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Incorporating Concern for Relative Wealth into Economic Models

by

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*Incorporating Concern for Relative Wealth into Economic Models**

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Abstract

We develop a simple model that captures a concern for relative standing. The concern for relative standing is instrumental in the sense that individuals do not get utility directly from their relative standing, but rather, the concern is induced because relative standing affects consumption of standard commodities. We investigate the consequences of a concern for relative wealth in models in which individuals are making labor-leisure choice decisions. Among the results, we show how individuals' decisions are affected by the aggregate income distribution and how the concern for relative wealth can generate behavior that can be interpreted as conspicuous consumption when wealth is not directly observable.

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1. Introduction

A standard, often implicit, assumption in economics is that people only accumulate wealth to fund consumption by themselves and their families. In this article, we will argue that, in many circumstances, people have other motivations for wealth acquisition. In particular, we will argue that people acquire wealth in order to be wealthier than other people. Moreover, while this desire to be wealthier than other people appears to capture a concern for relative status, it can be justified on narrow economic grounds.

This desire to be relatively wealth is similar to the social motivations for wealth acquisition mentioned by a number of prominent early economists. Broadly speaking, they argued that society views wealthy individuals positively and, furthermore, that this positive light serves as an important motivation for the acquisition of wealth.[1] Smith [11, pp. 112-113] wrote.

> For to what purpose is all the toil and bustle of this world? What is the end of avarice and ambition, of the pursuit of wealth, of power, and pre-eminence? Is it to supply the necessities of nature? The wages of the meanest labourer can supply them... From whence, then, arises that emulation which runs through the different ranks of men, and what are the advantages which we propose by that great purpose of human life which we call bettering our condition? To be observed, to be attended to, to be taken notice of with sympathy, complacency, and approbation, are all the advantages which we can propose to derive from it. It is the vanity, not the ease, or the pleasure, which interests us. But the vanity is always founded upon the belief of our being the object of attention and approbation.

Veblen [12] argued that there developed within societies a belief about the level of conspicuous consumption that is appropriate to a particular rank within a society, that this consumption level is increases with one's rank, and, further, that as the society became richer, the appropriate level for any given rank rises. Veblen also argued that since the primary purpose of these conspicuous consumptions is to signal one's success, they must be of a publicly observable nature or at least produce a publicly observable product.

The pervasive assumption in current economic models that people are not concerned with relative wealth stems not from a belief in its descriptive accuracy, but rather from methodological considerations. Economics has been successful

[1] This view was, of course, not one espoused by economists alone. See for example, Weber [13].

as a discipline because of the restrictions imposed by the assumptions of the models employed. A model can have predictive power only to the extent that some kinds of behavior are inconsistent with the assumptions of that model. Foremost among the assumptions that underlie economic models is that agents are rational: agents choose from the actions available that action which yields the highest utility. The assumption that agents maximize utility, however, puts no restrictions on behavior in the absence of restrictions on the nature of the utility function. Any observed pattern of behavior can be rationalized as utility-maximizing if utility functions can change arbitrarily through time. The force of the rational-agent assumption in economic models comes from the concurrent restrictions on the utility function, for example, the requirement that the utility function be either unchanging through time or changing in a well-defined way. Similarly, economists can assume that many variables affect individuals, but only at the cost of weakening the conclusions that can be drawn from the analysis. Typically, economists have restricted agents' utility functions to depend only on consumption for this reason: allowing agents' decisions to be affected by such things as feelings of competition, envy, or rivalry admits models that have no predictive power.[2]

We are interested in developing models that accommodate a concern for relative wealth in reduced-form models while maintaining the standard economic assumption that individuals ultimately care only about consumption. In these models, an agent's concern for relative wealth is instrumental: he or she cares about relative wealth only because final consumption is related not just to wealth, but additionally to relative wealth. In Cole, Mailath, and Postlewaite [4], we presented a model in which agents care about relative wealth because relative wealth affects mating. That model deals with an environment in which there is a succession of generations of men and women who match and jointly make a consumption/saving decision. The members of each sex differ only in their endowments. An immediate consequence of the assumption that consumption is joint is that each individual prefers to be matched with the richest member of the opposite sex, all other things being equal. If the matching in a particular period has no effect on how future generations will match, voluntary matching will be positively assortative on wealth; that is, the wealthiest men will match with the wealthiest women, and so on.[3] When matching is positively assortative

[2]A few modern economists have sought to explore the implications of including status concerns in preferences in formal economic models. They include Duesenberry [6], Frank [7], Abel [1], Zou [14], Robson [10], Campbell and Cochrane [3] and Bakshi and Chen [2]. However, none of their models seek to explain why agents might have a concern for status.

