus implies

$$V(d \,|\, \delta) \leq V(d \,|\, \psi^0) \text{ for } d \leq e^*. \tag{6}$$

[10] A unique debt contract satisfes (4). As ψ^* satisfies (LL), the RHS of (4) is no greater than $\pi \cdot p(e^*)$. The LHS increases continuously from $-\infty$ to $\pi \cdot p(e^*)$ as the face value of δ ranges from $-\infty$ to π_n.

Because the maximum of $V(\cdot \mid \delta)$ exceeds the maximum of $V(\cdot \mid \psi^0)$, we conclude that any effort that maximizes the former function must exceed the effort e^* that maximizes the latter. This proves the claim that $e > e^*$ for any $(\psi, e) \in A(\delta)$.

It is now easy to see that the principal is better off renegotiating δ than ψ^0. Let $(\psi, e) \in A(\delta)$. Hence, $e > e^*$. Since (ψ, e) satisfies (IR-δ), $\psi \cdot p(e) \geq \delta \cdot p(e)$. By (MLRP), $\delta \cdot p(e) \geq \delta \cdot p(e^*)$. These two inequalities and (4) imply

$$\psi \cdot p(e) \geq \psi^* \cdot p(e^*). \tag{7}$$

This is the desired result: the principal weakly prefers the outcome (ψ, e) of renegotiating δ to the outcome (ψ^*, e^*) of renegotiating ψ^0.

The principal strictly prefers to renegotiate the debt contract under the additional assumption that her payoff when ψ^0 is renegotiated exceeds the lowest possible profit: $\psi^* \cdot p(e^*) > \pi_1$. In this case, by (4), the face value of the debt exceeds π_1. Therefore δ is not riskless, and (MLRP) implies $\delta \cdot p(e) > \delta \cdot p(e^*)$. This makes the inequality in (7) strict, as claimed.

Theorem 1 builds on this logic and considers other cases. To handle cases in which $A(\psi^0)$ contains more than one outcome, denote the principal's greatest payoff from them as

$$P(\psi^0) \equiv \max_{(\psi,e) \in A(\psi^0)} \psi \cdot p(e). \tag{8}$$

Theorem 1. *Given any non-debt contract $\psi^0 \in \Psi$, a debt contract δ exists for which (i) $V(\delta) \geq V(\psi^0)$; (ii) $P(\delta) \geq P(\psi^0)$;*

(iii) All outcomes in $A(\delta)$ give the principal a payoff no less than $P(\psi^0)$ if $V(\delta) > V(\psi^0)$, or if (IR-ψ^0) binds in (P-ψ^0, d) for $d < e^*$; and

(iv) All outcomes in $A(\delta)$ give the principal a payoff greater than $P(\psi^0)$ if $V(\delta) > V(\psi^0)$ and $P(\psi^0) > \pi_1$.

Proof. Let (ψ^*, e^*) solve (8), so that $P(\psi^0) = \psi^* \cdot p(e^*)$. Let δ be the debt contract satisfying (4). Lemma 3 immediately implies (i). The argument in the text proves the first part of (iii), and (iv).

To prove (ii), note that the first part of (iii) implies $P(\delta) \geq P(\psi^0)$ if $V(\delta) > V(\psi^0)$. If instead $V(\delta) = V(\psi^0)$, then (ψ^*, e^*) solves (P-δ) because it is feasible for it. As this also implies $P(\delta) \geq P(\psi^0)$, (ii) is proved.

To prove the second part of (iii), let $(\psi, e) \in A(\delta)$ and assume (IR-ψ^0) binds in (P-ψ^0, d) for $d < e^*$. Then by Lemma 4 (ii), the inequality in (6) is strict for $d < e^*$. This shows that $e \geq e^*$. So (7) again holds, proving that the principal's payoff from (ψ, e) is not less than $P(\psi^0)$. ∎

6. Equilibrium Initial Contracts

To treat the adoption of an initial contract, consider a two-stage game in which the initial contract is determined in the first stage. For concreteness, assume the initial offer is also made by the agent as an ultimatum. (Results would be similar if the principal made the initial offer.) The game starts by the agent offering some $\psi^0 \in \Psi$. If the principal accepts it, she invests K and $\Gamma(\psi^0)$ is the ensuing subgame. If the principal rejects the offer, she receives a zero payoff, and the agent receives a reservation payoff $\bar{U}$. Denote this game as Γ.

The following is a rough construction of a perfect Bayesian equilibrium in which the initial contract is debt. For each $\psi^0 \in \Psi$, choose an outcome in $A(\psi^0)$ that gives the principal payoff $P(\psi^0)$, and let an equilibrium in subgame $\Gamma(\psi^0)$ be played that generates this outcome. The continuation payoffs if ψ^0 is accepted are then $V(\psi^0)$ and $P(\psi^0)$. Let the principal accept ψ^0 if and only if $P(\psi^0) \geq K$. The agent's best reply is then to offer initially a ψ^0 that maximizes $V(\psi^0)$ subject to $P(\psi^0) \geq K$. By Theorem 1 and Lemma 3, such an optimal contract is the acceptable debt contract with the smallest face value.

This construction has two problems. First, there may not be an initial contract the principal will accept, nor an acceptable initial contract that gives the agent his reservation payoff. The following assumption avoids this problem:[11]

$$\text{(A)} \quad V(\psi^0) > \bar{U} \text{ and } P(\psi^0) > K \text{ for some } \psi^0 \in \Psi.$$

Second, the set of acceptable offers, $\{\psi^0 \in \Psi \mid P(\psi^0) \geq K\}$, is not necessarily closed, and so may not contain a contract that maximizes V. However, on the set of debt contracts, V is nonincreasing in the face value of the debt. Hence, given any $\varepsilon > 0$, an acceptable debt contract δ exists such that $V(\delta) \geq V(\psi^0) - \varepsilon$ for all acceptable ψ^0. Letting the agent offer this δ yields a perfect Bayesian ε-equilibrium. This proves the following:

Theorem 2. *Assume* (A) *and let* $\varepsilon > 0$. *Then* Γ *has a perfect Bayesian* ε-*equilibrium in which a debt contract is initially offered and accepted.*

Remark 2. *Existence is not an issue if the number of contracts is finite. Then, if the grid of debt contracts is fine enough,* Γ *has an equilibrium in which the initial contract is debt. Even with a continuum of contracts, in all examples we have considered* Γ *has an equilibrium in which the initial contract is debt.*

An equilibrium (or ε-equilibrium) of Γ is generally not as efficient as would obtain if the parties could commit not to renegotiate. The following familiar program is a commitment benchmark:

$$(P) \quad U^* \equiv \max_{\psi \in \Psi, e \in E} U(\psi, e)$$
$$\text{subject to (IC) and } \psi \cdot p(e) \geq K.$$

[11] If all outcomes in $A(\psi^0)$ give the principal a payoff less than K, this construction yields an equilibrium in which the principal rejects any initial offer. It may then not be sensible to focus on the agent's best equilibria in the subgames. Other equilibria of some $\Gamma(\psi^0)$ may give the principal more than K, and so Γ may still have equilibria in which the initial offer is accepted.

This program retains the limited liability, monotonicity, and incentive constraints, and it requires the principal's payoff net of the start-up investment to be nonnegative. Assume $U^* > \bar{U}$. Then U^* is the agent's equilibrium payoff if he offers the initial contract as an ultimatum, and renegotiation is impossible. His equilibrium payoff in Γ may be less than this, as is shown in Section 7.

