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An Economic Model of Representative Democracy*

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Abstract

This paper develops a new approach to the study of democratic policy making where politicians are selected by the people from those citizens who present themselves as candidates for public office. Participation in the policy making process is, therefore, derived endogenously. The approach has a number of attractive features. First, it is a conceptualization of a pure form of representative democracy in which government is by, as well as of, the people. Second, the model is analytically tractable, being able to handle multidimensional issue and policy spaces very naturally. Third, it provides a vehicle for answering questions about the achievements of representative democracy. We study, in particular, whether representative democracy produces efficient outcomes.

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\The democratic method is that institutional arrangement for arriving at political decisions in which individuals acquire the power to decide by means of a competitive struggle for the people's vote," (Schumpeter (1954), page 269).

\In the real world, individuals, as such, do not make ¯scal choices. They seem limited to choosing \leaders," who will, in turn, make ¯scal decisions." (Buchanan (1967), page v).

1. Introduction

Understanding the determinants of policy choice in situations where policy makers are electorally accountable to the voters is a central task for political economy. Public choice theory should enable us both to predict policy choice in representative democracy and to assess its achievements in terms of normative criteria such as equity or e±ciency. However, in contrast to the analysis of markets, a satisfactory theoretical framework for analyzing policy choice in representative democracy has yet to be developed. This paper develops a new approach to the study of democratic policy making, whose novel feature is to break the arti¯cial distinction between political actors and citizens. In our model, policy makers are selected from the group of citizens who present themselves as candidates for public o±ce, and participation in the political process is derived endogenously.

Our model of representative democracy begins with a community of citizens, from which one is selected to make policy decisions via an election. All citizens can become candidates for o±ce, although running is costly. Citizens care about policy outcomes and are motivated to run by their desire to a®ect these outcomes and/or to hold the post of policy maker. The candidate who wins o±ce gains the right to choose policy and selects his preferred alternative. The citizens vote for candidates based on their policy preferences and other relevant characteristics, such as their competence.[1]

[1]This basic model of political competition was developed independently by Osborne and Slivinski (1994). Unlike us, they focus exclusively on a one-dimensional model with Euclidean preferences. They work with a continuum of citizens who vote sincerely, rather than a ¯nite number who vote strategically, as studied here. Their motivation is to compare the number and

The approach has a number of attractive features. First, it is a conceptualization of a pure form of representative democracy in which government is by, as well as of, the people. Political competition is among the citizens who wish to become policy makers, who are motivated by their desire to influence outcomes. Second, the model is analytically tractable. It is able to handle multidimensional issue and policy spaces very naturally. Most models from public economics fit within the framework, and it can be used to derive predictions about a host of policies, such as equilibrium levels of public goods, publicly provided private goods, and tax rates. Third, and perhaps most significantly, the model provides a vehicle for answering questions about the achievements of representative democracy. Its suitability for welfare analysis reflects the fact that the theory is built from the ground up. The primitives of the model are the set of feasible policy alternatives, citizens' preferences over these alternatives, and a constitution which specifies the rules of the decision making process. Policy outcomes are thus derived from the underlying tastes and policy technology.

The next section reviews the existing approaches to policy making in a representative democracy and explains how our approach fits in. Section 3 lays out the basic framework. There are three stages to the model. In the first stage citizens decide whether or not to declare themselves as candidates for public office. In the second, citizens vote over the declared candidates. Finally, the winning candidate selects a policy alternative. An equilibrium is a set of entry decisions such that each citizen's decision is optimal given the decisions of others. We show that an equilibrium exists, which may be either in pure or mixed strategies. Section 4 provides a fairly complete characterization of pure strategy equilibria. We provide necessary and sufficient conditions for one candidate equilibria, two candidate equilibria and equilibria involving three or more candidates. These results provide a tool kit for calculating pure strategy equilibria in applications.

Section 5 develops some illustrative applications of the approach. The first example is the standard one-dimensional policy model with Euclidean preferences. We show that there are a family of pure strategy equilibria which involve two candidates with counter balancing ideologies each of whom receives half the vote. The second example is a simple two-dimensional model with a continuous and a discrete policy variable. We derive a family of two candidate pure strategy equilibria, even though no Condorcet winner exists. The final example is a one-

type of candidates under plurality rule and majority rule with runoffs. We are more concerned with understanding the normative performance of representative democracy.

dimensional policy model with non-single peaked preferences. This is used to illustrate a mixed strategy equilibrium.