[3]The main point of Cole, Mailath, and Postlewaite [4] is to demonstrate that this is not the only possibility. There may be a social norm that prescribes a particular matching rule that

on wealth, individuals who are higher in the wealth distribution for their sex will end up with better matches (that is, richer mates). Thus individuals care about relative wealth, but in the instrumental way described above: they care about relative wealth because it leads to wealthier mates, which results in higher consumption.[4]

The purpose of the current article is twofold. First, we provide a simple exposition of the basic ideas contained in our earlier work and discuss in more depth the interaction between relative standing and economic behavior. Second, we apply these ideas to two economic problems of independent interest. We first develop an effort model with complete information and show how the concern about relative wealth affects individuals' effort decisions. We then develop a second model which extends the analysis to include private information about income, which induces signaling that can be interpreted as conspicuous consumption.

We should emphasize that the direct implications of these models in which agents care about relative wealth do not necessarily differ from those that would obtain if relative wealth were put directly into the utility function. There are, however, advantages to our approach. First, an agent's concern for relative wealth in reduced-form preferences is induced by the fundamentals of the environment. Changes in the fundamentals of that environment will lead to predictable changes in reduced-form preferences. Here, unlike the case in which relative wealth is put directly into the utility function, testable implications can be derived about the relationship between fundamentals and reduced-form preferences. The dependence of reduced-form preferences on the fundamentals provides for additional scope in explaining why seemingly similar agents behave differently.

2. An Effort Model With Complete Information

Consider a one-period model in which there are two types of agents, men and women. There exist a continuum of men indexed by $i \in [0, 1]$ and a continuum of women indexed by $j \in [0, 1]$. Male i is exogenously endowed with i units of good

is not necessarily positively assortative on wealth and, further, that it is in the self-interest of all individuals to follow the norm's prescriptions. In this way, there may naturally be multiple equilibria that provide agents with different incentives to save and invest.

[4]Of course, if individuals' decision problems were described in sufficiently rich detail, relative wealth wouldn't matter. For example, we might imagine treating the matching decision as a market transaction, with prices given for mates with particular characteristics. If all the decisions about which individuals care are determined through markets, then, obviously, prices and an individual's income completely determine that individual's utility. Hence the instrumental interest in relative wealth is linked to the absence of some relevant markets. We discuss this issue further in our concluding comments at the end of this article.

x, while female j can produce good y by expending effort. There is no trade; each agent seeks to match with an agent of the opposite sex in order to consume both goods. By assumption, both goods are jointly consumed by two matched individuals.[5]

All agents have identical utility functions over the joint consumption of a matched pair's bundle given by $u(y) + x$. We assume that the female agent has a disutility for effort given by $-v(l)$, where l denotes labor effort. Female outputof good y, is given by $a(j)l$, where the productivity function $a(j)$ gives female j's productivity per unit of effort. We allow the productivity levels, denoted by $a(j)$, to differ across females. We assume that the females are ordered so that $a(j)$ is increasing in j, the index or names of the females.

Matching is voluntary and, in this section, based on complete information. A given matching is voluntary if no two unmatched agents mutually strictly prefer each other to their current matches. Since all consumption is joint by assumption, agents desire to be matched with as wealthy a mate as possible. Consequently, in any voluntary matching, the wealthiest male will match with the wealthiest female and, more generally, the kth-percentile male in the wealth distribution of men will be matched with the kth-percentile woman in the female wealth distribution. Since the distribution of good x is fixed exogenously, women's effort decisions determine the matching of men and women, along with the consumption levels in the matches. In equilibrium, each female takes as given other women's effort decisions and, hence the endowment level of her equilibrium match is given by her rank in the distribution of y.

For a particular choice of outputs by women, we summarize the relationship between an individual female's output and her mate's endowment by the matching function $m(y)$, which indicates the endowment of the man who will match with a woman with wealth y. If female j produces y units of output, while half of the other females produce less than y and half more, then female j will be matched with the male with the median endowment, or an endowment of one-half. If female j's output is such that exactly three-quarters of the females produce less than she, then she will be matched in equilibrium with the male whose endowment is three-quarters. In other words, m is the distribution function of female output. If female output (wealth) is strictly increasing in j, then the matching function is simply the inverse of the output function: if female j produces $y = y(j)$ units of output, then

[5]We assume that only females can produce good y for tractability; for this case, the wealth distribution for only one sex needs to be endogenously determined. Similar considerations motivate our assumption that there are distinct goods x and y that cannot be traded. The assumption that all consumption within a matched pair is joint avoids distributional considerations between the two people. Models incorporating distributional considerations are analyzed in Cole, Mailath, and Postlewaite [5].

the index (and so endowment) of her mate is given by $m(y) = y^{-1}(y(j)) = j$.