However, renegotiation is not harmful if the start-up cost can be surely paid back, i.e., if $K \leq \pi_1$. In this case the riskless debt contract

$$\delta^K \equiv (K, \ldots, K),$$

satisfies limited liability and so is feasible. Suppose it is the initial contract. Since $\delta^K \cdot p(e) = K$ for any $e \in E$, and the principal has the option of not renegotiating, every equilibrium of $\Gamma(\delta^K)$ gives her a payoff of at least K. Hence, in any perfect Bayesian equilibrium, accepting the initial offer of δ^K is a best reply for the principal. Furthermore, $V(\delta^K) = U^*$, since the programs (P-δ^K) and (P) are identical. This almost proves the following:[12]

Theorem 3. *If $K \leq \pi_1$, then Γ has a perfect Bayesian equilibrium in which δ^K is initially adopted, and the agent's payoff is U^*.*

Proof. In every subgame $\Gamma(\psi^0)$, let an equilibrium with outcome in $A(\psi^0)$ be played. Let the principal accept precisely the initial contracts for which her continuation payoff is not less than K, and let the agent offer δ^K. Denote this assessment as σ^*. The principal's continuation payoff if she accepts δ^K and then σ^* is played is at least K. So σ^* requires her to accept δ^K. The agent's payoff from σ^* is thus $V(\delta^K) = U^*$. Offering δ^K is better than making an unacceptable

[12] Theorem 3 is like a result in Matthews (1995), which studies a game like Γ but without the (LL) and (MON) constraints, and with the principal making the initial offer. That game has equilibria in which a "sales contract" like δ^K is adopted and renegotiated to the same outcome as would occur if renegotiation were impossible.

offer, since $U^* > \bar{U}$. Any acceptable offer ψ^0 leads to an outcome $(\psi^*, e^*) \in A(\psi^0)$ such that $\psi^* \cdot p(e^*) \geq K = \delta^K \cdot p(e^*)$, and so Lemma 3 implies $V(\delta^K) \geq V(\psi^0)$. Hence, δ^K is the agent's best initial offer. ■

Theorem 3 says that the inability to prevent renegotiation is not harmful if the riskless debt contract satisfies limited liability. This is not to say that (LL), or for that matter (MON), do not bind in the commitment program (P).

7. Examples

In the first example, effort is a continuous variable and the first order approach to agency problems is valid (Rogerson, 1985). Debt is the only optimal initial contract in this example. The second example illuminates further the structure of the model, and shows that the principal may receive positive rent.

7.1. First-Order Approach

The set of efforts is now $E = [0, 1]$. There are two polar distributions of profit, p^a and p^b. Both put positive probability on all profits, and p^a is better in the sense of (MLRP): the ratio p_i^a / p_i^b increases in i. The effort choice determines a convex combination of these distributions: $p(e) = ep^a + (1 - e)p^b$. The agent's cost function is $c(e) = \frac{1}{2}e^2$.

We show that an optimal initial contract must be debt, assuming the corresponding effort is interior. Let $\psi^0 \in \Psi$ be a non-debt contract. For an interior e, the following is necessary and sufficient for (ψ, e) to satisfy (IC):

$$U_e(\psi, e) = u(\pi - \psi) \cdot (p^a - p^b) - e = 0. \tag{9}$$

Replacing (IC) by (9), we obtain the Lagrangian for $(\text{P-}\psi^0, e)$:[13]

$$L(\psi, \mu, \lambda, \alpha, \beta \,|\, \psi^0, e) \equiv U(\psi, e) + \mu U_e(\psi, e) + \lambda(\psi - \psi^0) \cdot p(e) + \sum_{i=1}^{n} \alpha_i(\pi_i - \psi_i) + \sum_{i=1}^{n-1} \beta_i(\psi_{i+1} - \psi_i).$$

Let $\beta_0^* = \beta_n^* = 0$. A necessary condition that a solution $(\psi^*, \mu^*, \lambda^*, \alpha^*, \beta^*)$ must satisfy is

$$\left[p_i(e) + \mu^*(p_i^a - p_i^b)\right] u'(\pi_i - \psi_i^*) = \lambda^* p_i(e) + \beta_{i-1}^* - \beta_i^* - \alpha_i^* \qquad (10)$$

for $i = 1, ..., n$. The multipliers λ^*, α_i^*, and β_i^* are nonnegative. By setting $i = 1$ in (10), we see that μ^* and λ^* are not both zero.

Consider an equilibrium outcome $(\psi^*, e^*) \in A(\psi^0)$, with $e^* \in (0, 1)$. Then ψ^* solves $(\text{P-}\psi^0, e^*)$. Make the (generic) assumption that the set of constraints that bind in $(\text{P-}\psi^0, e)$ is the same for all e in a neighborhood of e^*. The envelope theorem then implies $V_e(e^* | \, \psi^0) = L_e(* \,|\, \psi^0, e^*)$, where the asterisk denotes the optimal arguments. Hence,

$$V_e(e^* | \, \psi^0) = -\mu^* + \lambda^*(\psi^* - \psi^0) \cdot (p^a - p^b). \qquad (11)$$

Since (ψ^*, e^*) solves $(\text{P-}\psi^0)$, $V_e(e^* | \, \psi^0) = 0$. Thus, (11) implies that neither μ^* nor λ^* is zero. This shows that $\lambda^* > 0$ – the principal's individual rationality constraint $(\text{IR-}\psi^0)$ binds.

Let δ be the debt contract that gives the principal the same payoff as ψ^*, given effort $e^* : \delta \cdot p(e^*) = \psi^* \cdot p(e^*)$. Then $\delta \cdot p(e^*) = \psi^0 \cdot p(e^*)$, which shows that $(\text{P-}\delta, e^*)$ and $(\text{P-}\psi^0, e^*)$ are the same program. Contract ψ^* thus solves $(\text{P-}\delta, e^*)$, and so $V(e^* | \, \delta) = V(e^* | \, \psi^0)$. Also, the optimal values of the multipliers are the same in the two programs. The envelope theorem therefore implies

$$V_e(e^* | \, \delta) = -\mu^* + \lambda^*(\psi^* - \delta) \cdot (p^a - p^b).$$

[13] The multipliers are μ for (9), λ for $(\text{IR-}\psi^0)$, $\alpha = (\alpha_1, ..., a_n)$ for (LL), and $\beta = (\beta_1, ..., \beta_{n-1})$ for (MON).

Subtracting (11) yields

$$V_e(e^*|\,\delta) - V_e(e^*|\,\psi^0) = \lambda^*(\psi^0 - \delta)\cdot(p^a - p^b). \tag{12}$$

Recalling that $p(1) = p^a$, $p(0) = p^b$, and $\lambda^* > 0$, we conclude from (12) and Lemma 1 that

$$V_e(e^*|\,\delta) > V_e(e^*|\,\psi^0).$$

This and $V(e^*|\,\delta) = V(e^*|\,\psi^0)$ imply $V(\delta) > V(\psi^0)$. This proves that the agent strictly prefers to renegotiate δ rather than ψ^0. Furthermore, if the principal finds ψ^0 acceptable as an initial contract, she also finds δ acceptable, by Theorem 1 (*ii*). We conclude that any optimal initial contract is debt. Given assumption (A), the game Γ has a perfect Bayesian equilibrium in which the initial contract is debt, and the initial contract is debt in every equilibrium in which outcomes in $A(\psi^0)$ are played in the subgames $\Gamma(\psi^0)$.

