CARESS Working Paper #95-10
E±cient Policy Choice in a Representative Democracy:
A Dynamic Analysis¤

Timothy Besley
London School of Economics
Houghton Street
London WC2A 2AE

Stephen Coate
Wharton School
University of Pennsylvania
Philadelphia PA 19104

May 1995

Abstract

This paper studies the e±ciency of policy choice in representative democracies. It extends the model of democratic policy making developed in our earlier paper (Besley and Coate (1995)) to a simple dynamic environment. Equilibrium policy choices are shown to be e±cient in the sense that in each period, conditional on future policies being selected through the democratic process, there exists no alternative current policy choices which can raise the expected utilities of all citizens. However, policies which would be declared e±cient by standard economic criteria are not necessarily adopted in political equilibrium. The paper argues that these divergencies are legitimately viewed as \political failures".

¤The authors are grateful for comments from Avinash Dixit and Stephen Morris as well as seminar participants at Columbia, Penn, Melbourne, New South Wales, Northwestern, and Queens. The ¯rst author is also grateful for the hospitality of the Research School of Social Sciences, Australian National University, where his work on the ¯rst draft of this paper was completed.

JEL Classi¯cation Numbers: D61, D78, H11.
Key Words: dynamic e±ciency, democracy, political failure.

1. Introduction

One of the crowning achievements of neo-classical economics is a rigorous appreciation of the performance of markets in allocating resources. However, for resources allocated in the public sector, our understanding is much less complete.[1] In part this reflects the lack of a satisfactory theoretical framework to analyze policy choice in representative democracies. In Besley and Coate (1995), we introduced a new model of democratic policy making as a candidate framework for the systematic analysis of these questions. In this model, policy authority is delegated to particular citizens and individuals compete, through the electoral process, to acquire this power.[2]

Our model of representative democracy has two main merits for the investigation of efficiency issues. First, it is built from the ground up. The primitives of the model are the set of citizens, their preferences, the set of feasible policy alternatives and a constitution which specifies the rules of the decision making process. Policy outcomes are therefore derived directly from the underlying tastes and policy technology. Standard notions of efficiency are then straightforwardly applied.[3] Second, the model handles multi-dimensional policy choices very naturally; there is no need to work with special assumptions about preferences and/or technologies for equilibrium to exist.

This paper puts our model to work to investigate the efficiency of equilibrium policy choices in a world of repeated elections. Efficiency issues are then more subtle because preferences extend over the entire future policy sequence, while policy makers can control only what happens in their current term. We find that,

[1] Despite the vast volume of research on public choice, there is no clear consensus on the ability of representative democracy to produce efficient outcomes. At one extreme are writers in the Chicago tradition, such as Stigler (1982), Becker (1985) and Wittman (1989), who argue that political competition will bring about efficient policy choices. At the other are Buchanan, Tullock and their followers (the "Virginia School"), who see "political failures" as pervasive.

[2] A similar model, which shares the basic idea of viewing candidates for political office as citizens, was introduced independently by Osborne and Slivinski (1995).

[3] In models that begin with "group" preferences belonging to parties or pressure groups as primitives, it is less easy to think about efficiency issues. That is not say that these institutions are not an important part of the political landscape. However, they need to be built from the ground up when questions of efficiency are on the agenda.

while political equilibrium does satisfy a certain e±ciency property, this does not imply that policies which are e±cient according to standard economic criteria are necessarily undertaken. We o®er a comprehensive analysis of why such policies may not be undertaken in political equilibrium. We also discuss whether the non-implementation of economically e±cient policies can legitimately be viewed as \political failures", drawing the parallel with the market failure literature.

To analyze policy choices in a dynamic setting, we embed our model of representative democracy in a simple two period economic model. A single consumption good is produced using labor and citizens obtain utility from consumption and leisure, di®ering in their productive abilities. In each period, the citizen selected to be policy maker chooses the parameters of a negative income tax scheme. The ¯rst period incumbent must also decide whether to implement a public investment which raises individuals' second period abilities.

We begin the analysis by showing that equilibrium policy choices are e±cient in the following sense; in each period, conditional on future policies being selected through the democratic process, there exists no alternative current policy choices which can raise the expected utilities of all citizens. To show that this does not imply the selection of policies which are e±cient according to standard economic de¯nitions, we analyze the public investment decision. Following Samuelson (1950), we consider investments that shift out the utility possibility frontier. Investments satisfying this criterion are technologically e±cient in the sense that if they are not undertaken it is possible, with the available policy instruments, to make all citizens better o®. We identify three principal reasons why such investments may not be undertaken in political equilibrium, each illustrated with an example.

The ¯rst reason concerns non-payment of future compensation. To generate a Pareto improvement, a technologically e±cient public investment may require some individuals to be compensated, via the tax and transfer system, after the gains from the investment are realized. However, compensation may not actually be paid in political equilibrium, since future tax rates are determined by whichever policy makers govern at that time. Thus, if future policy makers are expected to have di®erent preferences, the current incumbent may be deterred from undertaking an investment for which future compensation is required to make it personally bene¯cial.

Our other two reasons apply to technologically e±cient public investments which are straightforward in the sense that they do not require future redistribu-

tion, via changes in tax rates, to generate Pareto improvements. Such investments may not be undertaken because they affect policy choices of future policy makers. In particular, an investment which changes the income distribution will influence the desired tax rates of future policy makers. If this change is unfavorable to the current incumbent, then this can negate the direct benefits of the investment and make undertaking the investment undesirable. Straightforward investments may also not be undertaken because they affect the identity of future policy makers. An investment can change citizens' policy preferences and thereby alter the incentives for individuals to run for office. If this results in the election of a citizen who would choose policies disadvantageous to the current incumbent then he may decide against undertaking it.

Whether the non-implementation of a technologically efficient investment is legitimately viewed as a "political failure", depends on how the latter is defined. Despite the widespread use of this term in the public choice literature, there appears to be no precise statement of what is meant by it. We argue that, in the interests of creating a level playing field for markets and governments, the ideal definition should parallel the conventional idea of market failure. Proceeding in this way, we conclude that a policy maker failing to undertake a technologically efficient investment does constitute a political failure. Thus, our analysis puts the notion of political failure on a rigorous footing and suggests three sources of such problems arising from periodic elections.

This paper contributes to our broader research agenda aimed at developing a comprehensive understanding of the efficiency of outcomes in representative democracy in the context of a single unified framework. The static analysis of our earlier paper identified two dimensions of efficiency issues; does representative democracy select competent leaders and, given the competences of the individuals selected, are their policy choices efficient? We found that political competition had a tendency to "sort in" competent policy makers and that policy choices were efficient in the economist's sense. The results of this paper are less positive; they show that the latter conclusion does not hold up in a dynamic environment with repeated elections.[4]

[4]Further inefficiencies arise when citizens have imperfect information about their leaders. Under these circumstances, incumbents have an incentive to care about their reputations. This, when combined with asymmetric information about the effects of policies, may lead to inefficient policy choices. In the context of a political agency model, Coate and Morris (1994) show that reputational considerations lead politicians to use inefficient "disguised" transfer mechanisms

Our paper brings together ideas from three fields. As noted above, it is directly concerned with issues that are central to public choice. However, the type of economic environment that we study and the definitions of efficiency that we use, are drawn from the normative public economics literature. Our arguments concerning the non-implementation of technologically efficient investments are related to a number of recent papers in the macro-policy literature. Persson and Svensson (1989) argue that debt policy can stray from the efficient path in political equilibrium. Current incumbents run budget deficits to manipulate the choices of future policy makers who do not share their policy preferences. This is one reason for political failure that emerges in our model.[5] Aghion and Bolton (1990) and Milesi-Ferretti and Spolaore (1994) develop models in which current policy choices affect which political party will win in the future. Aghion and Bolton show that the political party currently in power incurs debt because it knows that its rival party will default on this debt and this will make it unattractive to the majority of voters, while in Milesi-Ferretti and Spolaore, the party holding power is induced to raise revenue in an inefficient way because it guarantees that voters will prefer their party in the next election. This incentive to alter the probability distribution over future leaders by current policy choices is also present in our model. Our paper treats these ideas in a unified framework with a fully developed model of the political process.[6]

---

when redistributing to special interests. Rogoff (1990) argues that such considerations lead incumbents to distort the mix of public consumption and investment expenditures.

[5] In a related argument, Tabellini and Alesina (1990) argue that, in an environment of political instability, those currently holding political power have an incentive to borrow from the future (i.e., run deficits) because they can control how such resources are allocated. See also Glazer (1989).

[6] Most of the existing models rest on rather incomplete approaches to the electoral process. Persson and Svensson (1989) offer no political model, simply assuming that current policy makers anticipate that future policy makers will have different preferences. Aghion and Bolton (1989) and Milesi-Ferretti and Spolaore (1994) presume the existence of two political parties with exogenous policy preferences. They do not explain what determines these preferences or why there are only two parties. In addition, in their analysis of equilibrium policy choices, they do not explain how the party who is currently in office came to be in office. They simply point out that an inefficiency would arise if it were in office. Tabellini and Alesina (1990) provide the most complete analysis by assuming that decisions are made by majority rule. However, they must impose a great deal of structure on preferences to guarantee the existence of a median voter. Moreover, to get their argument to work, they must assume that the identity of the median voter may change over time. They argue that this can arise \due to: (i) random shocks to the costs of voting that affect the participation rate .....; or (ii) changes in the eligibility of

The organization of the remainder of the paper is as follows. The model is presented in section 2. In section 3 political equilibrium policy choices are characterized and the efficiency result is established. Section 4 introduces the definition of a technologically efficient investment and analyzes whether the public investment satisfying this condition is a sufficient condition for it to be introduced in political equilibrium. Section 5 discusses the interpretation of the results, while section 6 concludes.