Section 6 contains the normative analysis of our model. The social choice problem faced by the polity is to select a citizen to be policy maker and a policy to be implemented. Representative democracy, as we model it, represents one way of doing this. We consider whether the selection that it produces has desirable features. This is answered under two headings: e±ciency and equity. We ¯nd that when the task of policy maker involves no special skills and entails no personal costs, representative democracy always produces e±cient outcomes. If individuals di®er in their policy making competence, our results are less positive. While there is something to the idea that political competition will sort the appropriate candidates into o±ce, our analysis identi¯es a number of caveats to this argument. As regards equity, we show that political competition will tend to sort in more altruistic candidates. This casts doubt on the applicability of Leviathan models of public decision making, wherein policy makers are assumed to maximize the revenue that they extract from the economy.

2. Existing Approaches

There is a large body of work which analyzes policy choice in representative democracies, much of which is built on the seminal contribution of Downs (1957). His notion of representative democracy is of two parties competing for o±ce by o®ering voters di®erent \platforms". Parties care about winning and implement their proposed policies if elected. Under the assumption that the issue space is one dimensional and preferences are single peaked, both parties will o®er the policy preferred by the median citizen. This \median voter theorem" has signi¯cantly in°uenced work on economic policy making.[2] Numerous theoretical and empirical studies of taxes and expenditures are based upon it.

Despite its in°uence, the Downsian view of policy making in democratic societies has some serious short-comings. First, the model typically provides no predictions when either preferences are not single peaked or there are two or more

[2] This median outcome may also be derived in the context of a model of direct democracy in which citizens make proposals and vote on which proposal to implement via majority rule. The basic Downsian model therefore predicts no di®erence in the outcomes of direct and representative democracy. As will become apparent, our model di®ers in this respect. We are grateful to David Levy for this observation.

dimensions to policy choice. This re°ects the fact that, in such environments, there generally exists no Condorcet winner, i.e., a policy which is preferred by a majority of the polity to every other policy. Thus there exists no pair of platforms with which the two vote maximizing parties are satis¯ed. Obviously, this feature severely limits the usefulness of the model.

There are a number of responses to this problem. One promising avenue begins with the observation that the di±culties for obtaining existence of an equilibrium are created, in part, by the discontinuity of voters' behavior. If party A's platform o®ers a citizen even the smallest increment of utility over party B's, then that citizen will switch his vote to party A. To smooth this out, some have suggested modeling voters' decisions as probabilistic. In the modi¯ed Downsian model, two vote maximizing parties compete for political o±ce in an environment in which the probability of a particular individual voting for a party is increasing in the utility gain from having that party in power.[3] These models are able to handle multi-dimensional issue spaces and policy instruments and thus are an advance over the median voter model. However, it is necessary to make fairly restrictive assumptions about the probability of voting functions to guarantee that an equilibrium exists and these assumptions can dictate the policy outcome.[4]

A second problem with the Downsian view concerns the assumed motivation for governance. Political parties are supposed to care only about winning and are willing to implement any policy to do so. This precludes a government consisting of individuals with policy preferences, despite the fact that voters have such concerns. As Brennan and Buchanan (1980) note, \In these models, government is neither despotic nor benevolent; in a very real sense, \government," as such, does not exist", (page 15). Were the formation of parties explicitly modeled, it would seem unlikely that they would be pure vote maximizers. While a number of authors (see, for example, Alesina (1988) and Wittman (1983)) have analyzed models where parties have policy preferences, the theory begins with parties as primitives, without modeling their motives in relation to the voters at large.[5]

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[3]Ledyard (1984) provides a rigorous underpinning for this behavioral assumption. In his model voting is costly and individuals have private information about their policy preferences and their voting costs. Individuals vote only if the expected gain exceeds the cost. Ledyard analyzes the Bayesian equilibirum of the game in which parties ¯rst select platforms and then voters decide whether to vote.

[4]For further discussion of the probabilistic voting model see Coughlin (1992) and the references therein. For a well thought out criticism of the model see Usher (1994).