7.2. Edgeworth Box

In this section there are two possible profits, $\pi_1 = 1$ and $\pi_2 > \pi_1$, and two possible efforts, $\underline{e} = .3$ ("shirking") and $\bar{e} = .7$ ("working"). The effort is the probability π_2 occurs, so that $p_2(e) = e$. The cost of effort is $c(\underline{e}) = 0$ and $c(\bar{e}) = 1$. The agent's utility function is $u(y) = -1/y$ for $y > 0$.

A contract ψ gives the agent a contingent income vector

$$s = (s_1, s_2) = (\pi_1 - \psi_1, \pi_2 - \psi_2).$$

It is convenient to refer to either s or ψ as the contract. The Edgeworth box in Figure 1 depicts contracts.[14] The 45° line on the left depicts the *wage contracts* that perfectly insure the agent. The 45° degree line on the right depicts the

[14] Note that the horizontal axis denotes s_2, not s_1.

riskless debt contracts that perfectly insure the principal. Contracts in the agent's nonnegative orthant satisfy (LL); those to the left of the riskless debt line satisfy (MON). The riskless debt contracts above the horizontal axis are debt, as are the contracts on the horizontal axis to the left of the riskless debt line.

The agent prefers contracts to the northeast in Figure 1. The shallower (steeper) indifference curve is a level set of $U(\cdot, \underline{e})$ $(U(\cdot.\bar{e}))$, and it is relevant when the chosen effort is $\underline{e}$ $(\bar{e})$. The principal prefers contracts to the southeast. The steeper (shallower) downward-sloping line is an iso-return line for the principal when she believes the effort is $\bar{e}$ $(\underline{e})$.

The two indifference curves in Figure 1 yield the same payoff, labeled U^0. At their intersection, the agent's payoff is U^0 regardless of his effort. Refer to such contracts that make him indifferent about his effort as *IC contracts*. Their locus is the curve labeled IC.[15] The agent prefers $\bar{e}$ $(\underline{e})$ if the contract is below (above) the IC curve.

Consider Figure 2, and suppose s^0 is the initial contract. The unique equilibrium outcome of the renegotiation game $\Gamma(s^0)$ is $(\underline{s}, \underline{e})$. To see why, note first that $\underline{s}$ solves (P-$s^0, \underline{e}$), and $\bar{s}$ solves (P-$s^0, \bar{e}$). Given the indicated indifference curve, $U(\underline{s}, \underline{e}) > U(\bar{s}, \bar{e})$. Thus, the unique outcome in $A(s^0)$ is $(\underline{s}, \underline{e})$.[16]

Now consider shifting the initial contract in Figure 2 southeast from s^0, down the shallower iso-return line. As it shifts, the IC contract that has the same expected value as the initial contract, given $\bar{e}$, shifts upward along the IC curve. Eventually the agent prefers this IC contract to $\underline{s}$; this occurs in Figure 2 when the initial contract is s^{00}. The curve labeled WIC is the locus of such initial contracts.[17]

[15] For our parameters, the IC curve is the graph of $g(s_2) \equiv u^{-1}\left(u(s_2) + \frac{c}{\bar{e}-\underline{e}}\right) = \frac{.4s_2}{s_2+.4}$.

[16] It is easy to see that $(\underline{s}, \underline{e})$ is the only equilibrium outcome of $\Gamma(s^0)$. Regardless of her beliefs, the principal will agree to renegotiate s^0 to any wage contract below $\underline{s}$. Any equilibrium payoff for the agent is thus no less than $U(\underline{s}, \underline{e})$. Thus, $(\underline{s}, \underline{e})$ is the only equilibrium outcome.

[17] This curve is the set $\{(a(z), b(z))\}_{z \geq 0}$ determined by $u(\underline{e}a + (1-\underline{e})b) = \underline{e}u(z) + (1-\underline{e})u(g(z))$

If the initial contract is below the WIC curve, the agent's best equilibrium outcome is of the form $(s, \bar{e})$, with s on the IC curve. The further southeast the initial contract is on the indicated iso-return line, the greater is the agent's equilibrium payoff. The endpoint of this iso-return line is where either (LL) or (MON) binds, which is a debt contract. In Figure 2 (LL) binds, yielding the debt contract δ. When it is renegotiated, the equilibrium outcome in $A(\delta)$ is $(s^*, \bar{e})$. Both parties prefer it to $(\underline{s}, \underline{e}) \in A(s^0)$, as Theorem 1 requires.

Referring still to Figure 2, one equilibrium of $\Gamma(\delta)$ that yields $(s^*, \bar{e})$ starts with the agent proposing s^* and choosing $\bar{e}$. The principal's beliefs, given any proposal s, assign probability one to effort $\bar{e}$ ($\underline{e}$) if s is on or below (above) the IC curve. Thus, the principal thinks the agent would only propose a new contract if he thought it would be accepted. Given these beliefs, s^* is the agent's best proposal in the set that the principal will accept.

The equilibrium outcome $(s^*, \bar{e})$ of $\Gamma(\delta)$ shown in Figure 2 may or may not be the commitment outcome, the solution of (P). It is the commitment outcome if the (unshown) riskless debt contract δ^K lies on the steep iso-return line containing s^*, as then $(s^*, \bar{e})$ gives the principal a payoff of K. (Her individual rationality constraint in (P) binds.) In this case the inability to prevent renegotiation is not harmful, even if δ^K is infeasible.

An interesting case is shown in Figure 3.[18] The commitment outcome is $(s^*, \bar{e})$. Because δ^K violates (LL), it cannot be initially adopted to yield $(s^*, \bar{e})$. In fact, no s^0 that is feasible can be renegotiated to the commitment outcome. For, if $(s^*, \bar{e})$ is an equilibrium outcome of $\Gamma(s^0)$, then s^0 must be both above the WIC curve and, in order for the principal to agree to renegotiate it to s^*, to the right

and $\bar{e}a + (1-\bar{e})b = \bar{e}z + (1-\bar{e})g(z)$. Hence, $a(z) = \frac{8z+34z^2+42.875z^3}{8+34z+35z^2}$ and $b(z) = \frac{8z+14z^2-18.375z^3}{8+34z+35z^2}$.

[18]Figure 3 depicts the case $\pi_2 = 2.7$ and $K = 1.2$. The indicated contracts are $(\underline{s}_2, \underline{s}_1) = (.31, .31)$, $(\bar{s}_2, \bar{s}_1) = (1.143, .296)$, and $(s_2^*, s_1^*) = (1.284, .305)$. The δ is $(s_2, s_1) = (1.270, 0)$.

of the iso-revenue line containing s^*. Every such s^0 violates (LL). So in this case the inability to prevent renegotiation is harmful.

Suppose now in Figure 3 that the initial contract is the debt contract δ. As it is on the WIC curve, the unique equilibrium outcome in $A(\delta)$ is $(\bar{s}, \bar{e})$. (Uniqueness holds because $U(\bar{s}, \bar{e}) > (\underline{s}, \underline{e})$.) The game Γ thus has an equilibrium in which δ is initially offered and accepted, the effort chosen is $\bar{e}$, and δ is renegotiated to $\bar{s}$. The principal's gross payoff, $(\pi - s) \cdot p(\bar{e})$, is strictly greater than $(\pi - \delta^K) \cdot p(\bar{e}) = K$. That is, the principal's equilibrium rent is positive.