Given a matching function $m(\cdot)$, female j's optimal effort level will be the solution to the following problem:

$$\max_{l} u(a(j)l) - v(l) + m(a(j)l). \tag{2.1}$$

A female's total utility is determined by her direct utility from consumption of her own output, her disutility from effort, and the utility she derives from consuming her mate's endowment of x. It is not difficult to establish that in equilibrium a female's output level is increasing in her productivity. We establish this result in proposition 1 in the appendix.

The first-order condition which, under certain conditions,[6] characterizes the solution to the problem (1) is

$$a(j)[u'(a(j)l) + m'(a(j)l)] - v'(l) = 0. \tag{2.2}$$

We denote the value of effort l that solves (1) by $l(j)$. The first-order condition indicates how the impact of the equilibrium match quality affects a woman's effort decision. The concern about her relative output level induced by the tournament for males, reflected by m' in the first-order condition, leads to an increase in the effort level. When m' is relatively large, there is an incentive to work harder since the resulting increase in output has a greater impact on the quality of the resulting match. Since $m(y)$ is the fraction of females whose output level is below y, if the distribution of females' output is tight, m' is large; that is, a small change in an individual female's output can have a large effect on her rank; conversely, if females' outputs are disperse, the opposite is true.[7]

A female is concerned about her output rank only to the extent that males differ in the levels of the male good. If male j's endowment level was given by γj, then, if the effort levels of the other females are held fixed, the new matching function would be given by $\gamma m(a(j)l(j))$, and the impact on female j's effort decision would be larger or smaller as γ was greater or less than one. Note that in the extreme case where $\gamma = 0$, matching would be irrelevant from the females' perspective, and they would choose their effort levels so that $a(j)u'(a(j)l) - v'(l) = 0$.[8]

[6]The first-order condition characterizes any interior solution to (1) if the payoff function is differentiable. Such a solution exists if, for example, $u'(0) > 0, v'(0) = 0$, u is increasing and bounded, and v is strictly convex.

[7]Since the equilibrium matching function is just the inverse of the wealth distribution, $m'(c)$ is just the inverse of the slope of the female wealth distribution at c.

[8]There is one qualification to this discussion when considering the female at the bottom of the ability distribution. For the lowest-ability female there is no reason to distort upward her

An *equilibrium*, then, is an effort function $l : [0,1] \to \Re_+$ and a matching function $m : \Re_+ \to [0,1]$ such that

(3) $l(j)$ maximizes $u(a(j)l) - v(l) + m(a(j)l)$ and

(4) $m\left(a(j)l(j)\right) = j$.[9]

2.1. A Closed-Form Example

Suppose that $u(y) = y$, $v(l) = l^2$, and $a(j) = \alpha(2j)^{1/2}$. We will show that the equilibrium matching function is given by $m(y) = gy$, where g solves

$$g(1+g) = 1/\alpha^2. \tag{2.5}$$

If we assume that the equilibrium matching function is given by gy, female j's problem is given by

$$\max_l \; a(j)l + ga(j)l - l^2 \tag{2.6}$$

which implies that

$$l(j) = a(j)(1+g)/2. \tag{2.7}$$

In order to verify that the conjectured matching function is an equilibrium, we need to show that $m\left(y(j)\right) = j$, that is, that the jth-percentile female is being matched with the jth-percentile male in equilibrium. Using the conjectured form for g, and substituting for $y(j)$ and $a(j)$, yields

$$m(y(j)) = g(1+g)\alpha^2 j = j. \tag{2.8}$$

It follows that our conjectured matching rule and labor effort decisions constitute an equilibrium.

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effort level in response to matching considerations since in equilibrium she is already getting the worst quality mate. This implies that her effort level must be her optimal choice absent any matching considerations; that is, $l(0)$ will satisfy the equation $a(0)u'(a(0)l) - v'(l) = 0$. For this to be optimal, it must be the case that $m(a(0)l(0)) = 0$. Note that since female 0 is undistorted, the first-order cost of distorting upward her effort level is zero, which implies that the effort levels will rise very steeply in a neighborhood of $j = 0$ if $a(0) \neq 0$.

