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Portfolio Analysis of Latin American Stock Markets*

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Abstract

This paper examines the strategy of investing in seven Latin American emerging stock markets: Mexico, Brazil, Argentina, Colombia, Peru, Venezuela, and Chile. International Portfolio investment gradually increased during the late 1980s and the 1990s in this region. Investors willing to assume the additional risk present in these markets have been well compensated. Yet, many market analysts have indicated that such markest are some-what of an abnormality, in that they tend to be characterized as thin, narrow and driven by poorly informed individuals rather than by fundamentals. The optimization algorithms include Markowitz variance-covariance and lower partial moment. The optimal portfolios are evaluated using criteria such as terminal wealth, Sharpe utility measure, Treynor and Jensen measures, and reward to semi-variance. In addition, portfolios which employed first, second and third degree stochastic dominances are presented. It is shown that possesing a diversified international portfolio which includes Latin American stocks is beneficial.

Keywords: Portfolio diversification; Reward to semi-variance; Latin America; Sharpe, Traynor, Jensen Measures; Markowitz Covariance; Lower Partial Moment.

Jel classification: F3; F4; F21; G3; G11; G14; G15

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1. Introduction

This paper examines the strategy of investing in Latin American markets and the risk-return trade-off that is associated with such a strategy. An optimal portfolio is derived through historic observations and is subsequently evaluated utilizing performance measures.

International investment gradually increased during the late 1980s and the early 1990s with the emergence of markets in the newly industrialized countries of Latin America such as Mexico, Brazil, Argentina, Peru, Colombia, Venezuela and Chile. Investors willing to assume the additional risk present in these markets have been well compensated. Yet, many market analysts have indicated that such markets are somewhat of an abnormality, in that they tend to be characterized as thin, narrow and driven by poorly informed individuals rather than by fundamentals.

It cannot be assumed, however, that investing in emerging stock markets is more dangerous than investing in more progressive countries, given the expected returns. The average investors may increase his or her returns if they hold portfolios which include foreign stocks. Since stock markets are not highly correlated and consequently do not fluctuate in tandem, diversification leads to a higher return for a given risk.

To accurately measure the performance of Latin American stock markets, several standards may be employed. They include yearly rates of return, market capitalization and volume of shares traded. While the rate of return on the Latin American regional index was down for 1994, primarily due to the collapse of the Mexican peso, both market capitalization and volume of shares traded grew substantially.

The International Finance Corporation's (IFC) regional index for Latin America, which includes Mexico, Argentina, Brazil, Chile, Venezuela, Peru, and Colombia, performed poorly in 1994, falling 10.9 percent in US dollar terms. The collapse of the Mexican peso in December 1994, in part a result of massive capital outflows, led to large decreases in the Mexican stock market index: the IFC reported a 39.7 percent decline for the year. Comparable losses were experienced by investors in the stock markets of Argentina and Venezuela, which declined by 26.0 and 16.4 percent, respectively.

Not all Latin American markets, however, posted such dismal results. The IFC index for Brazil was up 64.9 percent for the year. Peru's market gained 47.5 percent, as did both Chile and

Colombia, with increases of 42.3 and 25.9 percent, respectively.

While the overall level of Latin American stock market performance was mixed in 1994, these exchanges were still able to outperform their counterparts in other developing regions. After solid results in 1993, the IFC Investable Composite Index (IFCI) for stocks in developing countries was off by 13.8 percent in US dollar terms in 1994. Asia was down 14.6 percent, while the Europe, Middle East and Africa (EMEA) Index fell 30 percent. Even Poland, the best performer in 1993, posted losses of 42.6 percent. Similarly, China lost 49.2 percent, as did Turkey with a realized drop of 42.7 percent.

Capitalization in Brazil grew from $99.4 billion in 1993 to $190.6 billion in 1994, an increase of 91.8 percent. Similar growth was recorded in the markets of Peru, Colombia and Chile, with increases of 63.5, 60.7, and 59.6 percent, respectively. Capitalization in Mexico, however, shrank by 2.3 percent in 1994, to $196.1 billion. The overall level of capitalization growth for the seven Latin American countries included in the IFC index - Mexico, Brazil, Chile, Colombia, Venezuela, Argentina and Peru - expanded by 28 percent between December 1993 and November 1994. Likewise, volume of shares traded experienced substantial increases. Colombia reported the largest gains, with an increase of 206.6 percent. Brazil expanded by 142.9 percent, Peru 101.9 percent and Chile 91.6 percent. Only Venezuela suffered a decrease, falling 35.9 percent for the period.