8. Random Contracts

Random contracts may be of value in this model, as we now discuss.

A *random contract*, a, specifies for each possible contract ψ a probability $a(\psi)$ that ψ will be the actual sharing rule. The same legal/wealth considerations that motivate the limited liability constraint require the agent to never pay more than the realized profit – any contract in the support of a random contract should then satisfy (LL). Assuming the random contract determines a final contract before the profit is known, the same ex post moral hazard motivations for imposing monotonicity also imply that contracts in the support of a should satisfy (MON). We thus consider a random contract to be a distribution of contracts satisfying these constraints: $a \in \Delta(\Psi)$.

Without loss of generality, attention can be restricted to random contracts with finite supports.[19] An allocation with a random contract, (a, e), generates payoffs for the agent and principal, respectively, of $U(a, e) \equiv \sum_{\psi \in \Psi} a(\psi) U(\psi, e)$ and $P(a, e) \equiv \sum_{\psi \in \Psi} a(\psi) (\psi \cdot p(e))$.

The standard argument for why a random contract lowers efficiency is the

[19] As the number of possible profits is $n < \infty$, Carathedory's Theorem implies there is no need to consider distributions that put positive probability on more than $n + 1$ contracts.

following. The certainty equivalent of a random contract a is the contract ψ^a that has components defined by

$$u(\pi_i - \psi_i^a) \equiv \sum_{\psi \in \Psi} a(\psi) u(\pi_i - \psi_i). \tag{13}$$

Since $U(\psi^a, e) = U(a, e)$ for all $e \in E$, the contract ψ^a provides the same incentives as does a. Since the risk-neutral principal gains the risk premium created by removing contract risk, $P(\psi^a, e) > P(a, e)$ for all $e \in E$. Both parties therefore gain by switching from a to ψ^a.

This insurance argument fails here because the certainty equivalent contract may not be monotonic. Whether it is depends on whether the agent exhibits increasing or decreasing absolute risk aversion. Let $r(y) = -u''(y)/u'(y)$.

Lemma 5. *Let ψ^a be the certainty equivalent of a random contract $a \in \Delta(\Psi)$. Then* (*i*) *ψ^a satisfies* (LL); (*ii*) *ψ^a satisfies* (MON) *if r is nondecreasing; and* (*iii*) *ψ^a does not satisfy* (MON) *if r is decreasing and, for some $i < n$, all ψ in the support of a satisfy $\psi_i = \psi_{i+1}$.*

Proof. As each ψ in the support of a satisfies (LL), (13) implies $u(\pi_i - \psi_i^a) \geq u(0)$. Hence $\pi_i \geq \psi_i^a$, proving (*i*). Now fix $i < n$. Define $g : [0, 1] \to \Re$ by

$$u[t\pi_{i+1} + (1-t)\pi_i - g(t)] \equiv \sum_{\psi \in \Psi} a(\psi) u[t(\pi_{i+1} - \psi_{i+1}) + (1-t)(\pi_i - \psi_i)]. \tag{14}$$

Then $g(0) = \psi_i^a$ and $g(1) = \psi_{i+1}^a$. Let

$$\lambda(\psi, t) \equiv \frac{u'[t(\pi_{i+1} - \psi_{i+1}) + (1-t)(\pi_i - \psi_i)]}{u'[t\pi_{i+1} + (1-t)\pi_i - g(t)].}$$

Differentiating (14) yields $g'(t) = T1 + T2$, where

$$T1 = (\pi_{i+1} - \pi_i) \left(1 - \sum_{\psi \in \Psi} a(\psi) \lambda(\psi, t) \right),$$

$$T2 = \sum_{\psi \in \Psi} a(\psi) \lambda(\psi, t)(\psi_{i+1} - \psi_i).$$

If r is nondecreasing (increasing), u' is concave (strictly convex) in u. So by (14),

$$\sum_{\psi\in\Psi} a(\psi)\lambda(\psi,t) \begin{cases} \leq 1 & \text{if } r \text{ is nondecreasing} \\ > 1 & \text{if } r \text{ is decreasing.} \end{cases}$$

Thus, $T1 \geq 0$ if r is nondecreasing, and $T1 < 0$ if r is decreasing. As $\lambda(\psi,t) > 0$ and each $\psi \in \Psi$ satisfies (MON), $T2 \geq 0$. Hence, $g' \geq 0$ if r is nondecreasing, and (ii) is proved. If $\psi_i = \psi_{i+1}$ for each ψ in the support of a, $T2 = 0$. In this case, if r is decreasing, $g' < 0$. This proves (iii). ■

Lemma 5 shows that random contracts may have value if, as is plausible, the agent exhibits decreasing absolute risk aversion. This motivates our interest in random contracts.

There are two ways of putting them in the model. The first is simply to let contracts be random. This does not change the results regarding initial contracts, largely because the single-crossing property of Lemma 1 still holds. To see this, let $\bar{\psi} = \sum_{\psi\in\Psi} a(\psi)\psi$, and note that the convexity of constraints (LL) and (MON), together with $a \in \Delta(\Psi)$, implies $\bar{\psi} \in \Psi$. Since $P(a,e) = \bar{\psi}\cdot p(e)$, Lemma 1 therefore implies its extension:

Lemma 6. *Let $a \in \Delta(\Psi)$ and $e \in [\underline{e},\bar{e}]$. If $\delta \neq \sum_{\psi\in\Psi} a(\psi)\psi$ is a debt contract such that $P(a,e) = \delta\cdot p(e)$, then $P(a,d) \lesseqgtr \delta\cdot p(d)$ for $d \lesseqgtr e$.*

The analysis now proceeds as before, with each instance of ψ^0 replaced by a^0, and ψ by a. A program (P-a^0) is obtained in which the agent chooses $(a,e) \in \Delta(\Psi)\times E$ to maximize his payoff subject to an incentive constraint, $U(a,e) \geq U(a,d)$ for all $d \in E$, and an individual rationality constraint, $P(a,e) \geq P(a^0,e)$.[20]

[20] Allowing the initial contract to be random is unimportant. The principal is willing to renegotiate a^0 to a if and only if she is willing to renegotiate the mean of a^0, say $\bar{\psi}^0$, to a (holding her beliefs fixed).

A solution is one of the agent's best equilibrium outcomes of the game $\Gamma(a^0)$. Program (P-ψ^0, e) becomes (P-a^0, e), with its choice domain expanded to $\Delta(\Psi)$. Because of Lemma 6, the value of this program, $V(\cdot \mid a^0)$, satisfies the single-crossing property of Lemma 4. This yields the precise analog of Theorem 1. Hence, for any initial contract $a^0 \in \Delta(\Psi)$, a debt contract δ exists such that both parties prefer some, and perhaps all, the equilibrium outcomes in $A(\delta)$ to any outcome in $A(a^0)$. The analog of Theorem 2 also holds, so that the game Γ (with random contracts) has a perfect Bayesian ε-equilibrium in which the initial contract is debt. Finally, the analog of the efficiency Theorem 3 also holds, except that now the feasible set of the commitment program (P) contains random contracts (and hence has a possibly higher value).

However, the writing and enforcement of a random contract – a probability device – may be problematic. *De facto* randomization is alternatively achieved by an equilibrium of $\Gamma(\psi^0)$ in which the principal uses a mixed strategy. If she accepts the equilibrium renegotiation proposal ψ with positive probability less than one, the outcome is a random contract putting positive probability on both ψ^0 and ψ.