2. The Model

The aim is to develop a simple, tractable model to investigate the efficiency of policy choice in a representative democracy. The basic framework we employ is standard in the literature on the political economy of income taxation, see for example Romer (1975) and Meltzer and Richard (1981). Our dynamic extension incorporates a first period public investment which makes individuals more productive in the second period. This investment can best be thought of as provision of infrastructure or education.

2.1. The Economic Environment

The economy consists of N citizens, indexed by $i \in N = \{1, ..., N\}$, and lasts for two time periods, indexed by $t \in \{1, 2\}$. There is a single (non-storable) consumption good denoted by x. The consumption good is produced using a single factor, labor, denoted by ℓ. Each citizen is endowed with one unit of labor in each period. However, citizens differ in their productive abilities: if citizen i provides ℓ units of labor in period t, he produces $a^i_t \cdot \ell$ units of x, where a^i_t is his period t "ability". The consumption good is produced by competitive firms, with each citizen's wage rate in each period being equal to his ability. Citizen i's utility in period t is given by $u(x^i_t, \ell^i_t)$, where x^i_t is his consumption and ℓ^i_t is the amount of labor he supplies. The common per period utility function $u(\cdot)$ is smooth, strictly quasi-concave, increasing in x and non-increasing in ℓ. Citizens do not discount their second period utility.

At the beginning of each period, a policy maker selects the parameters of a negative income tax.[7] These are the tax rate $t \in [0, 1]$ and the guarantee $T \in R$.

---

the voting population" (p.39).

[7] Thus, we are implicitly restricting the redistributive instruments the policy makers can

With policy choice $(t;T)$, a citizen with income y has a tax bill of $t¢y ¡ T$. We let $(t_¿;T_¿)$ denote the tax system in period ¿. The ¯rst period policy maker must also choose whether or not to undertake a discrete public investment. This decision is denoted by g, with $g = 1$ (0) meaning that the investment is (not) undertaken. The investment costs C units of the consumption good but is productive, raising all individuals' second period abilities. Thus, for all citizens i, $a_2^i = a_2^i(g)$ where $a_2^i(1) ¸ a_2^i(0)$.[8]

In each period, after the policy maker has selected the tax system, citizens decide how much labor to supply. With the tax system $(t_¿;T_¿)$, citizen i will supply $`(t_¿;T_¿;a_¿^i)$ units of labor, where

$$`(t;T;a) = \arg\max \{u((1 ¡ t)a` + T;`) : ` 2 [0;1]\}: \tag{2.1}$$

He will earn $y(t_¿;T_¿;a_¿^i)$ units of income, where

$$y(t;T;a) = a`(t;T;a); \tag{2.2}$$

and will enjoy a utility level $v(t_¿;T_¿;a_¿^i)$, where

$$v(t;T;a) = u((1 ¡ t)y(t;T;a) + T;`(t;T;a)): \tag{2.3}$$

The policy makers' tax and investment choices must be feasible in the sense that, given the labor supply decisions that they produce, net tax revenues equal public expenditures. The period one policy maker's feasibility constraint is

$$t_1 \sum_{i=1}^{N} y(t_1;T_1;a_1^i) ¡ NT_1 = Cg: \tag{2.4}$$

The left hand side represents net tax revenues and the right hand side public expenditures. Let Z_1 denote the set of period one policy choices $(t_1;T_1;g)$ which

use. This restriction will not be explicitly derived from some underlying constraints (such as imperfect information) faced by policy makers. It is therefore best interpreted as a constitutional restriction in the spirit of Brennan and Buchanan (1980). It should be noted, however, that for our purposes, the precise restrictions are not really important. Our aim is to discuss e±ciency given a set of policy instruments. One could undertake the same analysis with any given set of redistributive instruments.

[8] The productive investment may create incentives for individuals to borrow from one another: a citizen who expects to bene¯t greatly from the investment may gain from borrowing from a non-bene¯ciary in the ¯rst period. To focus directly on the e±ciency of public choices, however, we avoid the complications associated with introducing a private loan market.

satisfy this constraint. In period two there are no public expenditure decisions, so the policy maker's feasibility constraint is simply

$$t_2 \sum_{i=1}^{N} y(t_2; T_2; a_2^i(g)) - N T_2 = 0. \tag{2.5}$$

Let $Z_2(g)$ denote the set of period two policy choices $(t_2; T_2)$ which satisfy this constraint.

2.2. The Political Process

The model of policy making is based on Besley and Coate (1995).[9] We suppose that in each period one member of the community is selected to make policy choices, with an election determining the choice of citizen to do this. All citizens are able to run in these elections and each must choose whether or not to declare themself as a candidate. Running for office is not costly.[10] All individuals in the society then vote over the set of self-declared candidates. The candidate with the most votes wins (there is plurality rule). In the event of ties, the winning candidate is chosen randomly with each tying candidate having an equal chance of being selected. If only one individual runs for office then he is automatically selected. In the event that no citizen runs, taxes are set equal to zero and the laissez-faire outcome prevails. The winning candidate selects policy only in the period for which he is elected to govern.

To keep the notation simple, our formal account of the political process focuses only on the second period election. Thus, we assume that a particular citizen (say, citizen k) is already in office at the beginning of period one. Our description therefore begins after the first period election is over. In this sense, our framework is similar to a two period political agency model (see, for example, Austen-Smith and Banks (1992), Coate and Morris (1994) and Harrington (1993)). In our examples in Section 4, we will make assumptions about the ability type of the first period policy maker. We will therefore check that having a first period policy

[9] It should be noted that the standard Downsian model of democratic policy making in which two vote maximizing parties compete by offering platforms to the electorate is not applicable to the environment studied in this paper. This is because the first period policy choice is two dimensional; it involves both the choice of a tax rate and an investment decision. It is well known that in such environments, the Downsian model typically fails to produce a prediction.

[10] The results of the paper would be unaffected by the assumption that running for office has a small cost (say, in the form of reduced consumption) as in our earlier paper.

maker with the postulated ability type is consistent with first period political equilibrium.[11]

The timing of economic and political choices is illustrated in Figure 1. First, citizen k (the period one incumbent) selects a first period policy $(t_1; T_1; g) \in Z_1$. Then citizens choose their first period labor supplies. After this, the election is held and citizens decide whether or not to run for office. The winning candidate then selects a second period policy $(t_2; T_2) \in Z_2(g)$. Finally, citizens make their second period labor supply decisions.

The outcome of the political process is a particular policy sequence. A policy sequence consists of a first period policy choice $(t_1; T_1; g) \in Z_1$ and a probability distribution over second period policy choices $\pi : Z_2(g) \to [0; 1]$. Thus, $\pi(t_2; T_2)$ is the probability that the second period policy choice will be $(t_2; T_2)$. Probability distributions over second period policy choices arise from any uncertainty about the outcome of the election.[12] We will use the notation $\phi(\pi)$ to denote the support of the probability distribution π; that is, the set of second period policy choices such that $\pi(t_2; T_2) > 0$:

3. Equilibrium Policy Sequences

In this section we characterize the policy sequences that may arise in political equilibrium. The characterization proceeds in the obvious way. First, we describe the second period policy choice of the winning candidate and the election outcome, taking as given the public investment decision. Then we characterize the first period policy maker's optimal policy choice which anticipates the second period political competition.

3.1. Period Two Election and Policy Choice

The electorate knows that the citizen who wins the election will simply implement his preferred period two policy. While those who run for office may have

[11] The assumption that a particular citizen is in office in period one implicitly assumes that there is a first period political equilibrium which involves at least one citizen running with positive probability. Given that running for office is costless and that citizens have different abilities, this is indeed the case. It is not an equilibrium to have all citizens choosing not to run for office in the first period.

[12] This would occur, for example, if citizens randomized over which candidate to vote for or if two candidates tied.

an incentive to promise to do something other than this, such promises are not credible. When the investment decision in period one is g, citizen i's preferred policy is given by:

$$(t_2^i(g); T_2^i(g)) \in \arg\max \left\{ v(t_2; T_2; a_2^i(g)) \mid (t_2; T_2) \in Z_2(g) \right\}. \qquad (3.1)$$

We will assume that the solution to (3.1) is unique.[13] Associated with each citizen's election, therefore, will be a utility imputation $!_i(g) = (!_{1i}(g); \ldots; !_{Ni}(g))$; where $!_{ji}(g) = v(t_2^i(g); T_2^i(g); a_2^i(g))$ is individual j's period two utility if i is elected. If no citizen stands for office the second period policy will be $(t_2^0(g); T_2^0(g)) = (0; 0)$. We denote the utility imputation in this case as $!_0(g) = (!_{10}(g); \ldots; !_{N0}(g))$.

In their voting decisions, citizens anticipate the policy that would be chosen by each candidate and cast their vote for the candidate whose preferred alternative promises them the highest level of utility. If there are two or more candidates with this property, citizens vote for each of their preferred candidates with equal probability.[14] Thus successful candidates will be those whose preferred alternative is popular among the electorate. Suppose that the set of candidates that stands is $C \subseteq N$. Then we use $P^i(C)$ to denote the probability that candidate i receives the most votes and hence wins the election. Since if only one candidate runs he is automatically selected to choose policy, $P^i(C) = 1$ if $i \in C$ and $\#C = 1$.

Understanding how citizens will vote, each individual decides whether or not to run for the position of policy maker. Citizens are motivated to run for office by the possibility that they can move policy in their preferred direction. Whether they are able to do this depends on which other citizens decide to enter the race.

[13] While it is highly unlikely that (3.1) will have multiple solutions, it is difficult to come up with a set of simple sufficient conditions to rule it out. The conditions used by Meltzer and Richard (1981) to guarantee the inverse relationship between tax rates and productive ability ($u(\cdot)$ is strictly concave and satisfies the Inada conditions) are certainly sufficient, but are much stronger than we need.