[9]The equilibrium female output function c is the unique solution to the restricted initial value problem $c'(j) = \left[v'\left(c(j)/a(j)\right) - a(j)u'\left(c(j)\right)\right]^{-1}$, $0 < dc(j)/dj$, with $c(0)$ given by the solution to $v'(c(0)/a(0)) = u'(c(0))a(0)$.

This type of functional equation frequently arises in the study of signaling games. The question of the existence and uniqueness of solutions of this type is addressed in Mailath [8].

Making use of equations (5) and (7), we can derive the following expression for the impact of a change in α, which can be interpreted as a proportionate change in productivities:

$$(2.9) \qquad \frac{dl(j)}{d\alpha} = \frac{(1+g)(2j)^{1/2}}{2} + \frac{a(j)}{2}\frac{dg}{d\alpha} = \frac{(1+g(2j)^{1/2}}{2} - \frac{(2j)^{1/2}}{(1+2g)\alpha^2}.$$

The two terms in the above expression for the change in female j's effort level correspond to the effects of the change in her wage alone, with the matching function held fixed, and the effects of the change in the equilibrium matching function induced by the change in α. This example makes clear that female j would respond differently to a proportionate change in her own productivity than to a proportionate change in all the females' productivity. The first term is positive, demonstrating that increases in her own productivity increase a female's effort, while the second term is negative, indicating that when all females' productivities increase, the resulting change in the matching function diminishes each female's effort choice. The intuition behind the second effect is straightforward: when all females' productivities go up, the direct effect—if we ignore matching concerns—is to increase females' labor supplies. As a result, the wealth distribution becomes more dispersed, lowering the marginal value of an increase in wealth on matching. This lower marginal benefit negatively impacts females' effort decisions.[10]

It can also be seen from this example that it is competition from below that distorts individuals' effort decisions. If both the set of females and the set of males were truncated, by removing the males and females whose index is greater than one-half, the behavior of the remaining individuals would be unchanged. This follows from footnote 8. The female with least productivity has zero productivity and so chooses $l = 0$. This would not be affected by the removal of the upper-index individuals. However, truncating from the bottom would create a new lowest-productivity female who cannot be distorted. This is intuitive, since any female agent who is distorting her effort level upward is only doing so in order to avoid falling below the output level of the females just below her.

Finally, if we assumed that there were different societies, the members of which only mated with members of their own society, then differences in the distribution of productivities within these societies would generate differences in their effort decisions. For example, if the productivity multiplier in society A was greater than that in society B, $\alpha_A > \alpha_B$, then this would imply that society A's matching function was flatter, $g_A < g_B$, and females with identical ability

[10]It may be possible to construct examples in which the direct income effect of an increase in g is sufficiently stronger than the substitution effect so that female output levels become more concentrated, resulting in a rise in g.

levels would choose to work less in society A than in B. This is because output levels would be more disperse in society A than in B; hence the competition over matches would be more intense in B.[11]

2.2. Interpreting the Model

When females in this model make effort decisions, they take into account the effect of those decisions on their match, since their consumption will depend on that match. Precisely how a female's effort decision affects her match depends both on the effort choices of other women and on the distribution of wealth among men. Since men make no decisions in this model, they play no role other than to serve as prizes in the wealth tournament the females are engaged in. Any other exogenously given set of prizes that are to be awarded to females based on their relative rank in the final wealth distribution would serve the same purpose. The important property is that there is some prize (about which the females care) that is not allocated through standard markets, but rather can be obtained only through the wealth tournament. While the competition for mates has this property, we think there are a number of other goods and decisions that have the same property. We will return to this topic in the concluding remarks at the end of this article.

The model as presented has the females engaged in what is essentially home production; there is no market for labor. It is obvious, however, that if there were a competitive labor market which employed the females, the productivity function a would simply be the wage function, with each woman paid a wage equal to her marginal product.