Because random contracts may be of value, $\Gamma(\psi^0)$ may possibly have a mixed strategy equilibrium of this type that the agent prefers to any pure strategy equilibrium.[21] However, his equilibrium payoff may not be as high as when a random contract can be directly adopted, for two reasons. For if (a, e) is an equilibrium outcome of $\Gamma(\psi^0)$, then a puts probability on at most two contracts, ψ^0 and the proposal ψ. Also, if a is not degenerate the principal must be indifferent between ψ^0 and ψ, given e.

The first of these restrictions can be eliminated by allowing the agent to pro-

[21] We are not discussing equilibria in which the effort strategy is mixed. Their nature and existence are open questions.

pose more than one new contract. After he proposes a set of new contracts, the principal either accepts one of them, or she rejects them all in favor of the initial contract. This is a natural generalization of an ultimatum game, and its equilibrium payoffs may be higher if random contracts that put positive probability on more than two contracts are of value.

This generalized ultimatum game has another nice feature. Suppose an equilibrium of it generates a random contract a that puts positive probability on both the initial contract ψ^0 and the set of proposed contracts, a subset θ of Ψ. If the initial contract is instead not ψ^0, the random contract a may possibly still be obtained, as the agent can propose the larger set of contracts, $\theta \cup \{\psi^0\}$. This implies that for the generalized ultimatum game, the revealed preference result of Lemma 3 holds for equilibria with mixed selection strategies.[22]

The generalized ultimatum game is studied in the Appendix. Its equilibria are shown to satisfy the natural generalizations of our results. In particular, given any initial contract, a debt contract exists that both parties prefer to renegotiate instead, in the sense of Theorem 1.

9. Concluding Remarks

We have explored the effects of two restrictions on sharing rules in a moral hazard environment, limited liability for the agent, and a nonnegative dependence on the realized profit of the payment to the principal. Innes (1990) shows that under these restrictions, debt is the optimal incentive contract if the agent and the principal are risk neutral. In this paper we have shown that Innes' result generalizes, in a sense, to when the agent is risk averse, provided the parties cannot prevent

[22] Lemma 3 does not hold for mixed strategy equilibria of $\Gamma(\psi^0)$. If a mixed strategy equilibrium has an outcome a that puts positive probability on both ψ^0 and $\psi \neq \psi^0$, and if $\psi^1 \notin \{\psi^0, \psi\}$, then a is an infeasible outcome of game $\Gamma(\psi^1)$.

themselves from renegotiating. In this case the optimal initial contract, rather than the final contract, is debt. This debt contract is generally renegotiated to a non-debt contract that gives the agent better insurance, as when an entrepreneur of a start-up decides to sell equity in a public offer. The inability to prevent renegotiation is not harmful if the riskless debt contract that requires the start-up cost to be surely repaid satisfies the limited liability constraint. Otherwise the inability to commit to not renegotiating may be harmful.

These results depend on two features of the model. First, contracts are single sharing rules rather than menus, as discussed in the introduction. Menu contracts must in general be assumed infeasible, as they can sometimes perform better in our setting than do single sharing rules.[23]

Second, the agent has the bargaining power in the renegotiation. If instead the principal makes the renegotiation proposal, the agent's effort strategy is mixed in any equilibrium in which he does not take the least costly effort (Fudenberg and Tirole, 1990; Ma, 1991), and debt has no special role. The question of who actually makes (binding) offers in any particular application is thus important. Since renegotiation is less harmful if the agent has the bargaining power, (Theorem 3; Ma, 1994; Matthews, 1995), there are ex ante incentives to adopt, if possible, renegotiation mechanisms in which offers are made by agents. However, committing to such a renegotiation mechanism may be problematic. An interesting task for future research is to determine the effects of realistic bargaining rules in which both parties make offers, as in an alternating-offer game.

[23] In the Edgeworth box example of Section 7, if the initial contract can be the menu $(\underline{s}, \bar{s})$, the ensuing game has an equilibrium in which renegotiation does not occur, and the agent chooses $\bar{e}$ and selects $\bar{s}$ from the menu. This achieves the efficent outcome $(\bar{s}, \bar{e})$ even if the riskless debt contract δ^K is infeasible.

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A. Appendix

This appendix concerns the generalized ultimatum game in which the agent proposes a set of contracts, as introduced in Section 8. Proposals can be taken to be finite sets (see the proof of Lemma A1 (i) below). So the set of feasible proposals can be taken to be the set of all finite subsets of Ψ – denote it as Θ.

The generalized renegotiation game in which proposals can be sets, given an initial contract ψ^0, is denoted $\Gamma^s(\psi^0)$. It starts with the agent choosing an effort $e \in E$ and making a proposal $\theta \in \Theta$. The principal does not observe e. The principal then selects a contract from θ, or she selects ψ^0.

The principal's strategy is a selection rule $s(\cdot|\cdot)$: if θ is proposed, she selects $\psi \in \theta$ with probability $s(\psi|\theta) \geq 0$, where $\sum_{\psi \in \theta \cup \{\psi^0\}} s(\psi|\theta) = 1$ for all $\theta \in \Theta$. Her beliefs about the agent's effort, given that he proposed θ, are given by a probability distribution $\beta(\cdot\,|\theta)$ on E. An *assessment* is a profile of strategies and beliefs, (θ, e, s, β). A *perfect Bayesian equilibrium* is an assessment satisfying sequential rationality, and Bayes' rule on the equilibrium path.

It is convenient to focus on equilibria in which the principal surely selects from the equilibrium proposal. There is no loss of generality in doing this. Any equilibrium in which the principal responds to the equilibrium proposal θ by selecting ψ^0 with positive probability is equivalent to another equilibrium in which the proposal is $\theta \cup \{\psi^0\}$, and the principal surely selects from it. Accordingly, "equilibrium" henceforth refers to a perfect Bayesian equilibrium (θ, e, s, β) for which $\sum_{\psi \in \theta} s(\psi|\theta) = 1$ holds.[24]

An equilibrium (θ, e, s, β) determines a random contract, $a(\cdot) \equiv s(\cdot|\theta)$. By the previous paragraph, we may assume $a \in \Delta(\theta)$. The equilibrium's *outcome* is

[24] Of course, some non-equilibrium proposals are rejected in any equilibrium, i.e., $\sum_{\psi \in \hat{\theta}} s(\psi|\hat{\theta}) < 1$ holds for some $\hat{\theta} \neq \theta$.

(θ, a, e), and the set of possible outcomes is

$$\mathcal{A} \equiv \{(\theta, a, e) \,|\, \theta \in \Theta,\ e \in E,\ a \in \Delta(\theta)\}.$$

Outcome $A = (\theta, a, e)$ gives the agent and principal, respectively, payoffs of $U^A \equiv \sum_{\psi \in \theta} a(\psi) U(\psi, e)$ and $P^A \equiv \sum_{\psi \in \theta} a(\psi) (\psi \cdot p(e))$.