[14] Thus we are assuming that citizens vote sincerely. This should be contrasted with the hypothesis that citizens vote strategically, with their voting decisions being a best response to what others do. In some circumstances, such as when there are three candidates, two of which are tied for first place, sincere voting and strategic voting may be inconsistent. In our earlier paper, we addressed this by beginning with the assumption that citizens vote strategically, and then imposing the refinement that when sincere voting produced a clear winner and hence was a best response, individuals vote this way. All of the results and examples in this paper would go through if we adopted that approach here. However, we are able to avoid some notational complexity by assuming sincere voting up front.

The decision to enter is therefore strategic and is modeled as a game between the N citizens. We begin by de¯ning each citizen's pure strategy $s^j \in \{0,1\}$, where $s^j = 1$ denotes entry by citizen j. A pure strategy pro¯le will be denoted $s = (s^1, \ldots, s^N)$. If the pure strategy pro¯le is s, the set of candidates is $C(s) = \{i \mid s^i = 1\}$.

If citizen i enters then the probability that he/she wins is $P^i(C(s))$. Thus the payo® to any citizen j given their own entry decision and those of other citizens is given by

$$U^j(s,g) = \sum_{i \in C(s)} P^i(C(s))\,!_{ji}(g) + P^0(C(s))\,!_{j0}(g). \tag{3.2}$$

The notation $P^0(C(s))$ denotes the probability that the default outcome is selected. Thus, $P^0(C(s))$ equals one if $C(s) = \emptyset$ and zero otherwise. Citizen j's payo® is therefore the probability that each candidate i wins in the race times j's payo® from i's preferred policy. As (3.2) makes clear, the payo®s will depend upon the investment decision g.

To ensure the existence of an equilibrium, we need to allow the use of mixed strategies. Let σ^j be a mixed strategy for citizen j, with the interpretation that σ^j is the probability that j runs for o±ce. The set of mixed strategies for each citizen is then the unit interval $[0,1]$. A mixed strategy pro¯le is denoted by $\sigma = (\sigma^1, \ldots, \sigma^N)$ and citizen j's expected payo® under the mixed strategy pro¯le σ is denoted by $\pi^j(\sigma, g)$.[15] An equilibrium of the entry game is a mixed strategy pro¯le $(\hat{\sigma}^1, \ldots, \hat{\sigma}^N)$ such that for all $j \in \{1, \ldots, N\}$, $\pi^j(\hat{\sigma}^j, \hat{\sigma}_{ij}, g) \geq \pi^i(\sigma^j, \hat{\sigma}_{ij}, g)$ for all $\sigma^j \in [0,1]$. This game ¯ts the conditions of the standard theorem for proving the existence of an equilibrium due to Nash (1950). Thus in period two, an equilibrium of the entry game exists.[16]

Let $E(g)$ denote the set of second period equilibrium strategy pro¯les when the investment decision is g. Associated with any $\sigma \in E(g)$ will be a probability

[15] Citizen j's expected payo® from a mixed strategy pro¯le σ is given by

$$\pi^j(\sigma, g) = \prod_{i=1}^{N} \sigma^i U^j(1, \ldots, 1; g) + \prod_{i=2}^{N} \sigma^i (1 - \sigma^1) U^j(0, 1, \ldots, 1; g) + \ldots + \prod_{i=1}^{N} (1 - \sigma^i) U^j(0, \ldots, 0; g).$$

[16] For further discussion of properties of equilibria, see our earlier paper, which also develops some examples. We show that in many environments, there are equilibria with only a few candidates running, which turns on the fact that running for o±ce is a public good.

distribution over second period policy makers $r(\cdot) : N \cup \{0\} \to [0,1]$, where $r(\cdot)(i)$ denotes the probability that citizen i will be the policy maker and $r(\cdot)(0)$ denotes the probability that no-body runs. For example, if $\cdot = (1,1,0,\ldots,0)$, $r(\cdot)(1) = P^1(\{1,2\})$ and $r(\cdot)(2) = P^2(\{1,2\})$. The distribution over second period policy makers will, in turn, induce a probability distribution over second period policy outcomes. It will be convenient to introduce the notation $\pi^1(g,r)$ to denote the probability distribution over $Z_2(g)$ generated by the probability distribution over policy makers r. Clearly,

$$\pi^1(g,r)(t_2,T_2) = \sum_{j \in \{i \in N \cup \{0\} : (t_2^i(g),T_2^i(g)) = (t_2,T_2)\}} r_j. \tag{3.3}$$

Since there is a finite number of citizens, this probability distribution has finite support.

3.2. Period One Policy Choice

In choosing policy, the first period policy maker (citizen k) will anticipate the election outcome in the second period and the subsequent policy choices. Assume that he anticipates that the second period equilibrium will be $\cdot(g) \in E(g)$ when the investment decision is $g \in \{0,1\}$. He will then anticipate that the probability distribution over second period policy choices is $\pi^1(g, r(\cdot(g)))$ when the investment decision is g. Thus he will choose a first period policy to solve

$$\max_{(t_1,T_1,g)} v(t_1,T_1,a_1^k) + \sum_{(t_2,T_2) \in \phi(\pi^1)} \pi^1(g, r(\cdot(g)))(t_2,T_2) v(t_2,T_2,a_2^k(g)) \tag{3.4}$$

$$\text{subject to } (t_1,T_1,g) \in Z_1.$$

The period one incumbent therefore chooses the policy recognizing the endogeneity of second period policy choices. How this works in practice should be clearer in the context of the examples that we present below.

3.3. Equilibrium

A policy sequence is a political equilibrium if it could be generated by the political decision making process described above. Putting the different elements of the model together, we can now offer a formal definition.[17]

[17] The reader should be reminded that the definition that we offer does not impose the requirement that citizen k (the first period incumbent) be selected in a first period election. Thus our

De¯nition 1. A policy sequence $f(t_1^{\alpha}; T_1^{\alpha}; g^{\alpha}); \frac{1}{4}^{\alpha}g$ is a political equilibrium if there exists a function $\circ^{\alpha} : f0; 1g \to [0; 1]^N$ such that (i) $\circ^{\alpha}(g) \in E(g)$ for all $g \in f0; 1g$, (ii) $(t_1^{\alpha}; T_1^{\alpha}; g^{\alpha})$ satis¯es (3.4) given $\circ^{\alpha}(\phi)$, and (iii) $\frac{1}{4}^{\alpha} = {}^{1}(g^{\alpha}; r(\circ^{\alpha}(g^{\alpha})))$.

The function $\circ^{\alpha}(\phi)$ plays a key role in this de¯nition, representing the ¯rst period policy maker's beliefs about the equilibrium of the entry game, conditional on the period one investment decision. According to condition (i) of the de¯nition, these beliefs are consistent with equilibrium behavior | anticipated strategies are indeed equilibrium strategies. Part (ii) says that the ¯rst period policy maker maximizes his expected utility using his beliefs about the entry game. The ¯nal condition requires that the beliefs are correct along the equilibrium path | the actual probability distribution over period two policies is the same as that generated by the anticipated strategy pro¯le $\circ^{\alpha}(g^{\alpha})$.

3.4. An E±ciency Result

In political equilibrium, the right to choose policy in each period is vested in a particular citizen. That citizen will choose the policy which maximizes his own expected payo®. This guarantees that equilibrium policy choices have a certain e±ciency property. Our next step is to formalize this idea and to comment on its signi¯cance. We begin with some additional notation. For all $i \in N$, let

$$V^i(t_1; T_1; g; \circ(\phi)) = v(t_1; T_1; a_1^i) + \sum_{(t_2;T_2) \in \phi({}^{1})} {}^{1}(g; r(\circ(g)))(t_2; T_2)v(t_2; T_2; a_2^i(g)): \tag{3.5}$$

This is citizen i's expected lifetime utility if the equilibrium of the entry game is described by the function $\circ(\phi)$ and the ¯rst period policy choice is $(t_1; T_1; g)$.

Proposition 1. Let $f(t_1^{\alpha}; T_1^{\alpha}; g^{\alpha}); \frac{1}{4}^{\alpha}g$ be a political equilibrium policy sequence and let $\circ^{\alpha}(\phi)$ represent the associated beliefs about individuals' entry decisions. Then, (i) there exists no $(t_1; T_1; g) \in Z_1$ such that

$$V^i(t_1; T_1; g; \circ^{\alpha}(\phi)) > V^i(t_1^{\alpha}; T_1^{\alpha}; g^{\alpha}; \circ^{\alpha}(\phi)) \text{ for all } i \in N;$$

and (ii) for all $(t_2^{\alpha}; T_2^{\alpha}) \in \phi(\frac{1}{4}^{\alpha})$ there exists no $(t_2; T_2) \in Z_2(g^{\alpha})$ such that

$$v(t_2; T_2; a_2^i(g^{\alpha})) > v(t_2^{\alpha}; T_2^{\alpha}; a_2^i(g^{\alpha})) \text{ for all } i \in N:$$

---

de¯nition is, in this sense, too permissive. However, any properties possessed by all equilibrium policy sequences as de¯ned here, will also be shared by those with the additional restriction that the ¯rst period policy maker be electable.

Proof. See Appendix. ■

Part (i) of the Proposition says that, conditional on second period policy choices being chosen through the democratic process and the equilibrium of the entry game being $\circ^{\alpha}(g)$, there exists no alternative first period policy choice which gives every citizen more expected utility. Part (ii) says that for every second period policy choice which arises with positive probability, there is no alternative second period policy choice which, given the public investment decision, gives every citizen more second period utility.[18]

Proposition 1 parallels a result (Proposition 9) in Besley and Coate (1995). As we argued there, it is natural to find this when the utility of the policy maker is included in the notion of an efficient outcome. It represents the idea that politicians ought to be making the best use of their policy instruments to serve their ends. Here these ends are defined selfishly, although the model can easily handle more general preferences. As we shall see now, the fact that policies can not be found to make all citizens better off when future policies are determined in political equilibrium does not guarantee that public investment decisions satisfy reasonable economic criteria of efficiency.