While the aforementioned data indicate that the Latin American markets are attractive investment opportunities, they are not without their drawbacks. Among their deficiencies, the phenomenon of dollarization has been persistently observed in these economies.[1] Dollarization adversely affects the domestic economy because it intensifies the inflationary effects of a given fiscal deficit. Moreover, dollarization thwarts the ability of authorities to implement independent economy-wide monetary and fiscal policies because the foreign currency component of the total money supply is not subject to their control.

The methods by which firms in emerging markets secure funding and capital in the domestic financial markets is an issue that concerns international investors. In most developing countries, lack of funding is often the main obstacle to growth in the private sector. Banking systems are

[1]Dollarization refers to the holding of a large proportion of domestic financial assets in the form of interest-bearing foreign currency deposits. This strategy is undertaken to hedge against high and volatile inflation.

frequently unable to provide the required funds and capital markets have in the past failed to secure the desired instruments, both in terms of required quantities and preferred maturities. Moreover, government controls often restrict the types of instruments available and regulate the issuing and pricing of such tools.

However, with the advent of recent market reforms, firms in emerging countries are currently experiencing an increase in the choice of financing options. For many firms, the criterion that is used to determine the most desirable option available is a combination of minimized cost and minimized risk. In other words, cheaper financing is usually the first to be used and management is wary of overburdening the firm with debt secured financing due to the possibility of bankruptcy.[2]

The dynamic linkages among the world's major markets have been studied since the late 1960s (e.g., Grubel, 1968; Granger and Morgenstern, 1970; Levy and Sarnat, 1970; Grubel and Fadner, 1971; Agmon, 1972; Bertoneche, 1979; Hilliard, 1979), with increased scrutiny emerging in the last decade (e.g., Schollhammer and Sands, 1985; Eun and Shim, 1989; Meric and Meric, 1989; Von Furstenberg and Jeon, 1989, 1991; Hamao, Masulis and Ng, 1990; Koch and Koch, 1991; Birati and Shachmurove, 1992; Chan, Gup and Pan, 1992; Malliaris and Urrutia, 1992; Roll, 1992, Friedman and Shachmurove, 1996; and Shachmurove, 1996). While some have studied the Latin American economies (e.g., Bhagwati, 1993; Alonso, 1994; Gwyne, 1994), this study is among the first to investigate the dynamic linkages across national stock indexes of the seven newly emerging markets of Latin America.

Determining which country occupies the leading position in market size depends on how well each country's stock market and currency are performing at a given time. Hence, this process is not of great significance. Yet, the classification presented in Table 1 is true for the period studied in this paper. The Mexican stock market is the largest exchange, followed by the Brazilian, Chilean,

[2]While cost and risk are the two principal variables that are considered by most firms, a few firms believe there to be a third factor. This factor, particularly present in emerging markets where the tradition of family ownership is strong, is that of control of the firm. Control can dominate financial decision making to the extent that management is unwilling to relinquish its monopoly on firm decision making. Some firms avoid equity issues in apprehension of the loss of control that will inevitably result. Consequently, debt financing is often preferred over equity issues, even when the cost and risk features of such debt are below par, in order to prevent the dilution of control that follows equity issues.

Argentinean, Colombian, Peruvian, and the Venezuelan markets.

The remainder of the paper is organized as follows: Section II discusses theoretical issues. Section III presents the empirical results derived. Finally, section IV concludes

2. Theoretical Concepts

This section briefly surveys the theoretical concepts employed in this paper: optimization algorithms and portfolio evaluation techniques. Optimization algorithms are mathematical procedures that solve multiple variable problems simultaneously. The results are optimal given the information provided in the formulation of the problem. The allocation of funds into different investments is accomplished in such a way as to maximize returns and minimize variability. In order to screen investments according to their return and risk characteristics, several statistical measures are used. These statistics include the geometric mean, variance, beta and lower partial moment (LPM).[3] These procedures are employed to measure the return and risk inherent in investment. The ranking of assets by their risk/return statistics provides an initial screen of individual assets.