2.3. Relating the Model to Other Models

The model presented above has implications that differ from those of a more standard model for a wide range of questions. For example, standard models that analyze the impact of income taxes treat a proportional tax as a wage decrease. In such models, the impact of such a tax is the aggregate of the individual agents' responses to the lower wage. The main point of the model above, however, is that an agent responds differently to a lower wage when other agents' wages remain the same than she would if those agents' wages are also lowered. When all agents' wages are lowered, two things happen. First, people care less about whom they match with (unless people respond to the lower wages by increasing their effort sufficiently to keep their incomes from falling). Second, an individual

[11] As was pointed out in footnote 7, $m' = (c')^{-1}$, which here implies that $g = 1/c'(j)$.

will face a different wealth distribution following the aggregate wage change. Thus there will be a different mapping that associates a given wealth level with a particular mate. Standard models analyzing tax policies ignore the effect that a change in the wealth distribution may have on individuals' effort choices. A potentially interesting corollary of this is that there may be a component of a tax policy normally ignored—the effect the policy has on the distribution of income. This aspect of our model might be useful in investigating differences in economic performance between more egalitarian societies, such as Japan or Korea, and less egalitarian ones, such as India or the Philippines.

The example analyzed above and the discussion of the effects of tax policies are both illustrations of a more general point. When increases in wealth or income lead to secondary benefits from increases in rank in a society, individuals will respond differently to individual-specific and aggregate shocks. For problems in which these differences are significant, the common practice of using microeconomic data to try to draw inferences about responses to aggregate shocks presents difficulties that are usually overlooked. The micro data may represent responses to individual shocks, and those responses may systematically diverge from identical shocks that were aggregate (in the sense that all agents were subjected to the same shock). We discuss this point further in the concluding section.

Our model suggests that since people are in competition over their wealth rank, they might respond to the efforts of others to earn more by seeking to earn more themselves. Neumark and Postlewaite [9] examined the effects of other women's employment decisions on women with whom they might be in social competition. Neumark and Postlewaite assumed that siblings are likely to be in social competition over their relative wealth, perhaps because they are likely to know a lot about one another's economic circumstances. This study found some evidence that a woman's employment decisions are positively affected by her sister's decision to become employed.

3. Incomplete Information and Signaling

In the model presented above, an individual's wealth is observable. If wealth is not observable (but is still important to potential mates), individuals with relatively high wealth have an incentive to signal their situation. Building on this observation, we now develop a model of conspicuous consumption reminiscent of Veblen's [12]. In our model, however, agents are fully rational with standard preferences. Agents engage in conspicuous consumption because it is instrumental: in equilibrium, it results in wealthier mates and, consequently, higher consumption.

The underlying logic of the model is that of general signaling models: wealth-

ier agents consume expensive items that can be observed in order to signal the agents' greater wealth. The inferences to be drawn from such consumption are equilibrium inferences. It is not that poorer people cannot buy a pair of Gucci shoes, but rather that they choose not to in equilibrium. Richer individuals choose the signal because the opportunity cost to them in terms of foregone consumption of other types of goods is lower, since they are already consuming more of the other goods. To illustrate our point as starkly as possible, we will consider a variant of the original model in which individuals signal their wealth by destroying a portion of their wealth.

3.1. The Model with Incomplete Information

Assume now that female output levels cannot be observed, though for simplicity continue to assume that male endowments can. Assume also that females can destroy some of their output and that the amount they destroy can be observed. We are interested in equilibria where richer females destroy more of their output than do poorer females in order to signal that they are richer. Note that for reasons similar to those in the previous model, the woman who is destroying the least and hence receiving the worst match should in fact not be destroying any of her output. (Otherwise, lowering the amount destroyed cannot have a negative impact on her match quality, but her consumption would increase.) In equilibrium, the woman receiving the worst match and destroying nothing is the lowest-ability woman.

Since female wealth is unobservable, a male's evaluation of the attractiveness (in terms of contribution to consumption) of potential mates is determined only by observable characteristics of females: the amount of wealth destroyed in conspicuous consumption. Thus the match m is a function of the level of output that a female destroys d. In this case, female j's problem becomes

$$\max_{l,d} u(a(j)l - d) - v(l) + m(d). \tag{3.1}$$

The main difference between this problem and problem (1) is that previously there was a double benefit to wealth acquisition: it increased the quality of her match and increased her consumption. Here she derives no direct benefit from the portion of wealth that she allocates to improving her match quality. If we denote output al by y, the female's choice variables are y and d, and her objective function is

$$u(y - d) - v(y/a) + m(d). \tag{3.2}$$

We show in Proposition 2 in the Appendix that in equilibrium both y and d are nondecreasing in ability.