4. Public Investment in Political Equilibrium

It is common in economics to define efficiency without reference to the process for making policy choices. Standard notions of efficiency are technological in nature, taking no account of the institutional constraints which determine how actual choices are made. In this vein, perhaps the most obvious criterion for an investment to be efficient is that, if it were undertaken, the utility possibility frontier

[18] The notion of efficiency discussed here is weaker than is usual since it only requires that there exist no policy choice which makes all citizens strictly better off rather than all at least as well off and some strictly better off. Part (ii) of Proposition 1 can in fact be strengthened in this way since $(t_2^i(g); T_2^i(g))$ is, by assumption, a uniquely optimal choice for citizen i. This implies that changing $(t_2; T_2)$ must reduce citizen i's utility. Part (i) can similarly be strengthened if $(t_1^{\alpha}; T_1^{\alpha}; g^{\alpha})$ is a unique maximizer for citizen k. It is possible, however, that this is not the case. It might be that $g^{\alpha} = 0$ and citizen k is indifferent between $(t_1^{\alpha}; T_1^{\alpha}; 0)$ and $(t_1^{0}; T_1^{0}; 1)$, with the remaining agents strictly preferring $(t_1^{0}; T_1^{0}; 1)$: While examples of this form can be constructed, this is rather unlikely. Moreover, even in this case, there remains the possibility of citizen k commiting to the electorate in advance to choose $(t_1^{0}; T_1^{0}; 1)$ rather than $(t_1^{\alpha}; T_1^{\alpha}; 0)$. Such a commitment would be credible and would likely attract more votes.

would shift to the right.[19] It is an appealing criterion since for any utility allocation which could be reached without the investment, there exists a Pareto superior utility allocation available if it is undertaken. Hence, from a technological viewpoint the investment expands society's opportunity set. A social planner would choose to undertake the investment using any set of non-negative distributional weights.

Pursuing this formally, we begin with a particular policy sequence in which the investment is not undertaken. We then imagine that it is made, with a particular method of ¯nance and distribution of gains. Technologically e±cient investments are those for which feasible policy choices exist in which everyone gains compared with any policy sequence in which the investment is foregone.[20] This is embodied in the following de¯nition:

De¯nition 2. A public investment is technologically e±cient if for every policy sequence $f(t_1; T_1; 0); \mu g$ there exists $(t_1^0; T_1^0)$ and, for every $(t_2; T_2) \in \Phi(\mu)$, $(t_2^0; T_2^0)$ such that:

(i) $(t_1^0; T_1^0; 1) \in Z_1$ and $(t_2^0; T_2^0) \in Z_2(1)$, and

(ii) $v(t_1^0; T_1^0; a_1^i) + v(t_2^0; T_2^0; a_2^i(1)) > v(t_1; T_1; a_1^i) + v(t_2; T_2; a_2^i(0))$ for all $i \in N$.

The notion of a technologically e±cient investment abstracts from the fact that policy choices are made in political equilibrium. Our ¯rst result gives a su±cient condition for such investments to be implemented when a political process determines policy.

Proposition 2. Let $f(t_1^\alpha; T_1^\alpha; g^\alpha); \mu^\alpha g$ be a political equilibrium policy sequence and let $\circ^\alpha(\Phi)$ represent the associated beliefs about individuals' entry decisions. Suppose that $r(\circ^\alpha(1))(j) = 1$ for some j such that $a_2^j(1) = a_2^k(1)$. Then, if the public investment is technologically e±cient, $g^\alpha = 1$:

[19] This is the criterion suggested by Samuelson (1950). A weaker criterion is the Hicks-Kaldor criterion (Hicks (1939), Kaldor (1939)) that there exist a point on the utility possibility frontier when the investment is undertaken which Pareto dominates the utility allocation which would arise if it is not undertaken. Scitovsky (1941) pointed out that this criteria could be inconsistent if the relevant utility possibility frontiers with and without the investment crossed.

[20] Since policy sequences can involve randomly selected second period policy choices, there is a choice between ex ante or ex post domination. Our de¯nition requires that for every second period policy choice which might be selected, there is an alternative second period choice which makes all citizens better o® ex post.

Proof. See Appendix. ■

The key condition here is that the first period policy maker (citizen k) anticipates that the second period policy maker will share his preferences. All technologically efficient investments have, by definition, a way of redistributing the gains so that all individuals benefit. Thus a policy maker who knows that someone who shares his preferences will govern next period is sure that such redistribution as needed to make it worthwhile for him, will be forthcoming.

The condition of Proposition 2 is tantamount to complete political stability. The latter turns out to be crucial to the optimistic conclusions about the ability of representative democracy to implement technologically efficient investments. Political instability can lead the current incumbent to forgo the investment because he fears that he will not receive compensation for any costs that he bears in introducing it. The following example provides a simple illustration of this possibility.

Example 1

Suppose that the population is equally divided into two ability types | low and high. Let N_L denote the set of low ability individuals and let N_H the set of high ability individuals. Assume that individuals care only about their consumption and are risk neutral, so that $u(x; \ell) = x$. This assumption means that citizens will supply their one unit of labor inelastically in each period. Let a_L denote the first period ability of low ability individuals and a_H ($> a_L$) that of high individuals. If the investment is not undertaken, individuals' second period abilities equal those in the first period. If the investment is undertaken, it raises the abilities of high ability individuals. Thus, $a_2^i(1) = a_H + \pm$ for $i \in N_H$, while $a_2^i(1) = a_L$ for $i \in N_L$. We assume that $\pm/2 > C/N$, which can be shown to imply that the public investment is technologically efficient.

We now solve for the political equilibrium policy sequence. We will do so under the assumption that the first period policy maker is a low ability individual. Below, this will be shown to be consistent with first period political equilibrium. Whether or not the investment is undertaken, the only possible pure strategy equilibria in the second period election involve a single low ability individual running against a high ability individual. If elected, a high ability individual will always choose a zero rate of taxation and thus

$$\left(t_2^H(1); T_2^H(1)\right) = \left(t_2^H(0); T_2^H(0)\right) = (0;0):$$

Given the assumed labor supply inelasticity, a low ability individual will always choose a tax rate of 100%. Thus,

$$\left(t_2^L(1); T_2^L(1)\right) = (1; \bar{a} + \pm/2) \text{ and } \left(t_2^L(0); T_2^L(0)\right) = (1; \bar{a});$$

where $\bar{a} = (a_L + a_H)/2$ is the mean ¯rst period ability.

Since he is of low ability, the ¯rst period policy maker will always set a tax rate of 100%; the only question is whether he will undertake the investment. If he does so his expected payo® is

$$\bar{a} - C/N + \frac{1}{2}[a_L + \bar{a} + \pm/2];$$

while not introducing the investment yields an expected return

$$\bar{a} + \frac{1}{2}[a_L + \bar{a}]:$$

Clearly, he is better o® not introducing the investment if $\pm/4 < C/N$. Under this assumption, therefore, the equilibrium policy sequence involves a ¯rst period policy choice $(1; \bar{a}; 0)$ and a probability distribution over second period policy choices which selects $(1; \bar{a})$ with probability $\frac{1}{2}$ and $(0;0)$ with probability $\frac{1}{2}$. Its e±ciency not withstanding, the investment is not undertaken.

It only remains to check that the ¯rst period policy maker being of low ability is consistent with ¯rst period political equilibrium. This is clearly the case: there exists a ¯rst period pure strategy equilibrium involving a single low ability individual running against a high ability individual. The high ability individual would select the ¯rst period policy $(0; -C/N; 1)$. All individuals would vote for the candidate who shared their ability type.

The key feature of this example is that the investment harms the ¯rst period policy maker in period one, requiring period two compensation if he is to be induced to undertake it. Such compensation will be forthcoming if an individual with similar preferences is elected to o±ce, but not otherwise. Since there is

a possibility that an individual with opposing preferences will be elected, the first period policy maker is deterred from undertaking the investment.[21] Thus political turnover can induce policy makers to resist making investments which entail short-run costs, if the future gains are contingent on actions of future policy makers.

In this example, compensation is required to achieve a Pareto improvement over the status quo. We now turn to public investments with a structure of costs and benefits where compensations are not necessary to achieve Pareto gains. These are cases where it is not necessary to manipulate future taxes and transfers in order to make everyone better off. All benefit from the investment, provided only that the tax rate does not increase and the guarantee does not decrease. This will be the case for public investments which produce significant direct benefits for all citizens and/or those which have small costs.

To define formally such investments, we need two additional pieces of notation. First, for any $(t_2; T_2) \in Z_2(0)$, let $A(t_2; T_2)$ denote the set of second period policy pairs which are feasible if the investment is undertaken, where the income guarantee is higher and the tax rate is lower.[22] The set $A(t_2; T_2)$ is non-empty because the investment raises individuals' productive abilities. Second, let $Z_2^*(g)$ $(\subset Z_2(g))$ denote the set of period two policy choices which are not strictly Pareto dominated.[23] Using this notation, we have:

Definition 3. A technologically efficient public investment is straightforward if for every feasible policy sequence $\{(t_1; T_1; 0); \gamma g$ such that $\phi(\gamma) \subset Z_2^*(0)$ there exists $(t_1'; T_1')$ such that:

(i) $(t_1'; T_1'; 1) \in Z_1$, and

[21] A related argument is made by Dixit and Londregan (1994). They argue that democratic policy making deters efficient private sector decisions because of the inability of political parties to commit to let individuals suffer economic losses and/or keep economic gains. Thus, individuals in declining industries do not move to more productive jobs because they anticipate that the political process will either give their current industry transfers or tax away their gains in the more productive employment.

[22] Formally, for all $(t_2; T_2) \in Z_2(0)$,

$$A(t_2; T_2) = \{(t_2'; T_2') \in Z_2(1) : t_2' \leq t_2 \text{ and } T_2' \geq T_2\}.$$

[23] Thus, if $(t_2; T_2) \in Z_2^*(g)$ then reducing t_2 would not allow T_2 to be raised.