Optimization algorithms, however, provide only trade-off between risk and return. There will be optimized high return - high risk portfolios, optimized medium return - medium risk portfolios and optimized low return - low risk portfolios. At this point the owner of a portfolio has to decide which portfolio maximizes her utility. Evaluation techniques are applied to assess the optimal solutions derived by comparing them to other investment alternatives such as the S&P 500, or a portfolio consisting of equally weighted initial allocations of the assets present in the derived optimal portfolio.

2.1. Optimization Algorithms

Optimization algorithms are mathematical tools that solve multiple-variable problems using quadratic programming. In portfolio theory a dollar of investment is allocated among different securities in order to maximize return and minimize variability. In this case, the budget constraint requires that

[3] These statistics are described in the Appendix.

all allocations sum up to 100 percent of the available total investment. In addition to the variance, both beta and LPM statistics can be formulated and used in quadratic programming analysis.

As with any limited resource that is to be apportioned, decisions have to be taken regarding the division of these endowments. The construction of portfolios assumes two such decisions: choosing between asset classes such as stocks, bonds, foreign currency, etc. (strategic optimization) and choosing between securities in any given asset class (tactical optimization). The majority of investors prefer to participate in the former kind of allocation, mainly to optimize across asset classes. Few, however, optimize within a given asset class, omitting tactical optimization. This despite the fact that there is evidence to support the concept of tactical optimization. For example, an equity market index with optimized allocations will outperform indexes with equal or value weighted allocations (Haugan, 1990a, 1990b).

In this paper the Markowitz Variance-Covariance Analysis and the Lower Partial Moment Analysis are employed to derive the optimal portfolio.[4]

2.1.1. Markowitz Variance-Covariance Analysis

Markowitz (1959) developed the basic variance-covariance analysis. Low or negative correlations between assets are used to reduce the overall variability or risk of the portfolio. The variance of the portfolio is calculated as follows:

$$Vp = \sum_{i=1}^{k} \sum_{j=1}^{k} X_i \cdot X_j \cdot Cov_{ij}, \tag{2.1}$$

where Vp is the portfolio variance, k is the number of assets in the portfolio, X is the share of asset i or j within the portfolio, and Cov_{ij} is the covariance between assets i and j. It is calculated by:

$$Cov_{ij} = \sigma_i \cdot \sigma_j \cdot r_{ij}, \tag{2.2}$$

[4]Other algorithms are applied and tested as well. These are Nawrocki's (1991) Lower Partial Moment Heuristic Algorithm, Elton, Gruber, Padberg's (1976) Beta-Single Index Model, and Elton, Gruber, Padberg (1976) Average Correlation Heuristic Algorithm. The results obtained through the application of these algorithms are similar to those generated by the two algorithms mentioned above and are available from the author upon request.

where σ_i is the standard deviation for asset i and r_{ij} is the correlation coefficient between assets i and j.

The expected return of the portfolio is determined by:

$$Ep = \sum_{i=1}^{k} X_i \cdot E(R_i), \tag{2.3}$$

where Ep is the expected return of the portfolio and $E(R_i)$ is the expected return for asset i.

Using the preceding formulas, a quadratic programming method is constructed to maximize return and minimize variance as follows:

$$\begin{aligned} &\min \ z = Vp - \lambda \cdot Ep \\ &s \cdot t, \sum_{i=1}^{k} X_i = 1, \end{aligned} \tag{2.4}$$

where λ is the slope of the objective function. The term λ may be varied from zero to infinity in order to solve for various points on the efficiency frontier.[5] The outcome of these calculations is that the results map the efficiency frontier, where each point corresponds to a portfolio, which in turn represents the lowest risk for a given return or the highest return for a given risk (Markowitz, 1959).