Consider now the male's problem. Males are interested in matching females with high consumption, that is, females with high values of $y - d$. However, by assumption this consumption is not observable during the matching phase. Instead, males must draw inferences about this consumption from the level of destruction d. Suppose that the level of destruction is a perfect signal about the level of ability and thus consumption.[12] Since d is nondecreasing in ability, this requires that d be strictly increasing, which in turn requires that m be strictly increasing. Of course, m will only be strictly increasing if higher d is a signal of higher consumption, $y - d$, since only then will males prefer to match with females with higher levels of destruction. Equilibrium matching then implies that the female with the median level of conspicuous consumption is matched with the median male: that is, $m(d(j)) = j$.

A *signaling equilibrium* can then be described by an effort function $l : [0, 1] \to \Re_+$, specifying each female's effort choice, and a destruction function $d : [0, 1] \to \Re_+$, which gives each female's destroyed output, and a matching function $m : \Re_+ \to [0, 1]$ such that for all $j \in [0, 1]$,

(12) $(l(j), d(j))$ maximizes $u(a(j)l - d) - v(l) + m(d)$ subject to $d \in [0, a(j)l]$,

(13) d and $al - d$ are both strictly increasing functions,

and, for all j,

(14) $m(d(j)) = j$.

3.2. A Second Closed-Form Example

We now present a second example to illustrate a signaling equilibrium. In this example we take the output levels of the females to be exogenously given by the function $y(j) = e^{\gamma j}$, where $\gamma > 0$. Since their output is exogenous, the females no longer are concerned with their effort level in their preferences, so their utility function can now be taken to be the same as the males, that is, $u(c)+j$. Moreover, we take $u(c) = \ln c$.

The problem of female j is to choose $\hat{d}$ so as to solve

[12] A slightly weaker notion of signaling would only require that the level of destruction be a perfect signal of consumption. This would allow those with different abilities to choose the same level of y and d. Such equilibria can be eliminated using standard refinement arguments.

$$\max_{\hat{d}} \ln(y(j) - \hat{d}) + m(\hat{d}). \tag{3.6}$$

The first-order condition which characterizes the solution to this maximization is given by

$$\frac{-1}{(y(j) - d)} + m'(\hat{d}) = 0. \tag{3.7}$$

If $d(j)$ is the equilibrium level of destruction by female j, then $m(d(j)) = j$. Thus, in equilibrium, $m'(\hat{d}) = \left[d'(d^{-1}(\hat{d}))\right]^{-1}$, so (16) can be written as

$$d'(d^{-1}(\hat{d})) = y(j) - \hat{d}. \tag{3.8}$$

However, in order for $d(j)$ to be the equilibrium level of destruction of female j, it must be the case that $\hat{d} = d(j)$ solves (17). Substituting $\hat{d} = d(j)$ into (17) yields

$$d'(j) = y(j) - d(j). \tag{3.9}$$

Thus the equilibrium destruction function is the unique solution to the initial value problem given by (18) and the initial value condition, $d(0) = 0$. (Recall that the female destroying the least does not destroy any.) The solution is

$$d(j) = (1 + \gamma)^{-1} \left[e^{\gamma j} - e^{-j}\right]. \tag{3.10}$$

An interesting aspect of the signaling equilibrium is that wealthier females destroy a larger fraction of their wealth; that is, $d(j)/y(j)$ is increasing in j. This reflects the declining marginal rate of substitution between consumption and the quality of the match as consumption increases. Moreover, the fraction is decreasing in γ.[13] This illustrates the idea that, for small γ, the distribution of wealth is tight, and the competition for mates is intense, so a large fraction of wealth is destroyed. Conversely, if γ is large, the distribution of wealth is diffuse, the competition for mates is not intense, and a small fraction of wealth is destroyed.

[13]Differentiating $d(j)/y(j)$ with respect to γ yields $(1 + \gamma)^{-2}[(1 + j + \gamma j)e^{-j-\gamma j} - 1]$. This expression is negative, since $1 + j + \gamma j < e^{j+\gamma j}$ (a standard fact about ln is $\ln(1 + x) < x$).

3.3. Interpreting the model

Our simple incomplete information model readily generates the sort of conspicuous consumption behavior described by Veblen [12]. Consistent with Veblen's arguments, the equilibrium of our unobservable wealth model exhibits increasing conspicuous consumption as income rises. Note that in our example, if the parameter in the females' income function γ increases, the distribution of income shifts up and the equilibrium matching function shifts down. That is, when all females are wealthier, more wealth must be wasted in order to obtain the same quality mate.