[5]Endogenous party formation is considered in Baron (1993), where parties choose policies

A major alternative to the Downsian perspective on policy making is the pressure group approach of Stigler (1971), Peltzman (1976) and Becker (1983). This approach views policy as being determined by competing interesting groups who attempt to in°uence policy choices by providing support, either in the form of votes or money. A standard criticism of such theories is that they do not explicitly model the policy selection process or the nature of the \in°uence activities". Grossman and Helpman (1994) °esh out this story by modeling the in°uence process as a \menu auction" (based on Bernheim and Whinston (1986)) in which interest groups o®er conditional transfer schedules to the policy maker. The policy maker then chooses policy to maximize his utility, which depends on transfers and the level of social welfare. Viewing the policy maker(s) in isolation from political competition is, however, an unsatisfactory feature of these models, given that the former are usually electorally accountable.

Political agency models of policy formation represent a third approach. These were pioneered by Barro (1970) and Ferejohn (1986) and further developed by Austen-Smith and Banks (1991) and Banks and Sundaram (1993).[6] They focus on the choices of incumbent politicians with policy preferences in environments where future elections foster incumbent discipline, irresponsible or incompetent incumbents being thrown out of o±ce. This class of models has produced many novel insights. However, like models of parties with policy preferences, characteristics of the incumbent or challenger are left unexplained. Thus, while useful for thinking about qualitative features of incumbent's and voters' behavior, they are not altogether helpful for making policy predictions.

Departing from models where policy choices are made by a single politician or party, there is a literature which has sought to understand legislatures made up of representatives with diverse preferences. A key issue is how the cycling problem, which arises in the absence of a Condorcet winner, can be overcome. Three classes of solutions have been discussed: the development of norms of voting behavior among representatives (Weingast (1979)); the use of rules, specifying the way in which policy proposals can be made (Shepsle and Weingast (1981) and Baron and Ferejohn (1989))[7] and the formation of institutions, such as the committee

that maximize the average utility of their members. Party members are those who support it in equilibrium. Most models in the Downsian tradition take the number of parties to be ¯xed. Notable exceptions include Palfrey (1984) and Feddersen, Sened and Wright (1990).

[6] Recent applications of such models include Besley and Case (1995), Coate and Morris (1994), Harrington (1993) and Rogo® (1990).

[7] In this category falls the important work of Caplin and Nalebu® (1991). They investigate

system in the U.S. Congress (Weingast and Marshall (1988)). While this literature has yielded many interesting ¯ndings, it does not consider what determines the preferences of the legislators.

The approaches discussed so far are positive theories of policy choice. There is a parallel normative tradition which seeks to understand what policy should be. Normative analyses characterize those policy choices which maximize \social welfare", presumed to depend on the allocation of utilities in society. Classic analyses in this tradition are Ramsey's (1927) treatment of optimal taxation and Samuelson's (1954) discussion of public goods provision. (See Atkinson and Stiglitz (1980) for a thorough review.) The particular relationship between social welfare and individual utilities is speci¯ed by a social welfare function. While these could be viewed as summarizing the outcome of some political process, this is certainly not derived explicitly in most cases.[8] Hence, there is no obvious reason to think that any particular social welfare function captures the political economy of real policy choices and one cannot be certain that policies derived from normative models would ever be selected in social equilibrium.[9]

In discussing the relevance of normative models for understanding real world policy choice, it is important to recognize the distinction between e±ciency and social preferences over the distribution of well-being in society. One can think of policy choice in two stages. At the ¯rst stage, the e±cient set of policies is characterized.[10] (A policy is e±cient if it is feasible and if there exists no feasible policy which generates a Pareto dominant utility allocation.) The second stage involves selecting a policy from that set. The social welfare function is needed only at this stage.

Even without bringing in an exogenously determined social welfare function, normative models will be helpful for understanding actual policy choices, if such choices are e±cient. Writers in the Chicago tradition, such as Stigler (1982),

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the implications of introducing the requirement that proposals be approved by a fraction ® of the legislators where ® > 1=2 (so-called ®-majority rule): The larger is ® the less likely are there to be cycles. Caplin and Nalebu® (1991) ¯nd general conditions for ® = 64% to be cycle proof method of choice.

[8] Arrow's (1951) impossibility theorem tells us that any political process which generates a complete ordering over social alternatives must violate one or more of his axioms.