Any equilibrium outcome $A = (\theta, a, e)$ satisfies an incentive constraint requiring effort e to be a best reply:

$$(\text{IC}^s) \quad U^A \geq \sum_{\psi \in \theta} a(\psi) U(\psi, d) \text{ for all } d \in E.$$

It must also satisfy an acceptance constraint,

$$(\text{AC}) \quad P^A \geq \psi \cdot p(e) \text{ for all } \psi \in \theta,$$

in order for a to be a best reply, i.e., in order for a to select with positive probability only contracts in θ that give the principal the greatest expected payment given e. The principal must also not prefer the initial contract:

$$(\text{IR}^s\text{-}\psi^0) \quad P^A \geq \psi^0 \cdot p(e).$$

Refer to an equilibrium of $\Gamma^s(\psi^0)$ as an *E-equilibrium* if no other equilibrium gives the agent a greater payoff. As the following lemma shows, the E-equilibrium outcomes solve the following program:

$$(\text{P}^s\text{-}\psi^0) \quad V(\psi^0) \equiv \operatorname*{Max}_{A \in \mathcal{A}} U^A \ \text{ such that } (\text{IC}^s), (\text{AC}), \text{ and } (\text{IR}^s\text{-}\psi^0).$$

Let $A^s(\psi^0)$ be the set of outcomes that solve $(\text{P}^s\text{-}\psi^0)$.

Lemma A1. *For any $\psi^0 \in \Psi$, (i) $A^s(\psi^0) \neq \emptyset$, and (ii) $A \in A^s(\psi^0)$ if and only if A is an E-equilibrium outcome of $\Gamma^s(\psi^0)$.*

Proof of Lemma A1 (i)**.** As E is compact and U is continuous, $U(\psi^0, e)$ has a maximizer in E, say e^0. The constraint set of $(\text{P}^s\text{-}\psi^0)$ is thus non-empty, as $(\{\psi^0\}, e^0, a^0)$, where $a^0(\psi^0) = 1$, satisfies all the constraints.

We now show that proposals can be restricted to at most $n+1$ contracts. Suppose $A=(\theta,a,e)$ is a solution, and $|\theta|>n+1$. If $a(\psi)=0$ for some $\psi\in\theta$, then $(\theta\backslash\{\psi\},e,a)$ is solution. We can thus assume $a(\psi)>0$ for all $\psi\in\theta$. By Carathedory's Theorem, the n-vector $\sum_{\psi\in\theta}a(\psi)u(\pi-\psi)$ is a convex combination of at most $n+1$ of the vectors in $\{u(\pi-\psi)\}_{\psi\in\theta}$. Let $\theta'\subset\theta$ be the set of contracts receiving positive weight in this combination, and let the weights be $a'(\psi)$. Thus,

$$\sum_{\psi\in\theta'}a'(\psi)u(\pi-\psi)=\sum_{\psi\in\theta}a(\psi)u(\pi-\psi). \tag{A1}$$

Let $B\equiv(\theta',e,a')$. Since $a'\in\Delta(\theta')$, $B\in\mathcal{A}$. For any $d\in E$, (θ',d,a') gives the agent the same payoff as does (θ,d,a), by (A1). So B satisfies (IC^s) because A does. Since A satisfies (AC) and $a(\psi)>0$ for all $\psi\in\theta$, $P^A=\psi\cdot p(e)$ for any $\psi\in\theta$. It follows that $P^B=P^A$. Hence, B satisfies (AC) and (IR^s-ψ^0) because A does. This shows that B is feasible for (P^s-ψ^0), and hence solves it because $U^B=U^A$. We conclude that (P^s-ψ^0) has a solution iff it has a solution in which the proposal has no more than $n+1$ contracts.

The contract space can be bounded. For each $j\le n$, define

$$m_j\equiv\psi^0_j+\min_{e\in E}\,p_j(e)^{-1}\sum_{i\neq j}(\psi^0_i-\pi_i)p_i(e).$$

This m_j is well-defined, since p is continuous and positive on the compact set E. If ψ satisfies $\psi_j<m_j$ for some j, then for any $e\in E$,

$$\begin{aligned}\psi\cdot p(e)&<m_jp_j(e)+\sum_{i\neq j}\psi_ip_i(e)\\&\le\psi^0_jp_j(e)+\sum_{i\neq j}(\psi^0_i-\pi_i+\psi_i)p_i(e)\le\psi^0\cdot p(e),\end{aligned}$$

So the principal never selects such a contract. Thus, (P^s-ψ^0) has a solution iff the program obtained by restricting proposals to

$$\overline{\Theta}\equiv\{\theta\in\Theta\,|\,\psi_i\ge m_i\;\forall\psi\in\theta\text{ and }i\le n,\text{ and }|\theta|\le n+1\}$$

has a solution. The constraint set of this restricted program is closed and finite dimensional. It is bounded, since (a) E is compact; (b) the feasible selection rules a can be identified with the simplex Δ^n; and (c) $\psi_i \in [m_i, \pi_i]$ for all $\psi \in \overline{\Theta}$ and $i \leq n$. A solution therefore exists. ■

Proof of Lemma A1 *(ii)***.** First, preliminaries. For $\theta \in \Theta$, let $\theta^0 \equiv \theta \cup \{\psi^0\}$. Define a relation $\succcurlyeq_\theta$ on $\Delta(\theta^0)$ by $a \succcurlyeq_\theta a'$ iff for all $e \in E$,

$$\sum_{\psi \in \theta^0} a(\psi)(\psi \cdot p(e)) \geq \sum_{\psi \in \theta^0} a'(\psi)(\psi \cdot p(e)). \tag{A2}$$

Define the corresponding dominance relation by $a \succ_\theta a'$ iff $a \succcurlyeq_\theta a'$ and (A2) is strict for some $e \in E$. Let a^0 be the degenerate distribution selecting the initial contract: $a^0(\psi^0) = 1$. By Zorn's lemma, $\left\{a \in \Delta(\theta^0) \,|\, a^\theta \succcurlyeq_\theta a^0\right\}$ contains an undominated element; refer to it as a^θ. A separating hyperplane argument shows that a^θ is a best reply to some beliefs about effort, i.e., $\mu^\theta \in \Delta(E)$ exists such that

$$\int_E \sum_{\psi \in \theta^0} a^\theta(\psi)(\psi \cdot p(e))\, d\mu^\theta(e) \geq \int_E (\psi \cdot p(e))\, d\mu^\theta(e) \text{ for all } \psi \in \theta^0. \tag{A3}$$

Let $A = (\theta^A, a^A, e^A) \in A^s(\psi^0)$. Define a strategy s and belief β as follows: $s(\cdot|\theta^A) \equiv a^A(\cdot)$ and $\beta(e^A \,|\, \theta^A) \equiv 1$, and $(s(\cdot|\theta), \beta(\cdot|\theta)) \equiv (a^\theta, \mu^\theta)$ for $\theta \in \Theta \backslash \{\theta^A\}$. The assessment $(\theta^A, e^A, s, \beta)$ has outcome A, and satisfies Bayes rule on its path. We show it is an equilibrium.