In our model, no female's wealth is observable. An interesting extension of the logic of the example would include the possibility that some individuals' wealth levels are known to others while other individuals' wealth levels are not known. It is clear that no individual whose wealth is known has any incentive to engage in conspicuous consumption. The sole reason for an individual to conspicuously consume is to alter others' perceptions about that individual's wealth. The cost of conspicuous consumption is independent of what others know, but the benefit of such consumption is limited by their initial uncertainty. Thus an implication of a model with differentially known wealth levels would be that the more certainly known an individual's wealth is, the less that individual will conspicuously consumpe ceteris paribus. In a multiperiod model in which an individual's wealth is learned by others over time, one would then see the newly rich more likely to engage in conspicuous consumption than people with old money.

In comparing our two models, it is ambiguous whether individuals work harder in the observable or unobservable wealth models because there are two opposing forces. In the observable wealth model, an increase in a person's wealth increases both consumption and the quality of her match. In the unobservable wealth model, a person can use wealth for one or the other of these purposes but not both. So there is a sense in which an additional unit of wealth may be more valuable in the obsevable wealth model if one's marginal utility of consumption is held fixed. However, if an individual's wealth is held fixed, her marginal utility will generally be lower in the unobservable wealth model since she does not enjoy all the direct consumption benefits that this wealth would imply in the other model. A higher marginal utility of consumption would encourage her to work harder.[14]

[14]Of course, the lowest-productivity female is working the same amount in the two economies.

3.4. Relating the model to other models

The main point of this model is that because people care about whom they are matched with, they will compete to appear to be desirable matches. Wealth makes one more desirable, and when wealth is incompletely known by others, there is an incentive by the relatively wealthy to make that fact known. In our model, goods that might serve as signals of wealth (because, for example, they are known to be expensive, such as Rolex watches, Gucci shoes, and BMWs) will have qualitatively different demand structures than in standard models. For example, if prices are too low, the good may not support an equilibrium in which the wealthy can use it as a signal: the (relatively) poor may be willing to buy the good and thus destroy its signaling value.

A second important difference between our model and other models is suggested by the remarks in the previous section about new wealth versus old wealth. We pointed out how incomplete information about others' economic characteristics could be a factor in the demand for goods of a certain type. The logic of the model, then, suggests how changes in the information structure can influence economic decisions such as effort and spending choices in ways that differ from standard models. Models of the sort analyzed in this section suggest how changes in the environment that affect the informational structure (increased geographic mobility, for example) might affect economic decisions in ways that standard models cannot capture.

4. Concluding Comments

The models presented above induce a concern for relative rank. This concern arises because there are utility-relevant decisions—in these models, matching decisions—that are affected by one's relative position in the wealth distribution. We want to make several points regarding the manner in which an individual's utility is affected by relative position.

As noted in footnote 4, if an individual's decision problem were described in sufficiently rich detail, relative wealth wouldn't matter: an individual's income and the prices of all utility-relevant objects and decisions would completely determine utility. The concern for relative wealth in our models arises because of the existence of a utility-relevant decision which (in our model) is not mediated by prices—specifically, the matching decision. This raises the question of whether there is a simple reinterpretation of the equilibrium in which an implicit price can be put on the scarce objects. In such a reinterpretation, every man can be associated with a wealth level that is necessary to assure matching with him. One could then think of this as the price function women face for mates. But this is not

quite correct. Unlike the situation in which women work to buy some inelastically supplied good of varying quality like land, women in our models don't really pay for mates. A woman who generates the highest wealth in the first period does match with the wealthiest man, but she also continues to consume the wealth she accumulated. To make the land example analogous to our models, we should have the land simply given away, with the best given to the wealthiest, and so on. The allocation of desirable goods or decisions in accordance with economic performance can substantially differ from the allocation of those goods through normal markets. In particular, we should note that when the desirable goods or decisions are allocated as prizes rather than sold, the standard welfare theorems regarding the Pareto optimality of the outcomes no longer apply.

We chose the present models rather than alternative models in which all goods and decisions are mediated through markets for reasons of descriptive accuracy: it seems obvious to us that there are myriads of goods and decisions about which people care (sometimes passionately) that (1) individuals don't purchase through standard markets and (2) wealthier individuals are better at obtaining than the less wealthy. Country club memberships, charity board invitations, university trusteeships, invitations to chic parties, and assigned seats in churches and synagogues come easily to mind as examples. To be sure, these decisions are often accompanied by money changing hands, but not in the form of a simple purchase of a good or service. Whenever an increase in an individual's position in the wealth distribution by itself increases the likelihood of obtaining desirable outcomes, optimal individual behavior will exhibit some of the qualitative features exhibited in the models analyzed above. We should emphasize that our choice of matching as the decision that causes women to adjust their decisions from what the decisions would otherwise have been is to illustrate the more general effect of utility-relevant decisions that are not mediated by markets. There are presumably many important details of real-world matching that we have abstracted from. We think, however, that while this may not be a particularly compelling model of matching, it clearly illustrates our general point.