[9] The probabilistic voting literature has demonstrated that, under some conditions, equilibrium policy choices maximize some form of social welfare function (see, for example, Ledyard (1984)).

[10] Hammond (1979) was among the ¯rst papers to look at policy choice in this way.

Becker (1985) and Wittman (1989), have argued that political competition should give rise to efficient policy choices. If this view is correct, then normative models should have predictive power. However, its legitimacy remains unresolved, in part because the literature lacks a satisfactory theoretical model of political competition to rigorously investigate these arguments. The basic Downsian model is deficient in two main respects. First, in order to guarantee the existence of equilibrium, the policy maker is restricted to using only a one dimensional policy instrument. Efficiency in policy choice is thus either trivial (if the feasible set is one-dimensional) or generically impossible (if the feasible set is not so constrained).[11] Second, many features of the underlying economic environment are incompletely specified. How, for example, are we to account for the utilities of the winning party members? The absence of a theoretical framework to explore the efficiency of public choices has created a gulf between positive and normative economics which appears wider than it need be on theoretical (and possibly even practical) grounds.

The approach developed here is an alternative to the Downsian model of policy making, rejecting all of the latter's key assumptions. Most fundamentally, it does not assume the pre-existence of political parties. Candidates in our model are citizens who have policy preferences and run for office to influence policy outcomes, rather than parties that maximize votes. Citizens weigh up costs and benefits of political involvement, with their number and type being endogenous. Our approach is complementary with the other positive models of policy choice discussed above. The pressure group and political agency approaches share our assumption that office holders have policy preferences and that policy is not committed to in advance. It should be straightforward to incorporate into our model interest groups which offer transfers to the policy maker who is selected, with the effects on incentives to run for office and voter preferences over candidates being of paramount interest. Making our model dynamic, with repeated elections, would raise many issues, such as reputation formation, that are considered in the existing

[11] This is brought out clearly in Bergstrom (1979) who uses the Downsian model to analyze whether political competition will produce an efficient level of public goods. He shows that strong restrictions are needed for the median voter's desired level of a public good to satisfy the Samuelson condition. However, his analysis assumes that the policy maker must employ a given method of financing, whereas the Samuelson rule is derived under the presumption of the existence of lump sum taxes and transfers. If the method of financing is taken as a constraint, then the Downsian outcome is trivially efficient: any change in the level of public goods must reduce the utility of the median voter.

political agency literature. This would be enriched by allowing the characteristics of incumbents and challengers to be derived endogenously, so that the sorting and disciplinary role of elections could be considered in tandem.[12] Legislative models also assume that representatives have diverse policy preferences. Our model would be a natural vehicle to make the composition of legislators endogenous, with each community's choice of a representative being studied. Added richness would come from citizens having to form beliefs about the type of representatives elected in other communities.

We also see our approach as a bridge between positive models and that part of normative economics devoted to the characterization of efficient policies. Our model permits a rigorous analysis of the view that representative democracy produces efficient policy choices. It therefore provides a theoretical underpinning for viewing the prescriptions of normative economics as predictions about policy choices in political equilibrium.

3. The Model

Consider a community made up of N people, labeled $i \in \mathcal{N} = \{1, \ldots, N\}$, which must choose a policy maker to select and implement a policy alternative. We denote a generic policy alternative by the vector x. Alternatives could be N-tuples of consumption bundles, one for each citizen in the community, or levels of conventional policy instruments, such as taxes and public expenditures. It is unnecessary to be specific at the moment. The set of policy alternatives available if individual i is the policy maker is denoted by A^i. This set may take account of informational and other feasibility constraints on policy. Differences in A^i across citizens reflect varying levels of policy-making competence. Let $A = \bigcup_{i=1}^{N} A^i$ be the set of all possible policy alternatives.

Each citizen's utility depends on whether he is selected to be policy maker and on the policy selected. Citizen i's utility if he is (is not) the policy maker and the policy is x is given by $V^i(x; 1)$ ($V^i(x; 0)$). This specification allows for possibilities other than individuals caring only about how policies affect their own consumption bundle. Citizens may, for example, be altruistic. They might also be paternalistic, viewing policies such as education or health care provision as “merit

[12] The issue of whether, in dynamic environments, political competition will sort in candidates with policy preferences reflective of their constituency is the subject of much discussion in the public choice literature. See, for example, Lott and Reed (1989) and the references therein.

goods". This specification also permits individuals to differ in their personal costs or benefits of being the policy maker.