(ii) for every $(t_2; T_2) \in \phi(\frac{1}{4})$, $(t_2^0; T_2^0) \in \alpha(t_2; T_2)$ implies that

$$v(t_1^0; T_1^0; a_1^i) + v(t_2^0; T_2^0; a_2^i(1)) > v(t_1; T_1; a_1^i) + v(t_2; T_2; a_2^i(0)):$$

The earlier definition required that, for any given policy sequence in which the investment were not undertaken, there existed a way of financing the investment and redistributing the proceeds in the second period to make all citizens better off. This definition requires that there exists a way of financing the investment which makes all citizens better off for all second period policy pairs which involve a lower tax rate and a higher guarantee. Observe that the investment considered in Example 1 violates this condition for the policy sequence involving a first period policy choice $(1; \bar{a}; 0)$ and a second period policy choice $(0; 0)$. There is no way of financing the investment in the first period to make low ability individuals better off, if the second period tax rate is zero.[24]

It would seem more hopeful that straightforward investments would be chosen in political equilibrium. The next result gives some conditions for this to be the case.

Proposition 3. Let $\{(t_1^\alpha; T_1^\alpha; g^\alpha); \frac{1}{4}^\alpha\}$ be a political equilibrium policy sequence and let $\circ^\alpha(\phi)$ represent the associated beliefs about individuals' entry decisions. Suppose that (i) $r(\circ^\alpha(1)) = r(\circ^\alpha(0))$ and (ii) $(t_2^i(1); T_2^i(1)) \in \alpha(t_2^i(0); T_2^i(0))$ for all i such that $r(\circ^\alpha(1))(i) > 0$. Then, if the public investment is technologically efficient and straightforward, $g^\alpha = 1$:

Proof. See Appendix. ■

This proposition weakens the condition of complete political stability from Proposition 2. It requires only that the investment does not affect the probability distribution over future policy makers or cause them to select a policy pair involving a higher tax rate or lower guarantee. In Example 1, both of these conditions were satisfied. Nonetheless, the conditions for straightforward, technologically efficient investments to be made in political equilibrium are strong. Our next two

[24] The definition also restricts comparisons to feasible policy sequences which select only Pareto undominated tax rates in period two (i.e., which satisfy $\phi(\frac{1}{4}) \frac{1}{2} Z_2^\alpha(0)$). This, for example, rules out comparisons with policy sequences which set t_2 so high that individuals are deterred from supplying any labor in period two. In that case, no investment, no matter how much it enhanced productivity, could satisfy the requirements of the definition.

examples illustrate cases where the conditions of Proposition 3 fail and technologically e±cient and straightforward investments are not undertaken. They are very stylized, intending to give pure cases of each violation. We start, in reverse order, with an example illustrating a case where condition (ii) of the proposition fails.

Example 2

Suppose that the population is divided into three groups; poor, middle class and rich. Let $N_R; N_M; N_P$ denote the sets of rich, middle class, and poor individuals respectively. Suppose that the number of poor citizens equals the number of rich and middle class citizens; i.e., $\#N_P = \#N_M + \#N_R$. Assume that individuals' preferences take the following form: $u(x; l) = x ¡ l^2=2$. Let a_j denote the ¯rst period ability of individuals in social class j (j 2 $fP; M; Rg$)and assume $a_P < a_M < a_R$. The public investment raises each middle class individual's ability by an amount ± 2 $(0; a_R ¡ a_M)$. It does not a®ect poor or rich individuals' abilities. We suppose that the public investment is costless, which implies that it is both technologically e±cient and straightforward.[25]

The political equilibrium policy sequence will be derived under the assumption that the ¯rst period policy maker is poor. It will also be assumed that the ¯rst period policy maker anticipates that, whatever his investment decision, the second period election will involve a poor individual running against a middle class individual. This is consistent with political equilibrium. The middle class candidate will be supported by both the middle class and the rich and hence each candidate will win with probability 1=2. No rich citizen has an incentive to enter, since his entry will result in the poor candidate winning for sure.

It is straightforward to solve for the second period policies which would be selected by a successful poor or middle class candidate. The assumed utility function implies that labor supply, and hence earnings, depend only on the tax rate t. This means that if $(t_2; T_2)$ 2 $Z_2(g)$, then it must be the case that $T_2 = t_2 ¢ \bar{y}(t_2; g)$ where $\bar{y}(t_2; g)$ is mean second period earnings when the tax rate is t_2 and the investment decision is g. We can therefore simplify the optimization in (3.1) to a

[25] This is strictly speaking inaccurate since when $t_2 = 0$, it is only possible to weakly pareto dominate the allocation without the investment. However, if we allowed the investment to give a very small gain to the rich and poor, then the de¯nition would be satis¯ed even in this case and the analysis would be unchanged.

choice of the tax rate alone; i.e.,

$$t_2^i(g) = \arg\max v(t_2; t_2 \cdot \bar{y}(t_2; g); a_2^i(g)).$$

Using the envelope theorem, we obtain the following first order condition:

$$\bar{y}(t_2^i; g) - y(t_2^i; a_2^i(g)) + t_2^i \cdot \partial \bar{y}(t_2^i; g)/\partial t_2 \cdot 0 \ (= 0 \text{ if } t_2^i > 0).$$

Using the fact that $y(t; a) = (1 - t)a^2$, the above condition can be used to solve for the optimal tax rates of the two types of candidates. These are

$$t_2^j(g) = \max\left(0; \frac{A(g) - a_2^j(g)^2}{2A(g) - a_2^j(g)^2}\right) \quad j \in \{P; M\};$$

where $A(g) = \frac{[\#N_P a_2^P(g)^2 + \#N_M a_2^M(g)^2 + \#N_R a_2^R(g)^2]}{N}$.

Now consider the investment decision of a poor individual who is in power in period one. Since the investment is costless, all that matters is how it affects the policy that he expects in period two. Using the fact that $v(t; T; a) = T + [(1 - t)^2 \cdot a^2]/2$, it is straightforward to calculate period two expected utility as:

$$\tfrac{1}{2}\left[t_2^M(g) \cdot \bar{y}(t_2^M(g); g) + (1 - t_2^M(g))^2 \cdot a_P^2/2\right]$$
$$+ \tfrac{1}{2}\left[t_2^P(g) \cdot \bar{y}(t_2^P(g); g) + (1 - t_2^P(g))^2 \cdot a_P^2/2\right]$$

The investment raises the income generating abilities of the middle class, so that if a poor candidate were elected in the second period, the first period policy maker would be better off in the second period if he had invested. However, there is no guarantee that investing will increase the policy maker's second period utility if a middle class candidate is elected in the second period. Provided that $t_2^M(0) > 0$, undertaking the investment will reduce the tax rate that would be chosen by the middle class candidate; i.e., $t_2^M(1) < t_2^M(0)$. The investment raises the relative productivity of the middle class and hence the middle class will favor a lower rate of taxation. Despite the increase in the economy's income generating ability, this reduction in tax rate may be sufficient to reduce the guarantee ($t_2^M(1) \cdot \bar{y}(t_2^M(1); 1) < t_2^M(0) \cdot \bar{y}(t_2^M(0); 0)$). If this reduction is large enough, the first period policy maker is better off foregoing the investment.

Intuitively, it seems likely that the smaller the fraction of middle class individuals in the economy, the more likely is the poor person to forego the investment. When $\#N_M/N$ is small, the gains to the poor person in terms of raising mean earnings are small, but the costs in terms of less redistribution remain. This logic is verified in the following result:

Lemma 1. Suppose that $(a_P^2 + a_R^2)/2 > a_M^2$. Then there exists $\beta^* > 0$ such that a poor first period policy maker would forgo the investment if $\#N_M/N < \beta^*$.

Proof. See Appendix. ■

If the investment is not undertaken, the second period is identical to the first. Thus the first period policy maker will choose a first period tax rate $t_2^P(0)$. The equilibrium policy sequence will therefore involve a first period policy choice $(t_2^P(0); t_2^P(0) \cdot \bar{y}(t_2^P(0); 0); 0)$ and second period choices of $(t_2^P(0); t_2^P(0) \cdot \bar{y}(t_2^P(0); 0))$ and $(t_2^M(0); t_2^M(0) \cdot \bar{y}(t_2^M(0); 0))$ with equal probability. Since the poor form one half of the population, it is clearly consistent with equilibrium to have a poor first period policy maker. For example, there exists a first period political equilibrium with a poor candidate running against a middle class candidate who is also supported by the rich.

The possibility that current policy makers may manipulate policies to affect the policy choices of future leaders arises in any environment in which there is political instability and current policies have future (direct or indirect) consequences. As noted in the introduction, this idea has been used in the macro-policy literature to explain why governments run budget deficits. Our final example illustrates a case in which the investment alters the probability distribution over future policy makers in a way that is harmful to the first period policy maker. Again, the result is that the investment is not undertaken despite being technologically efficient and straightforward.

Example 3

The population is divided into three groups of individuals: high types, low types and movers. Let N_H denote the set of high types; N_L the set of low types and N_M the set of movers. It is assumed that $\#N_L + \#N_M = \#N_H$ and that

$\#N_M < N/3$. As in Example 1, assume that individuals care only about their consumption and are risk neutral, so that $u(x;\ell) = x$. High types have ability a_H in both periods and low types have ability a_L in both periods, where $a_H > a_L$. For both of these groups, there is no gain from having the public investment implemented. Movers have ability a_L in the first period but, if the investment is undertaken, they will have ability $a_L + \varepsilon$ in period two. It is assumed that

$$N(a_H - a_L)/2(N - \#N_M) < \varepsilon < 3(a_H - a_L)/4.$$

The investment is presumed to be free, so that $C = 0$. This assumption implies that the investment is technologically efficient and straightforward.