2.1.2. Lower Partial Moment (LPM) Analysis

In Lower Partial Analysis (LPM), the variance is replaced with the lower partial moment or the semivariance, which is a special case of lower partial moment with $n = 2$. The same expected return and risk equations hold true as does the following quadratic formulation:

[5] The algorithm used is the Critical Line Algorithm. It begins with the highest return portfolio which, by definition, includes the highest return asset. Each asset is then evaluated using a critical value (pivot conditions) to determine which is the next asset to enter the portfolio. As assets enter into the portfolio, it becomes more diversified and will have lower risk as well as return. Each portfolio derived is called a corner portfolio. A corner portfolio is generated when an asset either enters or exits the portfolio. The result of these corner portfolios is that they map the efficient frontier, where each portfolio represents the lowest risk for a given return or the highest return for a given risk.

$$LPM_{2,p} = \sum_{i=1}^{k} \sum_{j=1}^{k} X_i \cdot X_j \cdot SD_i \cdot SD_j \cdot r_{ij} \tag{2.5}$$

$$\min \; z = LPM_{2,p} - \lambda \cdot Ep, \tag{2.6}$$

where $LPM_{2,p}$ is the semivariance of portfolio p, k is the number of assets, SD_i is the semideviation (square root of the semivariance) for asset i, and r_{ij} is the correlation between assets i and j (e.g., see Bawa, 1975; Fishburn, 1977; Nawrocki, 1991).

2.2. Portfolio Evaluation

After the selection of a portfolio, its performance is then evaluated. Performance measures that account for both risk and return need to be computed. Portfolio evaluation measures consist of Terminal Wealth, Sharpe's Utility Measure, Sharpe, Treynor and Jensen Measures, Reward to Semivariance, and Stochastic Dominance.

2.2.1. Terminal Wealth

The Terminal Wealth measure replies to the following query: How much money did the investor make? It is a ratio that indicates the amount of money generated for each dollar of initial investment. Terminal Wealth is the k-th power of the geometric mean, or simply the product of the individual returns. It is the only important, long term evaluation performance measure. This is because risk-return measures are not accurate, since as the investment horizon increases, the importance of liquidity risk decreases. Terminal Wealth is given by the following expression:

$$\text{Terminal Wealth} = \prod_{t=1}^{k} (R_t) \tag{2.7}$$

where Π is the multiplication operator, k is the number of periods and R_t is the rate of return at time period t.

2.2.2. Sharpe (1966) Utility Measure

The Sharpe Utility Measure employs an estimate of the investor's risk tolerance rather than the riskless rate of return as an indicator of the portfolio holder's utility function. The risk tolerance ranges from zero to one. The higher the risk tolerance, the higher the proportion of the portfolio invested in the riskier assets (Sharpe and Alexander, 1990). The measure is defined as follows:

$$Utility = Return - (Variance/Risk\ Tolerance). \tag{2.8}$$

Risk tolerance is defined as the level of risk an investor is disposed to bear. This characteristic is unique for each individual investor. Investors who are risk-averse accept only low amounts of risk compared to their risk-neutral and risk-loving counterparts. Risk-averse investors penalize the expected rate of return of a risky investment by a certain percentage to reflect the inherent danger. Risk-neutral investors, on the other hand, judge investments solely on the basis of the expected return, thus eliminating risk considerations from their strategy. Finally, risk-loving investors adjust expected returns upwards when risk is present (Bodie, Kane and Marcus, 1993).

2.2.3. Sharpe (1966), Treynor (1965) and Jensen (1968) Measures

The Sharpe (1966), Treynor (1965), and Jensen (1968) Measures are defined as follows:

$$Sharpe = (R_p - R_f)/\sigma_p \tag{2.9}$$

$$Treynor = (R_p - R_f)/ß_p \tag{2.10}$$

$$Jensen(a_p) = R_p - R_f - ß_p(R_m - R_f) - e_t, \tag{2.11}$$

where R_p is the return on the portfolio, R_f is the riskless rate of return, σ_p is the standard deviation and $ß_p$ is the beta of the portfolio.