The polity selects the policy maker in an election.[13] Citizens wishing to represent the community present themselves as candidates for office. All citizens can run for office, although there is a (possibly small) cost δ of doing so. This might represent the cost of running a campaign or the disutility of being in the public eye. The candidate who receives the most votes selects and implements policy. We assume a specific constitution governing the operation of elections. It specifies that in the event of ties, the winning candidate is chosen randomly with each tying candidate having an equal chance of being selected. If only one candidate runs for office then this candidate is automatically selected to make policy choices. Finally, if no-one runs a default policy x_0 is implemented.[14]

The social decision process has three stages. At stage one candidates declare themselves. At stage two voters choose whom to vote for among the declared candidates, with the candidate gaining the largest number of votes being elected. At the final stage, the selected candidate makes a policy choice. We analyze these three stages in reverse order.

3.1. Policy Choice

The citizen who wins the election implements his preferred policy. While candidates may have an incentive to promise something other than this, such promises are not credible. Citizen i's preferred policy is given by

$$x_i^* \in \arg\max_x \left\{ V^i(x;1) \mid x \in A^i \right\}: \tag{3.1}$$

[13] There are two possible interpretations of the model. The view adopted in most of this paper is of the community selecting a policy maker, charged with the task of selecting and implementing policy. This can be viewed as an occupation, with incumbents foregoing other opportunities in order to do it, and differing in their ability to be effective in the job. Hence $V^i(x;1) \neq V^i(x;0)$ and $A^i \neq A^j$: The model could also be interpreted as the community selecting a policy alternative by picking a representative to make that decision, rather than directly voting over policy alternatives. Thus one individual is assigned the right to control policy. On this interpretation, there is no reason to think that policy making consumes real resources. The assumptions $A^i = A$ for all $i \in N$ and that $V^i(x;1) = V^i(x;0)$ then make more sense. We focus on the first interpretation here as it raises a broader range of efficiency questions, particularly those pertaining to the competence of elected policy makers.

[14] Other constitutional rules could also be considered.

We will assume a unique solution to (3.1). Associated with each citizen's election, therefore, will be a utility imputation $v_i = \{v_{1i}, \ldots, v_{Ni}\}$; where v_{ji} is individual j's utility if i is elected. Obviously, $v_{ji} = V^j(x_i^*; 0)$ if $j \neq i$ and $v_{ii} = V^i(x_i^*; 1)$. If no citizen stands for office the default policy x_0 will be selected. We denote the utility imputation in this case as $v_0 = (v_{10}, \ldots, v_{N0})$.

3.2. Voting

Given a candidate set $C \subset N$, each citizen j makes a voting decision. He may vote for any candidate in C or he may abstain. Let $\alpha_j \in C \cup \{0\}$ denote his decision. If $\alpha_j = i$ then citizen j casts his vote for candidate i, while if $\alpha_j = 0$ he abstains. A vector of voting decisions is denoted by $\alpha = (\alpha_1, \ldots, \alpha_N)$.

Given C and α, let $F^i(C, \alpha)$ denote the number of votes that candidate i receives.[15] Then the set of winning candidates is

$$W(C, \alpha) = \{j \in C \mid F^j(C, \alpha) \geq F^k(C, \alpha) \text{ for all } k \in C\}. \tag{3.2}$$

These are the candidates who get at least as many votes as any other. Since if only one candidate runs he is automatically selected to choose policy, we adopt the convention that $W(C, \alpha) = C$ (for all α) when $\#C = 1$. The probability that candidate i wins is

$$P^i(C, \alpha) = \begin{cases} \frac{1}{\#W(C,\alpha)} & i \in W(C, \alpha) \\ 0 & \text{otherwise.} \end{cases} \tag{3.3}$$

This reflects the assumption that those candidates with the most votes have an equal chance of being chosen.