We pointed out above the difference between an individual's response to an individual-specific shock and an aggregate shock. In general, one should expect a difference. In an environment in which there are many agents, an individual-specific shock should have no effect on prices, while an aggregate shock generally will. Hence an aggregate shock will affect prices, prompting a response different from that induced by an individual-specific shock. Our model generates different responses to individual and aggregate shocks for similar reasons. Any shock will have a primary effect on an individual, resulting in a change in effort expended. If the shock is an aggregate shock, all individuals will adjust, and as a result, the

mapping that associates a given wealth level with a particular mate will change. The change in this mapping is analogous to the price change one expects in a general equilibrium model that is subjected to an aggregate shock. We point to this difference in response to individual and aggregate shocks in our model because, while economists are accustomed to thinking about general equilibrium price effects that might accompany an aggregate shock, it would be easy to overlook the general equilibrium effects on goods or decisions that are not mediated by standard economic markets, but are affected by relative wealth position.

5. Appendix

Proofs of Propositions 1 and 2

Here we develop the proofs for the two propositions discussed in the preceding paper.

Proof of Proposition 1

PROPOSITION 1 *In the complete information model, output,* $c(j) \equiv a(j)\,l(j)$, *is increasing in* j.

Proof. Consider two arbitrary female agents j and j', where $j' > j$, and suppose (en route to a contradiction) that the optimal output levels of the female good are c and c', respectively, with $c > c'$. Then it must be the case that female j weakly prefers $(c, m(c))$ (an output of c and matching with $m(c)$) to $(c', m(c'))$; that is,

$$u(c) + m(c) - v(c/a(j)) - [u(c') + m(c') - v(c'/a(j))] \geq 0. \qquad \text{(A1)}$$

Similarly, female j' weakly prefers $(c', m(c'))$ to $(c, m(c)))$; that is,

$$u(c') + m(c') - v(c'/a(j')) - [u(c) + m(c) - v(c/a(j'))] \geq 0. \qquad \text{(A2)}$$

Adding (A1) to (A2) yields

$$v(c/a(j')) - v(c'/a(j')) - [v(c/a(j)) - v(c'/a(j))] \geq 0. \qquad \text{(A3)}$$

But the convexity of v implies that $v(c/a) - v(c'/a)$ is decreasing in a when $c > c'$, a contradiction. □

Proof of Proposition 2

PROPOSITION 2 *In the incomplete information model, both the equilibrium output and destruction levels are nondecreasing in ability.*

Proof. Let (y, d) denote an optimal choice for female j and (y', d') an optimal choice for j', and suppose that $j < j'$. Then, (with $a = a(j)$ and $a' = a(j')$),

$$u(y-d)-v(y/a)+m(d)\geq u(y'-d')-v(y'/a)+m(d') \tag{A4}$$

and

$$u(y'-d')-v(y'/a')+m(d')\geq u(y-d)-v(y/a')+m(d). \tag{A5}$$

Adding and canceling yield

$$v(y/a')-v(y'/a')\geq v(y/a)-v(y'/a). \tag{A6}$$

Then, since $a < a'$, we have that $y \leq y'$.(If not, convexity of v implies that $v(y/a)-v(y'/a)$ is decreasing in a.) If $d' \geq y$, we have that $d \leq y \leq d'$, and both y and d are nondecreasing. So suppose $d' < y$. In this case, female j can destroy the same amount as j' while still producing y. Then,

$$u(y-d)-v(y/a)+m(d\,)\geq u(y-d')-v(y/a)+m(d'), \tag{A7}$$

that is,

$$u(y-d)-u(y-d')\geq m(d')-m(d). \tag{A8}$$

Moreover, since $d \leq y \leq y'$,

$$u(y'-d')-v(y'/a')+m(d')\geq u(y'-d)-v(y'/a')+m(d), \tag{A9}$$

that is,

$$m(d')-m(d)\geq u(y'-d)-u(y'-d'), \tag{A10}$$

so that

$$u(y-d)-u(y-d')\geq u(y'-d)-u(y'-d'). \tag{A11}$$

The concavity of u together with $y \leq y'$ then implies that $d \leq d'$. □

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