Both the Sharpe and the Treynor measures use reward to risk ratios. The Sharpe measure employs standard deviation in its denominator, while the Treynor measure applies the beta value. The Jensen Measure, which is based on the Capital Asset Pricing Model (CAPM), investigates the investments performance by calculating the intercept a_p of the regression line: $R_p - R_f = a_p + ß_p(R_m - R_f) + e_t$. If the portfolio fares better than the market, a_p is greater than zero. When it under-performs, a_p is less than zero. If a_p is significantly different than zero and it is positive, the portfolio is considered successful. On the other hand, if a_p is less than zero, the portfolio is a failure. Therefore, the higher the value of a_p, the greater the abnormal rate of return achieved by the portfolio in excess of the market (Jensen, 1968, and Levy and Sarnat, 1984). These three measures, however, are not fully accurate since they are statistically biased (Ang and Chua, 1979). The effect of the bias is that each of the measures may rank the performance of a group of portfolios differently from its counterparts.

2.2.4. Reward to Semivariance

The Reward to Semivariance Ratio is defined as follows:

$$Reward\ to\ Semivariance = (R_p - R_f)/SD_p, \tag{2.12}$$

where SD_p is the semideviation of the portfolio. This ratio is preferred over other alternatives because studies have revealed that the Sharpe (1966), Treynor (1965), and Jensen (1968) measures are statistically biased. Various causes of the biases have been proposed. They include the existence of unequal borrowing and lending rates, the failure to account for higher moments of return distributions and the elusive "true" holding period (Ang and Chua, 1979). The deficiency of this ratio is that it assumes a fixed utility function by setting $n = 2$. This shortcoming can be overcome by employing the more general Reward-to-LPM Ratio. The degree, n, can then be manipulated in this case to match the utility function of the investor (Klemkosky, 1973).

2.2.5. Stochastic Dominance

Stochastic dominance is an effective evaluation technique for judging the performances of portfolios, because it does not make any assumptions concerning the underlying probability distribution of security returns, and is based on a very general utility function. First Degree Stochastic Dominance (FSD) places no restrictions on utility functions except that they should be non-decreasing. Thus, FSD acts as a preliminary screen that eliminates those options that no rational investor would choose. Second Degree Stochastic Dominance (SSD) applies only to risk-averse investors by assuming a concave utility function. All efficient sets included in SSD are also present in FSD, but not necessarily vice versa. Finally, Third Degree Stochastic Dominance (TSD) further assumes decreasing absolute risk aversion, and hence is applicable to yet a smaller group of investors. Decreasing absolute risk aversion means that the risk premium an investor is willing to pay to be rid of a given risk, declines as his wealth increases. This implies that, at higher levels of wealth, the portfolio owner becomes more risk-neutral (Porter, Wart and Ferguson, 1973; Francis and Archer, 1979; Francis, 1980; Saunders 1980; Elton and Gruber, 1984; Levy and Sarnat, 1984).

3. Empirical Results

The data cover the period from December 31, 1987 through December 30, 1994. For the purposes of this paper, an optimal portfolio for the period ranging from July 7, 1994 to December 30, 1994 is used as the basis of the discussion. The average Treasury Bill return, which is the assumed risk free rate for the period, is 3.00 percent. The performance of the Argentinean, Mexican, Brazilian, Chilean, Venezuelan, Colombian, and Peruvian exchanges are recorded and compared with the S&P 500.[6] The aforementioned time frame has been chosen for several reasons. First, it is more likely to reveal the optimal asset allocation because studies employing this interval have obtained the largest number of optimal portfolios in comparison with any other period. Second, the T-bill rate for the period is the lowest interest rate over the span of the study. Such an environment encourages investors to consider more diversified investment strategies in hopes of securing higher

[6] The S&P 500 is the composite index of 500 US stocks, and is commonly regarded as an accurate representation of the US stock market.

yields.