We solve for the political equilibrium policy sequence under the assumption that a low type is in office in the first period. The consistency of this assumption will be verified below. It is straightforward to solve for the second period political equilibrium. If no investment is undertaken in the first period, then the low types and movers have the same preferences: they would both like to redistribute as much as possible and thus $t_2^L(0) = 1$. For their part, the high types would like no redistributive taxation so that $t_2^H(0) = 0$. The unique pure strategy equilibrium therefore involves a low type or a mover running against a high type and the second period policy will be $(1;\overline{a})$ with probability 1/2 and $(0;0)$ with probability 1/2; where $\overline{a} = (a_H + a_L)/2$ denotes mean first period income. To make our point cleanly, we will assume in what follows that a low type will run.

If the first period investment is made. Then the second period political equilibrium is different, since now the movers and high types are period two allies: they both want no redistributive taxation. To see this observe that if the tax rate is set equal to one, the movers receive consumption $\overline{a} + \varepsilon\#N_M/N$. If it is set equal to zero, then they receive $a_L + \varepsilon$. Under our assumptions on ε, setting taxes equal to zero is the best strategy. The only pure strategy equilibria, involve one of either of these types running unopposed and a second period policy $(0;0)$. We will assume that a high type will run. Note here that if a low type were in power, $t_2^L(1) = 1$. This example is therefore one in which condition (ii) of Proposition 3 is satisfied, but condition (i) is violated. Undertaking the investment ensures that a high type will be the second period policy maker.

We can now discuss the implications of this for period one policy choice. Since he is of low ability, the first period policy maker will always set a tax rate of 100%; the only question is whether he will undertake the investment. Since it is costless, the issue is how it affects second period utility. If he undertakes the investment his

expected second period payoff is a_L; if he does not it is $(3a_L + a_H)/4$. It is clear that the investment is not worthwhile. Thus the political equilibrium policy sequence involves a first period policy choice of $(1; \bar{a}; 0)$ and an equal probability of the second period policy choices $(1; \bar{a})$ and $(0; 0)$. The investment is not undertaken despite being straightforwardly desirable.

It only remains to check that it is possible for a low type to be in office in the first period. We claim that there is a two candidate pure strategy equilibrium involving a low type running against a high type in period one. The only thing to check here is that movers would vote for the low type. Provided this is true, then both candidates will win with probability 1/2 and will be willing to run against each other. No mover would have an incentive to enter since he would ensure the loss of the low candidate. If a high type is elected in the first period, he would select the policy $(0; 0; 1)$. This would mean that the second period policy would be $(0; 0)$. Thus, under the governance of the high type, a mover would receive lifetime utility $2a_L + \pm$. Under a low type he would receive $3a_H/4 + 5a_L/4$. Our assumptions on $\pm$ imply that a mover prefers a low type in office as required.[26]

The interpretation of the result in this example is interesting. The incentive that incumbents have not to implement the project even though nobody apparently loses from it, is the fact that it leads to loss of control. The beneficiaries of the reform switch allegiance and "gang" up on the low types, reducing the amount of redistribution that they get. This makes low types antithetical to social mobility, which destroys their ability to redistribute. We believe that this is a general phenomenon, which could retard efficient investments. Another example, worthy of investigation in light of this is the mass privatization efforts that have occurred throughout the world. These can be viewed as policies along these lines. By changing the traditional alliances embodied in state run enterprises, they change future voting patterns in favor of those supporting the introduction of privatization.

[26] This example nicely illustrates the difference between our model of the political process and the Downsian framework. Having two competing parties offer the first period policies $(1; \bar{a}; 0)$ and $(0; 0; 1)$ would be inconsistent with equilibrium. The party offering the platform $(0; 0; 1)$ would change its policy to attract the votes of the movers. In our model, while the high type candidate may have an incentive to promise more redistribution in the first period, such promises are not credible. The Downsian model circumvents this difficulty by assuming that parties just care about winning and thus can commit to implement any platform they propose. For more discussion of the differences between the two approaches see Besley and Coate (1995).

The three examples of this section all illustrate cases where an investment is not undertaken, even though technological considerations suggest that it is worthwhile. What unifies the three examples is that future policies are important. In the first example, this takes the form of requiring future compensation for the investment to be undertaken by an incumbent. In the second and third, it is the endogeneity of future policy outcomes to current policies which yields the results. This suggests that the latter two arguments are most applicable to larger programs of economic reform which affect economy wide variables, rather than to small projects.

5. Discussion

The results of the previous section leave no doubt that investments satisfying standard criteria of efficiency will not necessarily be adopted in political equilibrium. However, in view of the efficiency result in Proposition 1, it is not clear whether these should be viewed as "political failures" or whether technological definitions of efficiency should be thrown into question. Here, we address this issue.

Despite the widespread use of the term "political failure" in public choice writings, it is hard to find any precise definition of this idea. We must begin, therefore, by developing one. Our objective is find a definition which parallels as closely as possible the usual definition of a market failure. Recall, therefore, the textbook analysis of market efficiency. First, the set of efficient allocations is defined, usually represented graphically by a utility possibility frontier. This notion of efficiency is purely a technological one, since the frontier depends only on the tastes and technologies of the members of the economy. Next, a model of how markets allocate resources is developed. The Arrow-Debreu model has primacy of place among those that are typically used. That model yields the first fundamental theorem of welfare economics, that under certain tightly specified conditions, equilibrium utility allocations are on the utility possibility frontier. The idea of market failure, then comes from observing that deviations from the Arrow-Debreu ideal do not result in allocations that are on the frontier. Using the term "failure" is justified by the observation that if the market does not produce an allocation on the utility possibility frontier, then, in principle, all citizens could be made better off.

To provide a parallel definition of a political failure, one must similarly begin by defining the set of technologically feasible utility allocations. This should

re°ect the available policy instruments. It is unreasonable, for example, to hold representative democracy to the standards of what might be achieved were lump sum taxes and transfers available, while restricting policy makers to using only distortionary or uniform taxation.[27] At the second stage, political institutions are modeled, with a focus on whether equilibrium policy choices result in utility allocations on the frontier. By analogy with markets, a political failure arises when equilibrium policy choices leave the possibility of feasible, Pareto improving policy choices.

By this standard, the non-implementation of technologically e±cient investments described in the previous section represents a political failure; if such an investment is not implemented then there exists a Pareto improving policy sequence.[28] There is no con°ict between this ¯nding and Proposition 1. That result just says that, conditional on second period policies being selected through the democratic process, there exists no alternative ¯rst period choices which can raise the expected utilities of all citizens. Thus, it implies that if a technologically e±cient investment is not undertaken then it must be that the resulting second period equilibrium choices would have been such as to make some citizens worse o®. Hence, it is the fact that future policy choices are made through the democratic process that is constraining society to the interior of the set of technologically feasible utility allocations.

We now anticipate two kinds of objection to our claims of political failure. The ¯rst accepts the de¯nition that we put forward, but disputes the way in which we have analyzed the issue. The second takes issue with the de¯nition. In the former category, we include a Coasian-style challenge to our analysis, which argues that bargaining between citizens and policy makers should be able to eliminate

[27] It is common in the literature to overlook this point. Consider, for example, the textbook argument that representative democracy produces an ine±cient level of public goods. The level of public goods demanded by the median voter is claimed to be "ine±cient" because, in general, it does not satisfy the Samuelson rule. The latter is, however, derived under the assumption that lump sum taxes and transfers are feasible.

[28] In Example 1 all citizens would be better o® under the policy sequence which involves a ¯rst period policy choice $(1;\overline{a};1)$ and a probability distribution over second period policy choices which selects $(1;\overline{a}+\frac{\pm}{2})$ and $(\frac{\pm}{2\overline{a}+\pm};\frac{\pm}{2})$ with equal probability. In Example 2, all citizens would prefer the policy sequence which involves a ¯rst period policy choice $(t_2^P(0);t_2^P(0)\diamond\overline{y}(t_2^P(0);0);1)$ and second period choices of $(t_2^P(0);t_2^P(0)\diamond\overline{y}(t_2^P(0);1))$ and $(t_2^M(0);t_2^M(0)\diamond\overline{y}(t_2^M(0);1))$ with equal probability. In Example 3, the policy sequence which involves a ¯rst period policy choice of $(1;(a_L+a_H)=2;1)$ and an equal probability of the second period policy choices $(1;(a_H+a_L)=2+\pm\#N_M=N)$ and $(0;0)$ makes all citizens better o®.

the ine±ciency. It is straightforward to see that a current policy maker would implement the investment in return for future policy makers setting second period policies which would compensate them. By ignoring such gains from bargaining, our analysis might be regarded as misleading, just as Coase (1960) criticized the usual approach to externalities in a market context.

In response, we observe that the kind of bargain needed to restore e±cient policy would require a signi¯cant amount of commitment; future policy makers would have to commit to the policies that they would select in the second period, and citizens would have to commit to elect these individuals in the future. Thus the Coasian bargain must involve the entire polity. However, if such bargains were possible, then representative democracy would surely be redundant. If transactions costs were low enough, then the citizens could get simply get together at a \Coasian town meeting" and bargain with each other over current and future policies.[29] It is true that in this utopian vision, all technologically e±cient investments would be undertaken.[30] Nonetheless, it is hard to take it seriously as a model of government, except perhaps in very small groups.

A further objection to the analysis is the observation that the political failures that we have identi¯ed could be eliminated if policy makers were elected for two periods. Proposition 2 then implies that technologically e±cient investments would always be undertaken. However, our model provides no justi¯cation for holding a second period election. While technically correct, this critique takes the model's structure too much at face value. In reality, democracy is characterized by periodic rather than one time elections and our model captures this as simply as possible. Given that it is a commonly observed feature of democratic government, it is reasonable to suppose that periodic elections have a rationale.[31] This suggests that any bene¯ts from this exceed the social cost from the policy distortions identi¯ed here. However, this is di®erent from arguing that the distortions that we have identi¯ed do not exist. They will necessarily arise in any system with periodic elections where government policies have e®ects that last beyond

[29] This is like an application of the Coase theorem, with the policy instruments (which a®ect the utility of all members of the community) being the public good. One suspects that transactions costs are the main reason why societies resort to more delegated forms of decision making. See Dixit (1995) for a discussion of transactions costs politics more generally.