Since A satisfies (AC) and (IR^s-ψ^0), and (a^θ, μ^θ) satisfies (A3), (s, β) is sequentially rational. So we need only show that (θ^A, e^A) is a best reply to s. The agent's payoff if he does not deviate is U^A. His payoff is no higher if he proposes θ^A and chooses any $e \neq e^A$, since A satisfies (IC^s). Suppose he instead deviates to (θ, e), where $\theta \neq \theta^A$. Letting $\theta^0 \equiv \theta \cup \{\psi^0\}$, his resulting payoff is

$$U = \left[\sum_{\psi \in \theta^0} a^\theta(\psi) u(\pi - \psi)\right] \cdot p(e) - c(e). \tag{A4}$$

Let $\bar{\psi} \equiv \sum_{\psi \in \theta^0} a^\theta(\psi)\psi$. Constraints (LL) and (MON) define a convex set, and so $\bar{\psi} \in \Psi$. By Jensen's inequality,

$$U \leq u(\pi - \bar{\psi}) \cdot p(e) - c(e). \tag{A5}$$

Consider outcome $B \equiv (\bar{\psi}, \bar{a}, \bar{e})$, where $\bar{a}(\bar{\psi}) = 1$, and $\bar{e}$ is the agent's best effort given $\bar{\psi}$. By (A5), $U \leq U^B$. Clearly, B satisfies (IC^s). It satisfies (AC) vacuously. It satisfies (IR^s-ψ^0) because

$$\begin{aligned} P^B &= \sum_{\psi \in \theta^0} a^\theta(\psi)(\psi \cdot p(\bar{e})) \\ &\geq \sum_{\psi \in \theta^0} a^0(\psi)(\psi \cdot p(\bar{e})) = \psi^0 \cdot p(\bar{e}), \end{aligned}$$

where the inequality is due to $a^\theta \succsim_\theta a^0$. This shows B is feasible for (P^s-ψ^0), and so $U^B \leq U^A$ (since A solves (P^s-ψ^0)). Thus, $U \leq U^A$.

We conclude that $(\theta^A, e^A, s, \beta)$ is an equilibrium. Its outcome, A, is one of the agent's best equilibrium outcomes, since any equilibrium outcome satisfies the constraints of (P^s-ψ^0), and A solves it. Thus, A is an E-equilibrium outcome.[25]

To prove the converse, let A be an E-equilibrium outcome of $\Gamma^s(\psi^0)$. Then it satisfies the constraints of (P^s-ψ^0). Let B be a solution of (P^s-ψ^0), which exists by part (*i*). As just shown, B is an E-equilibrium outcome. The agent is therefore indifferent between A and B, and so A solves (P^s-ψ^0). ■

As in the proof of Lemma 2 in the text, conservative beliefs were used in the proof of Lemma A1, but other beliefs could have been used as well.

If the principal uses a pure strategy in some equilibrium, its outcome is also generated by an equilibrium of the game considered in the text in which the renegotiation proposal must be a singleton. The following proposition indicates

[25] But recall the restriction to equilibria in which the agent plays a pure strategy. An open question is whether he could prefer a mixed strategy equilibrium to A.

two sufficient conditions for this to be the case, both of which are restrictive. Given outcome $A = (\theta, a, e)$, say the principal *selects deterministically* if she surely selects just one contract: $a(\psi) = 1$ for some $\psi \in \theta$. If she selects deterministically, dropping the non-selected contracts yields an equivalent equilibrium outcome with a singleton proposal, $\theta = \{\psi\}$.

Proposition A1. *(i) If* $-u''/u'$ *is nondecreasing, then* (P^s-ψ^0) *has a solution in which the principal selects deterministically. (ii) If* $n = 2$*, the principal selects deterministically in every solution of* (P^s-ψ^0).[26]

Proof. (*i*) Let $A = (\theta, a, e) \in A^s(\psi^0)$. By Lemma 5 (*i*) and (*ii*), the certainty equivalent contract ψ^a defined by

$$u(\pi_i - \psi_i^a) \equiv \sum_{\psi \in \theta} a(\psi) u(\pi_i - \psi_i) \tag{A6}$$

is in Ψ. Let $B = (\psi^a, a^B, e)$, where $a^B(\psi^a) = 1$. We show that B solves (P^s-ψ^0). By Jensen's inequality and (A6), $P^B = \psi^a \cdot p(e) \geq P^A$. Thus, since A satisfies (AC) and (IR^s-ψ^0), so does B. Since A satisfies (IC^s), and (A6) implies that

$$U(\psi^a, d) = \sum_{\psi \in \theta} a(\psi) U(\psi, d) \text{ for all } d \in E,$$

B also satisfies (IC^s). So B is feasible for (P^s-ψ^0). Thus, as $U^B = U^A$ and A solves (P^s-ψ^0), so does B. This proves (*i*).

(*ii*) Let $A = (\theta, a, e)$ solve (P^s-ψ^0). Assume $S(a) \equiv \{\psi \,|\, a(\psi) > 0\}$ is not a singleton. For $\psi \in S(a)$, (AC) implies $\psi \cdot p(e) = P^A$, and so (MON) implies $\psi_1 \leq P^A$ and $\psi_2 \geq P^A$. Both inequalities are strict for some $\psi \in S(a)$, else $S(a)$ would be a singleton.

[26] Because the examples in Section 7 have $n = 2$, this shows that they also illustrate equilibrium outcomes of $\Gamma^s(\psi^0)$.

Again let (A6) define the certainty equivalent contract ψ^a. By Lemma 5 (i), it satisfies (LL). Since $\psi_1 \leq P^A$ for all $\psi \in S(a)$, and $\psi_1 < P^A$ for some $\psi \in S(a)$,

$$\begin{aligned} u(\pi_1 - \psi_1^a) &= \sum_{\psi \in \theta} a(\psi) u(\pi_1 - \psi_1) \\ &> u(\pi_1 - P^A). \end{aligned}$$

So $\psi_1^a < P^A$. A similar argument proves $\psi_2^a > P^A$. So ψ^a satisfies (MON) strictly. Thus, $\psi^a \in \Psi$.

Let $B = (\psi^a, a^B, e)$, where $a^B(\psi^a) = 1$. As in the proof of (i), B must solve (P^s-ψ^0). Since a is not degenerate and u is strictly concave, (A6) and Jensen's inequality imply $P^B = \psi^a \cdot p(e) > P^A$. Thus, as A satisfies (IR^s-ψ^0), $P^B > \psi^0 \cdot p(e)$. We conclude that the solution B of (P^s-ψ^0) satisfies its constraint (IR^s-ψ^0) with slack. This is easily shown to be impossible, since ψ^a is strictly monotonic (see Remark 1 in the text). Hence, $S(a)$ must be a singleton, and this proves (ii). ∎

We now show that the results in the text hold for this generalization. The revealed preference result, Lemma 3, becomes the following.

Lemma A2. *Let ψ^0 and ψ^1 be any contracts, and let $A = (\theta, a, e) \in A^s(\psi^0)$. Assume $P^A \geq \psi^1 \cdot p(e)$. Then $V(\psi^1) \geq U^A$.*

Proof. By hypothesis, A satisfies (IR^s-ψ^1). It satisfies (IC^s) and (AC), as it solves (P^s-ψ^0) by Lemma A1. So A is feasible for (P^s-ψ^1) The result follows. ∎

The analysis now proceeds as in the text. Define

$$(\text{P}^s\text{-}\psi^0, e) \quad V^s(e \,|\, \psi^0) \equiv \max_{\theta, a} U(a, e) \text{ subject to } a \in \Delta(\theta),$$
$$(\text{IC}^s), \text{ (AC), and } (\text{IR}^s\text{-}\psi^0).$$

Then $V^s(\psi^0) = \max_{e \in E} V^s(e \,|\, \psi^0)$. The function $V^s(\cdot \,|\, \cdot)$ satisfies the single-crossing properties of Lemma 4, by virtually the same proof. This leads to the following analog of Theorem 1, again with virtually the same proof. Let

$$P^s(\psi^0) \equiv \max_{A \in A^s(\psi^0)} P^A.$$

Theorem A1. *Given any non-debt contract* $\psi^0 \in \Psi$*, a debt contract* δ *exists for which* (*i*) $V^s(\delta) \geq V^s(\psi^0)$; (*ii*) $P^s(\delta) \geq P^s(\psi^0)$;

(*iii*) *All outcomes in* $A^s(\delta)$ *give the principal a payoff no less than* $P^s(\psi^0)$ *if* $V^s(\delta) > V^s(\psi^0)$, *or if* ($\mathrm{IR}^s$-$\psi^0$) *binds in* ($\mathrm{P}^s$-$\psi^0, d$) *for* $d < e^*$; *and*

(*iv*) *All outcomes in* $A^s(\delta)$ *give the principal a payoff greater than* $P^s(\psi^0)$ *if* $V^s(\delta) > V^s(\psi^0)$ *and* $P^s(\psi^0) > \pi_1$.