Table 2 indicates that the optimization frontier contains seven different portfolios, each optimizing one or more particular criteria. These criteria are annual return, periodic return, standard deviation, probability of loss, utility, and the Reward to Semivariance Ratio (R/SV). The method used is the Markowitz Critical Line Algorithm which computes corner portfolios on the efficiency frontier, (Markowitz, 1959). Of the seven optimal portfolios, the one with the highest Reward to Semivariance ratio is deemed appropriate for the purposes of this study. Portfolio number 4 conforms to this characteristic. Its R/SV ratio of 0.38 surpasses the corresponding R/SV ratios of the six other candidates. Portfolio number 4 yields an annual return of 74.91 percent on investment. Table 3 exhibits the component securities of this optimal portfolio: Colombia 60.38 percent, Chile 35.10 percent, Peru 4.35 percent and Brazil 0.17 percent. The standard deviation of the portfolio is 0.96 percent. The shortfall probability of realizing a return below the risk-free rate is 0.41.

Table 4 provides a short summary of individual assets. The annualized return in Argentina is -8.71 percent and the standard deviation is 2.29 percent; in Mexico, the return is -33.58 percent with a standard deviation of 2.14 percent. In Brazil, the return is 34.33 percent and the deviation 4.01 percent; in Chile, the return is 50.35 percent with a standard deviation of 1.49 percent. In Venezuela, the return is -32.04 percent with a deviation of 2.80 percent; in Colombia, the return is 91.40 percent with a standard deviation of 1.35 percent. Finally in Peru, the return is 71.62 percent with a standard deviation of 2.37 percent.

It may seem surprising that the Colombian market is not the most volatile of the markets, given the high return that it offers. This leads to the conclusion that there must be some additional risk inherent in the Colombian market that is not reflected in the calculated standard deviation. There are two kinds of risk omitted by the numerical observations. Mainly, foreign exchange and sovereign risks. Foreign exchange risk is defined as the risk that a return denominated in a foreign currency will have a lower value in the domestic currency due to a fluctuation between the two currencies. Sovereign risk refers to the danger of a government interfering in its domestic market in a manner that has an adverse impact on investments (Grabbe, 1991). These risks are present in the Colombian market and to a lesser extent, in the other Latin American markets as well. Shortfall probabilities are 0.51 in Argentina, 0.53 in Mexico, 0.49 in Brazil, 0.46 in Chile, 0.52 in Venezuela, 0.42 in Colombia, and 0.46 in Peru.

Table 5 shows that the portfolio beta is 0.13, well below the market (S&P 500) beta of one. The Sharpe measure is 0.22, the Treynor measure is 2.63 and the Jensen Alpha value is 0.002. In order to provide a complete analysis of the findings, these statistics are compared to the analogous market values. The Sharpe measure for the S&P 500 is -0.04, the Treynor measure is -0.03 and the Jensen value is, by definition, 0. The portfolio, therefore, provides substantially more reward per unit of risk, whether variance or beta, than the S&P 500. The results are also compared to statistics derived from a portfolio consisting of equally weighted initial allocations to all securities in the optimal portfolio. Portfolio number 4 outperforms the equally weighted portfolio on all counts. The performance measures considered are Periodic Return, Sharpe Measure, Treynor Measure, Jensen Alpha, Beta, T-test, R-squared, Terminal Wealth, Utility and the Reward/Semivariance (R/SV) ratio. Furthermore, portfolio number 4 provides a higher return than that predicted by the Capital Asset Pricing Model (CAPM), given its beta and the average market return. Since the Jensen measure is greater than zero, this means that the portfolio performs better than the market. These results are summarized in Table 5.

The optimal portfolio is subject to another test in addition to the variance-covariance analysis. The Lower Partial Moment Algorithm calculates the LPM/CLPM (Lower Partial Moment/Covariance Lower Partial Moment) matrix, given the investor's level of risk aversion. The optimal portfolio derived through the application of this algorithm is listed in Table 6. The portfolio yields an annual return of 76.71 percent and a R/SV ratio of 0.38. These statistics are almost identical to the return and R/SV ratio generated by the Critical Line Algorithm. Table 7 presents the allocations within the portfolio. The results generated by the Lower Partial Moment Algorithm (65.53 percent Colombia, 31.43 percent Chile, 2.69 percent Peru and 0.35 percent Brazil) are again similar to those obtained from the Critical Line Algorithm described previously.