[30] This vision of policy making accords well with Wicksell's vision of perfect democracy (see Wicksell (1896)).

[31] In line with our earlier paper, one rationale is the need to throw out leaders who are shown to be incompetent by subsequent events.

the time of the next election.

The major objection to our definition of political failure is that it is insufficient to demonstrate such a failure by showing that there exists some technologically feasible policy choices which produces a Pareto superior utility allocation. Instead, it behooves the analyst to specify an alternative institutional arrangement for making policy decisions which actually selects such policy choices. Absent this, the term \failure" is misleading. Similar objections can, of course, be made about the notion of a market failure. In particular, there is a need to demonstrate that the government (or some other institution for allocating resources) could do better (see Williamson (1994) for some discussion along these lines).

In response to this, we wish to make clear that we are not attempting to answer the broader question of whether representative democracy is desirable in the class of institutions that could be used to make collective choices (although answering this would constitute an interesting research agenda). Clearly, the ability to implement technologically efficient investments represents only one dimension of this question. What is important is to reach a consistent position on markets and governments. Our definition is intended to achieve that. Just as the presence of market failures does not imply abandonment of the market system, the existence of political failures does not mean that representative democracy should be forsaken.[32] Indeed, the revealed preference for representative democracy, confirmed even more strongly in recent years, suggests that the benefits of democratic policy making outweigh its costs.

There are, however, still important reasons to ask whether representative democracy results in efficient policy choices. First, it is central to many issues in public economics. In the absence of political failures, the prescriptions of normative economics (particularly those concerning efficiency) may be interpreted as predictions about actual policy choices. If observed policy choices do not correspond to these predictions, then, instead of berating policy makers for their foolishness, it is more constructive to provide explanations for their choices (a conclusion reminiscent of Stigler (1982)). A better model for public economics is then industrial organization or labor economics, where trying to explain observed

[32] In the market failure literature, it is all too common to reach the conclusion that the existence of market failure implies that the government could do better. This stems from the bad habit of viewing policy as being selected by a social planner. Showing that a planner could do better than the market is really no advance over pointing out that there exists a technologically feasible, Pareto superior allocaction. See Buchanan (1972) for further elaboration of this argument.

contractual arrangements is central. The second reason is more practical: if we develop an understanding of why political failures occur then we may be able to suggest e±ciency enhancing, piecemeal institutional reforms.

6. Concluding Remarks

The main achievements of this paper are twofold. First, it o®ers a precise de¯nition of political failure. The notion put forward takes the standard technological de¯nition of what is feasible for the citizens. This, we argued, is the most natural parallel to the literature on market failure. Second, the paper identi¯es three distinct classes of failures which arise when elections are repeated. Technologically e±cient policies may not be introduced because of fears that compensation needed to cover current costs will not be delivered and the fact that such policies may change the policy choices and/or identities of future leaders. These reasons can be traced to the fact that preferences over policy extend into the future, while political control does not.

We acknowledged that a Coasian argument could be levied against our claims of political failure; bargaining between citizens over current and future policies could eliminate the ine±ciencies. However, this would require the ability to reach binding agreements about future policy, as well as a forum to bring the whole polity together. This vision is perhaps too utopian, especially given the transactions costs involved. More generally, however the issue of how institutions might evolve to minimize the economic costs associated with repeated elections is worthy of further study.

Much remains to be done to understand comprehensively the e±ciency of resource allocation in a democracy, using the approach developed here and in our earlier paper. We have not yet modeled private sector accumulation decisions and the e®ect of policy on these. Introducing imperfect information also seems like an interesting avenue for future exploration. Finally, it would be interesting to incorporate other political institutions, such as pressure groups and legislative decision making, into the analysis.

References

[1] Aghion, Philippe, Bolton, Patrick: Government debt and the risk of default: A politico-economic model of the strategic role of debt. In Rudiger Dornbusch and Mario Draghi (eds), Public Debt Management: Theory and History, Cambridge University Press, Cambridge: England (1990).

[2] Austen-Smith, David, Banks, Je®rey: Electoral accountability and incumbency. In Peter C. Ordeshook (ed), Models of Strategic Choice in Politics, Ann Arbor: University of Michigan Press (1989).

[3] Becker, Gary: Public policies, pressure groups and dead weight costs. J. Public Econ., 28, 329-347 (1985).

[4] Besley, Timothy, Coate, Stephen: An economic model of representative democracy. University of Pennsylvania, CARESS Working Paper #95-02 (1995).

[5] Brennan, Geo®rey, Buchanan, James M.: The Power to Tax: Analytical Foundations of a Fiscal Constitution, Cambridge: Cambridge University Press (1980).

[6] Buchanan, James M.: Toward an analysis of closed behavioral systems. Chapter 2 in James Buchanan and Robert Tollison (eds), Theory of Public Choice, Ann Arbor: University of Michigan Press (1972).

[7] Coase, Ronald: The problem of social cost. J. Law and Econ., 3, 1-44 (1960).

[8] Coate, Stephen, Morris, Stephen: On the form of transfers to special interests. University of Pennsylvania, CARESS Working Paper #94-21, J. Pol. Econ., forthcoming (1994).

[9] Dixit, Avinash: Economic policy and the political process. Princeton University, typescript (1995).

[10] Dixit, Avinash, Londregan, John: Redistributive politics and economic e±ciency. CEPR Discussion Paper No. 1056 (1994).

[11] Glazer, Amihai: Politics and the choice of durability. American Econ. Rev., 79, 1207-13 (1989).

[12] Harrington, Joseph E. Jr.: Economic policy, economic performance and elections. American Econ. Rev., 83, 27-42 (1993).

[13] Hicks, John R.: The valuation of social income. Economica, 7, 105-24 (1939).

[14] Kaldor, Nicholas: Welfare propositions and interpersonal comparisons of utility. Econ. Journal, 44, 549-52 (1939).

[15] Meltzer Allan H., Richard, Scott F.: A rational theory of the size of government. J. Pol. Econ., 89, 914-27 (1981).

[16] Milesi-Ferretti, Gian-Maria, Spolaore, Enrico: How cynical can an incumbent be? Strategic policy in a model of government spending. J. Public Econ., 55, 121-40 (1994).

[17] Nash, John: Equilibrium points in n-person games. Proceedings of the National Academy of Sciences, 36, 48-49 (1950).

[18] Osborne, Martin J., Slivinski, Al: A model of political competition with citizen-candidates. McMaster University, Department of Economics Working Paper #95-04 (1995).

[19] Persson, T., Svensson, L.: Why a stubborn conservative would run a de¯cit: Policy with time-inconsistent preferences. Quar. J. Econ., 104, 325-346 (1989).

[20] Rogo®, K.: Equilibrium political budget cycles. American Econ. Rev. 80, 21-36 (1990).

[21] Romer, Thomas: Individual welfare, majority voting and the properties of a linear income tax. J. Public Econ., 4, 163-85 (1975).

[22] Samuelson, Paul: Evaluation of real national income. Oxford Econ. Papers, 2, 1-29 (1950).

[23] Scitovsky, Tibor: A note on welfare propositions in economics. Rev. Econ. Studies, 9, 77-88 (1941).

[24] Stigler, George: Economists and public policy. Regulation, May/June, 7-13 (1982).

[25] Tabellini, Guido, Alesina, Alberto: Voting on the budget de¯cit. American Econ. Rev., 80, 37-49 (1990).

[26] Wicksell, Knut: A new principle of just taxation. Translated and reprinted in Alan Peacock and Richard Musgrave (eds) Classics in the Theory of Public Finance, London: MacMillan (1896).

[27] Williamson, Oliver: The politics and economics of redistribution and e±-ciency. University of California-Berkeley, typescript (1994).

[28] Wittman, Donald: Why democracies produce e±cient results. J. Pol. Econ., 97, 1395-426 (1989).

7. Appendix

Proof of Proposition 1: Part (i) of the Proposition follows immediately from part (ii) of the De¯nition which tells us that $(t_1^{\alpha}; T_1^{\alpha}; g^{\alpha})$ maximizes $V^k(t_1; T_1; g; °^{\alpha}(¢))$. Part (ii) follows from part (iii) of the De¯nition. If $(t_2^{\alpha}; T_2^{\alpha}) \in ¢(¼^{\alpha})$ then $(t_2^{\alpha}; T_2^{\alpha}) \in ¢(¹(g^{\alpha}; r(°^{\alpha}(g^{\alpha}))))$. This means either that $(t_2^{\alpha}; T_2^{\alpha}) = (t_2^i(g^{\alpha}); T_2^i(g^{\alpha}))$ for some $i \in N$ or that $(t_2^{\alpha}; T_2^{\alpha}) = (0; 0)$. In either case, part (ii) of the Proposition holds. ■

Proof of Proposition 2: We know that $(t_1^{\alpha}; T_1^{\alpha}; g^{\alpha})$ solves the problem:

$$\text{Max } V^k(t_1; T_1; g; °^{\alpha}(¢)) \text{ s:t: } (t_1; T_1; g) \in Z_1.$$

Suppose that $g^{\alpha} = 0$. Then, since the investment is technologically e±cient, we know that there exists $(t_1^0; T_1^0)$ and, for every $(t_2; T_2) \in ¢(¹(0; r(°^{\alpha}(0))))$, $(t_2^0; T_2^0)$ such that

(i) $(t_1^0; T_1^0; 1) \in Z_1$ and $(t_2^0; T_2^0) \in Z_2(1)$,

and

(ii) $v(t_1^0; T_1^0; a_1^k) + v(t_2^0; T_2^0; a_2^k(1)) > v(t_1^{\alpha}; T_1^{\alpha}; a_1^i) + v(t_2; T_2; a_2^k(0))$.