Now let Γ^s denote the two-stage game in which the agent offers the initial contract $\psi^0 \in \Psi$ as an ultimatum, and if the principal accepts it she invests K and $\Gamma^s(\psi^0)$ is played. If the principal rejects the offer, her payoff is zero and the agent's is $\bar{U}$. The analog of assumption (A) is

$$(A) \quad V^s(\psi^0) > \bar{U} \text{ and } P^s(\psi^0) > K \text{ for some } \psi^0 \in \Psi.$$

The following is the analog of Theorem 2, and it too has virtually the same proof.

Theorem A2. *Assume* (A) *and let* $\varepsilon > 0$. *Then* Γ^s *has a perfect Bayesian* ε-*equilibrium in which a debt contract is initially offered and accepted.*

Finally, the analog of Theorem 3 is

Theorem A3. *If* $K \leq \pi_1$, *then* Γ^s *has a perfect Bayesian equilibrium in which* δ^K *is initially adopted, and the agent's payoff is no less than* U^*.

This theorem, unlike Theorem 3, does not rule out the possibility of an equilibrium payoff for the agent which exceeds that of the commitment benchmark (P). This is of course because an equilibrium outcome of Γ^s, unlike a solution of (P), can indirectly entail the use of a random contract.

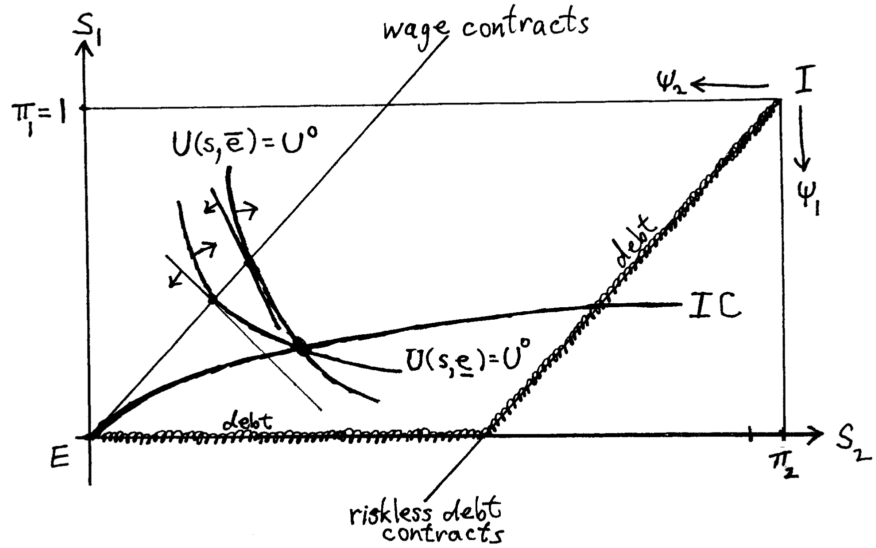

Figure 1: The two-profit two-effort example.

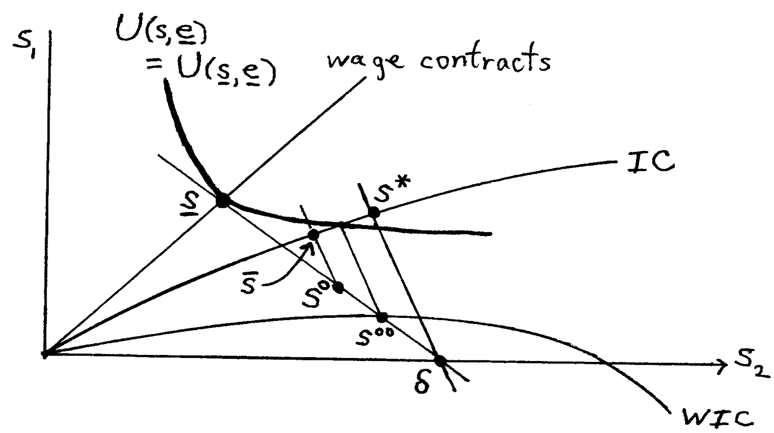

Figure 2: Contract s^0 is renegotiated to $\underline{s}$, but the debt contract δ is renegotiated to s^*. The entrepreneur chooses $\underline{e}$ in the former case, $\bar{e}$ in the latter.

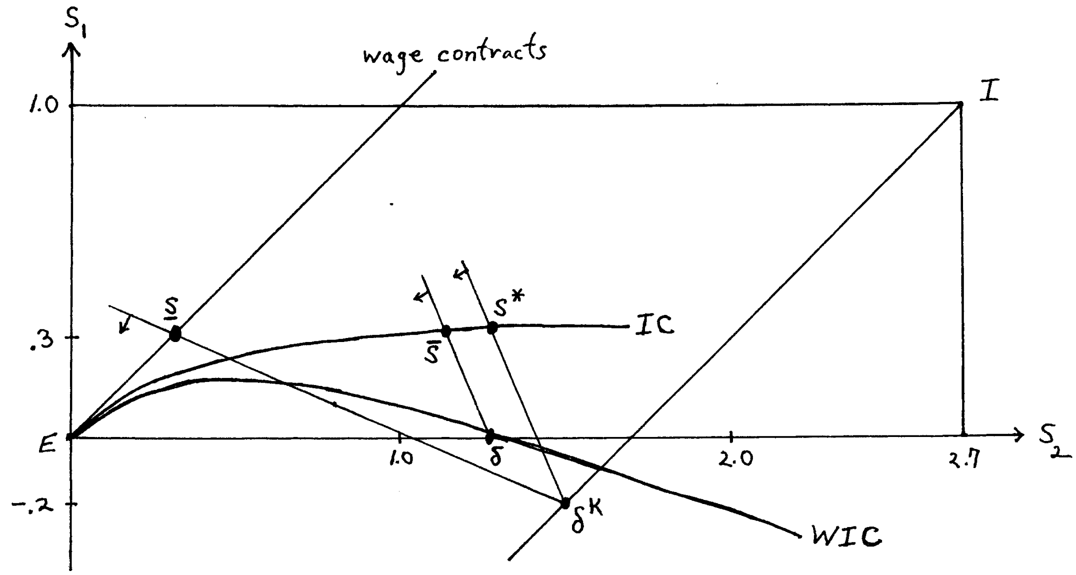

Figure 3: An equilibrium of Γ in which the investor receives rent. First the debt contract δ is adopted. Then the entrepreneur chooses $\bar{e}$, and renegotiates δ to $\bar{s}$.