Finally, to complete the analysis, the risk/return performance of the securities in the portfolios is evaluated by using First, Second, and Third Degree Dominance techniques. The assets for each degree of dominance and their corresponding statistical variables are exhibited in Table 8. The best risk/return performance is provided by those securities listed under Third Degree Dominance. Under First Degree Dominance all the assets except Peru are included. Peru is exempt because it registers a lower probability of achieving the same level of return as the other markets, given a specific level of risk. Under Second and Third Degree Dominance, only Colombia, Chile, and the S&P 500 are listed. The other securities are excluded from Second Degree Dominance because the cumulative probability of either Colombia, Chile or the S&P achieving a given return, each taken separately, minus the cumulative probabilities of the other securities achieving the same return, also taken separately, are consistently non-negative.

4. Conclusion

This paper studies the daily stock market returns of seven Latin American countries, and the prospect of investment for the purposes of diversification. The period July 7, 1994 to December 30, 1994 is used as the basis of the analysis. An optimal portfolio is generated and then evaluated with appropriate performance measures. The optimal portfolio, acquired through the application of the Markowitz Critical Line Algorithm, allocates 60.38 percent of the funds in the Colombian exchange, 4.35 percent in the Peruvian market, 0.17 percent in Brazil and 35.10 percent in Chile. It achieves an annualized return of 74.91 percent, a R/SV ratio of 0.38, a standard deviation of 0.96 percent and a shortfall probability of 0.41. The portfolio's beta is 0.13, well below the corresponding market beta of one. Hence, it is far less volatile than the market, as represented by the S&P 500. The Sharpe measure is 0.22, the Treynor measure is 2.63 and the Jensen Alpha is 0.002.

In addition, the Lower Partial Moment Algorithm is applied to the optimal portfolio. The portfolio allocations obtained though the use of the Lower Partial Moment Algorithm are very similar to those generated by the Critical Line Algorithm.

The focus of this analysis is a select group of Latin American markets. While the Reward-to-Risk ratios, based on stock return volatilities, might be appealing, additional risk factors need to be both examined and accounted for. There are intrinsic dangers in foreign investment. The risk manifests itself in two forms: foreign exchange risk and sovereign risk. The former implies that a foreign currency denominated return will have a lower real value following an adverse change between the relative values of two currencies. Sovereign risk refers to the possibility of intervention in foreign markets by the domestic governments, producing an environment that has a negative impact on investments. The optimal portfolio derived above incorporates both of these risks, since it is based on the allocation of funds into foreign securities. Therefore, investors are rewarded for the additional risk they are bearing by higher premiums.

Nevertheless, it is beneficial for the contemporary investor to possess a well diversified portfolio, rather than to limit his investments to a single market. The low correlation among stock markets implies that their movements are not perfectly synchronized. Consequently, investing in a portfolio consisting of allocations in several foreign exchanges permits an investor to negate the risk that an adverse fluctuation in any given market will have a considerable effect on the return of his or her portfolio.

Appendix

Statistical Measures

The statistical measures used are: geometric mean, variance, beta, and lower partial moment (LPM).

1. Geometric Mean

For the k numbers a, b, c, d, e, and f, the geometric mean is:

$$[a \cdot b \cdot c \cdot d \cdot e \cdot f]^{(1/k)} \tag{4.1}$$

For the purpose of determining rates of return, the method of computing a geometric mean is more accurate than a simple arithmetic mean, since it takes into account the compounding nature of interest over time.

2. Variance

$$\sigma_i^2 = (1/k) \cdot \sum_{t=1}^{k} [R_{it} - E(R_i)]^2, \tag{4.2}$$

where R_{it} is the return to asset i in period t, and $E(R_i)$ is the expected geometric mean return for asset i. Variance measures the magnitude of deviations from the mean. The greater the deviations, the greater the level of risk. Variance plays an important role in the evaluation of potential investments. For a risk-averse individual choosing between two investments with equal expected returns, the investment with the lower variance is more attractive. Consequently, investments with higher risk - i.e. higher variance - must offer higher expected returns to compensate investors for the additional risk, see Markowitz (1959).