We will show that $V^k(t_1^0; T_1^0; 1; °^{\alpha}(¢)) > V^k(t_1^{\alpha}; T_1^{\alpha}; 0; °^{\alpha}(¢))$, which will contradict the hypothesis that $g^{\alpha} = 0$.

By hypothesis, the second period policy maker will share citizen k's preferences if the investment is undertaken. Hence:

$$V^k(t_1^0; T_1^0; 1; °^{\alpha}(¢)) = v(t_1^0; T_1^0; a_1^k) + \max\{v(t_2; T_2; a_2^k(1)) : (t_2; T_2) \in Z_2(1)\}:$$

Thus, by (ii) above, for all $(t_2; T_2) \in ¢(¹(0; r(°^{\alpha}(0))))$,

$$V^k(t_1^0; T_1^0; 1; °^{\alpha}(¢)) > v(t_1^{\alpha}; T_1^{\alpha}; a_1^i) + v(t_2; T_2; a_2^k(0)):$$

It follows that

$$V^k(t_1^0; T_1^0; 1; °^{\alpha}(¢)) > v(t_1^{\alpha}; T_1^{\alpha}; a_1^i) + \sum_{(t_2;T_2)\in ¢(¹)} ¹(0; r(°^{\alpha}(0)))(t_2; T_2)v(t_2; T_2; a_2^k(0));$$

as required. ■

Proof of Proposition 3: We know that $(t_1^{¤}; T_1^{¤}; g^{¤})$ solves the problem:

$$\text{Max } V^k(t_1; T_1; g; °^{¤}(¢)) \text{ s:t: } (t_1; T_1; g) \text{ 2 } Z_1.$$

Suppose that $g^{¤} = 0$. Then, since $f(t_1^{¤}; T_1^{¤}; g^{¤}); ¼^{¤}g$ is a political equilibrium policy sequence, Proposition 1 tells us that $¢(¼^{¤})$ ½ $Z_2^{¤}(0)$. Thus, since the investment is technologically e±cient and straightforward, we know that there exists $(t_1^0; T_1^0)$ such that:

(i) $(t_1^0; T_1^0; 1)$ 2 Z_1,

and

(ii) for every $(t_2^{¤}; T_2^{¤})$ 2 $¢(¼^{¤})$, $(t_2^0; T_2^0)$ 2 $^{a}(t_2^{¤}; T_2^{¤})$ implies that

$$v(t_1^0; T_1^0; a_1^i) + v(t_2^0; T_2^0; a_2^i(1)) > v(t_1^{¤}; T_1^{¤}; a_1^i) + v(t_2^{¤}; T_2^{¤}; a_2^i(0)):$$

We will show that

$$V^k(t_1^0; T_1^0; 1; °^{¤}(¢)) > V^k(t_1^{¤}; T_1^{¤}; 0; °^{¤}(¢));$$

which will contradict the hypothesis that $g^{¤} = 0$.

By hypothesis, $r(°^{¤}(1)) = r(°^{¤}(0))$: Thus,

$$V^k(t_1^0; T_1^0; 1; °^{¤}(¢)) = v(t_1^0; T_1^0; a_1^k) + \sum_{i2N[f0g} r(°^{¤}(1))(i)v(t_2^i(1); T_2^i(1); a_2^k(1))$$

$$= \sum_{i2N[f0g} r(°^{¤}(0))(i)\left[v(t_1^0; T_1^0; a_1^k) + v(t_2^i(1); T_2^i(1); a_2^k(1))\right]:$$

We may also write,

$$V^k(t_1^{¤}; T_1^{¤}; 0; °^{¤}(¢)) = \sum_{i2N[f0g} r(°^{¤}(0))(i)\left[v(t_1^{¤}; T_1^{¤}; a_1^k) + v(t_2^i(0); T_2^i(0); a_2^k(0))\right]:$$

Thus, it su±ces to show that for all i such that $r(°^{¤}(0))(i) > 0$,

$$v(t_1^0; T_1^0; a_1^k) + v(t_2^i(1); T_2^i(1); a_2^k(1)) > v(t_1^{¤}; T_1^{¤}; a_1^k) + v(t_2^i(0); T_2^i(0); a_2^k(0)).$$

But this follows from the fact that, by hypothesis, $(t_2^i(1); T_2^i(1))$ 2 $^{a}(t_2^i(0); T_2^i(0))$ for all i such that $r(°^{¤}(1))(i) > 0$. ■

Proof of Lemma 1: The idea of the proof is to treat the fraction of middle class individuals in the population as a parameter and to show that as this fraction becomes very small, the gains from making the investment go to zero while the costs remain positive. The result will then follow from continuity. With this in mind, de¯ne $\lambda = \#N_M/N$. Given our earlier assumptions, we have that $\#N_P/N = 1/2$ and $\#N_R/N = 1/2 - \lambda$. We now de¯ne the variables of the example as functions of λ. Thus,

$$A(g;\lambda) = a_P^2/2 + \lambda(a_M + \pm g)^2 + (1/2 - \lambda)a_R^2;$$

$$t_2^P(g;\lambda) = \frac{A(g;\lambda) - a_P^2}{(2A(g;\lambda) - a_P^2)};$$

$$t_2^M(g;\lambda) = \max\{0; \frac{A(g;\lambda) - (a_M + \pm g)^2}{(2A(g;\lambda) - (a_M + \pm g)^2)}g;$$

and,

$$\overline{y}(t;g;\lambda) = (1 - t)A(g;\lambda).$$

Observe for future reference that

$$\lim_{\lambda \to 0} A(1;\lambda) = \lim_{\lambda \to 0} A(0;\lambda) = (a_R^2 + a_P^2)/2;$$

$$\lim_{\lambda \to 0} t_2^P(1;\lambda) = \lim_{\lambda \to 0} t_2^P(0;\lambda) = \frac{(a_R^2 - a_P^2)/2}{a_R^2};$$

and,

$$\lim_{\lambda \to 0} t_2^M(0;\lambda) = \frac{(a_R^2 + a_P^2)/2 - a_M^2}{a_R^2 + a_P^2 - a_M^2} > \lim_{\lambda \to 0} t_2^M(1;\lambda) = \max\{0; \frac{(a_R^2 + a_P^2)/2 - (a_M + \pm)^2}{a_R^2 + a_P^2 - (a_M + \pm)^2}g:$$

Letting,

$$!_{PJ}(g;\lambda) = t_2^J(g;\lambda)\cdot\overline{y}(t_2^J(g);g;\lambda) + (1 - t_2^J(g;\lambda))^2\cdot a_P^2/2; \; J \in \{P;M\};$$

we know that the investment will not be undertaken if

$$\frac{1}{2}[!_{PP}(1;\lambda) + !_{PM}(1;\lambda)] < \frac{1}{2}[!_{PP}(0;\lambda) + !_{PM}(0;\lambda)]:$$

Note that for $J \in \{P, M\}$,

$$\omega_{PJ}(0, \varepsilon) - \omega_{PJ}(1, \varepsilon) = [t_2^J(0, \varepsilon) \cdot \overline{y}(t_2^J(0, \varepsilon); 0, \varepsilon) + (1 - t_2^J(0, \varepsilon))^2 \cdot a_P^2/2]$$

$$- [t_2^J(1, \varepsilon) \cdot \overline{y}(t_2^J(1, \varepsilon); 0, \varepsilon) + (1 - t_2^J(1, \varepsilon))^2 \cdot a_P^2/2].$$

or,

$$\omega_{PJ}(0, \varepsilon) - \omega_{PJ}(1, \varepsilon) = (1 - t_2^J(0, \varepsilon))[t_2^J(0, \varepsilon)A(0, \varepsilon) + (1 - t_2^J(0, \varepsilon))a_P^2/2]$$

$$- (1 - t_2^J(1, \varepsilon))[t_2^J(1, \varepsilon)A(1, \varepsilon) + (1 - t_2^J(1, \varepsilon))a_P^2/2].$$

It is clear from our claims above that

$$\lim_{\varepsilon \to 0} \omega_{PP}(1, \varepsilon) = \lim_{\varepsilon \to 0} \omega_{PP}(0, \varepsilon).$$

However,

$$\lim_{\varepsilon \to 0}[\omega_{PM}(0, \varepsilon) - \omega_{PM}(1, \varepsilon)] = f(\lim_{\varepsilon \to 0} t_2^M(0, \varepsilon)) - f(\lim_{\varepsilon \to 0} t_2^M(1, \varepsilon)),$$

where

$$f(t) = (1 - t)[t(a_R^2 + a_P^2)/2 + (1 - t)a_P^2/2].$$

Di®erentiating we obtain,

$$f'(t) = (a_R^2 - a_P^2)/2 - ta_R^2.$$

Observe that $f'(t) > 0$ for all $t \leq (a_R^2 - a_P^2)/2a_R^2$. Thus, since

$$\lim_{\varepsilon \to 0} t_2^M(1, \varepsilon) < \lim_{\varepsilon \to 0} t_2^M(0, \varepsilon) < (a_R^2 - a_P^2)/2a_R^2,$$

we have that

$$\lim_{\varepsilon \to 0} \omega_{PM}(1, \varepsilon) < \lim_{\varepsilon \to 0} \omega_{PM}(0, \varepsilon).$$

By continuity, therefore, for su±ciently small ε,

$$\frac{1}{2}[\omega_{PP}(1, \varepsilon) + \omega_{PM}(1, \varepsilon)] < \frac{1}{2}[\omega_{PP}(0, \varepsilon) + \omega_{PM}(0, \varepsilon)],$$

and the investment will not be undertaken. ■