3. Beta

The beta (ß) of an asset measures the variability of an asset relative to the market index. It is a popular risk measure, and has been widely used for the past 25 years. The standard was developed to make the Modern Portfolio Theory (MPT) model operational, which is computationally complex when the variance is used. The ß statistic is determined using the following regression:

$$R_{it} = a_i + \beta_i \cdot R_{mt} + e_r \tag{4.3}$$

$$\sigma_e^2 = (1/k) \cdot \sum_{t=1}^{k} e_t^2, \tag{4.4}$$

where,

$$e_r = R_{it} - [a_i + \beta i \cdot R_{mt}], \tag{4.5}$$

R_{it} is the return on asset i for period t, a_i is the intercept of the line, ß$_i$ represents the slope of the line, and is defined as the tendency of the asset's returns to respond to swings in the broad market, R_{mt} is the return to the market index for that same period t, and e_t measures the deviation of R_{it} from the regression line for period t. There are k observations, such that $t = 1$, 2, ..., k.

The beta of the market index ß$_n$, is arbitrarily set at 1.0 and serves as a reference value with which to compare individual asset betas. If the beta of an asset is equal to 1.0, then both the asset and the market are equally risky, and will tend to move together. If ß$_i$ is greater than 1.0, then the asset is more volatile than the market, and hence, more risky. If ß$_i$ is less than 1.0, then the asset is less volatile than the market, and hence, less risky. Furthermore, beta serves to determine the incremental risk an individual asset brings to a well diversified portfolio.

$$\sigma_i^2 = ß_i^2 \cdot \sigma_m^2 + \sigma_e^2 \tag{4.6}$$

The first component of the variance of an asset ($ß_i^2 \cdot \sigma_m^2$) is termed the systematic or non-diversifiable risk component and is the risk inherent in the general market. The second component (σ_e^2) is termed the unsystematic or diversifiable risk component and can be diversified away as it is due not to the market in general, but rather, only to that particular asset (Sharpe, 1964).

4. Lower Partial Moment (LPM)

Both variance analysis and the use of betas to estimate risk levels presuppose a normally distributed set of security returns and investors with quadratic utility functions. In order to address risk levels when these assumptions cannot confidently be made, the Lower Partial Moment (LPM) was developed.[7] Semivariance is a special case of LPM analysis (Bawa, 1975; Fishburn, 1977). Semivariance is defined as an n-degree LPM with $n = 2$. The variable n refers to the degree that deviations below a target return are raised to. The lower Partial Moment is defined as follows:

$$LPM_n(h) = (1/k) \cdot \sum_{t=1}^{k} \max \; [0, h - R_t]^n, \tag{4.7}$$

where n is the degree of the LPM, h is the target return the investor does not wish to go below, k is the number of periods used to calculate the LPM, and R_t is the return for the asset for period t. A problem that often occurs when determining asset riskiness is that security returns have non-normal distributions. For two distributions, one positively skewed and the other negatively skewed, it is possible that they both have the same mean and variance; that is, the variance measure might not differentiate between the two distributions. However, the LPM measure can handle non-normal distributions, and is able to differentiate between the two. In LPM analysis, $n = 1$ is the boundary between risk-averse and risk-loving investors. If n is greater than 1, the investor is risk-averse and attempts to minimize risk for a given return, while for values of n less than 1, the investor is risk-loving and prefers higher to lower risk. Furthermore, the use of LPM is less restrictive on assumptions of the investor's behavior than beta and variance analysis. It has been shown that the LPM can match the utility functions of investors who have been described in utility function literature. Decision makers in investment contexts frequently associate risk with failure to attain a target return. Examination of published utility functions which use the maximization of expected utility criterion lends support to the notion of a target return at which the utility undergoes a noticeable change. Depending on the context, the change point may be negative, zero or positive (Fishburn, 1977).

It is important to note that in equation (A.7), the above-target returns ($R_t > h$) provide negative numbers. Given the choice between a zero or a negative number, the maximization operator will select the zero. Only below-target returns ($R_t < h$) will provide a positive deviation that is raised to the n power and added to the LPM calculation. LPM will, therefore, only provide non-negative values.

[7] Markowitz (1959) was the first to offer the use of semivariance as a substitute for beta and variance to handle both skewed return distributions and investors who have non-quadratic utility functions.

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