We assume that citizens correctly anticipate the policies that would be chosen by each candidate and vote strategically, with their voting decisions being a best response to what others do.[16] Hence, assuming that citizens are expected utility maximizers, we define a vector of voting decisions $\alpha^* = (\alpha_1^*, \ldots, \alpha_N^*)$ to be a voting equilibrium if for all $j \in N$,

$$\alpha_j^* \in \arg\max \left\{ \sum_{i \in C} P^i\left(C, \left(\alpha_j, \alpha^*_{-j}\right)\right) v_{ij} \mid \alpha_j \in C \cup \{0\} \right\}. \tag{3.4}$$

[15] Formally, $F^i(C, \alpha) = \#\{j \in N \mid \alpha_j = i\}$.

[16] For other models of voting behavior see Myerson and Weber (1993) and Palfrey and Rosenthal (1983).

This requirement is actually very permissive. There are many voting equilibria; in most of these, one individual's vote has no effect on the probability that any candidate wins. Thus the best response requirement has relatively little bite. We therefore introduce two refinements to help narrow the set of equilibria.

Our first refinement is standard in the voting literature: we require that no individual uses a weakly dominated voting strategy.[17] This eliminates voting equilibria in which individuals cast votes for their least preferred candidate. The weak dominance refinement has particular power in two candidate elections, since it implies that citizens vote sincerely; i.e., cast their votes for their most preferred candidate.

In elections with more than two candidates, the refinement of weak dominance does not have much power. Our second refinement is helpful here. Essentially, it says that individuals will vote sincerely in races where this produces a clear cut winner. Thus we respect the time-honored tradition of assuming that voters vote sincerely, except when such voting behavior fails to produce a definite winner. In this case voting sincerely need not be a best response.

To formally state our second refinement, we need the notion of a sincere partition. Given a candidate set C a partition[18] of the electorate $(N_i)_{i \in C \cup \{0\}}$ is said to be sincere if and only if (i) $\ell \in N_i$ implies that $v_{\ell i} \geq v_{\ell j}$ for all $j \in C$ and (ii) $\ell \in N_0$ implies that $v_{\ell i} = v_{\ell j}$ for all $i, j \in C$. Intuitively, a sincere partition divides the electorate among the candidates so that every voter is voting for his/her preferred candidate. There are many such partitions if some voters are indifferent between candidates. A candidate $k \in C$ is said to be dominant in the set of candidates C if, for all sincere partitions $(N_i)_{i \in C \cup \{0\}}$,

$$\#N_k - 1 > \mathrm{Max}\{\#N_i \mid i \in C / \{k\}\}.$$

Thus a dominant candidate gets one more vote than any other no matter how one assigns the indifferent voters to the candidates. If there is a dominant candidate

[17] A voting decision α_j is weakly dominated for for citizen j if there exists $\hat{\alpha}_j \in C \cup \{0\}$ such that

$$\sum_{i \in C} P^i(C, (\hat{\alpha}_j; \alpha_{-j}))v_{ij} \geq \sum_{i \in C} P^i(C, (\alpha_j; \alpha_{-j}))v_{ij}$$

for all α_{-j} with the equality holding strictly for some α_{-j}.

[18] A partition is a collection of disjoint, non-empty subsets of N; $(N_j)_{j \in J}$; such that $\cup_{j \in J} N_j = N$.

and individuals vote sincerely, then the race will not be close enough for any citizen to a®ect the outcome by switching his vote. Thus, voting sincerely will be a best response. Our second re¯nement, therefore, says that when there exists a dominant candidate individuals will vote for their preferred candidates.

In the sequel, we impose these two re¯nements on the set of voting equilibria and call an equilibrium which survives them a sincerely re¯ned voting equilibrium. We denote the set of such equilibria by $E(C)$. Our ¯rst proposition establishes, by construction, that this set is non-empty. The proof of this, and all subsequent results, can be found in the Appendix.

Proposition 1. For all non-empty candidate sets C ½ N, a sincerely re¯ned voting equilibrium exists.

3.3. Entry

Each citizen must decide whether or not to run for o±ce. The \campaign cost" incurred if he runs is ±. The potential bene¯t of running is either directly from winning o±ce and gaining the right to choose policy or indirectly from a®ecting which candidate wins and moving policy in a preferred direction. An individual's bene¯t from standing depends upon who else decides to enter, making the entry decision strategic. Thus, we model entry as a game between the N citizens.

Each citizen's pure strategy is s^i 2 f0; 1g, where $s^i = 1$ denotes entry by citizen i. A pure strategy pro¯le is denoted by $s = (s^1; :::; s^N)$. Given s, the set of candidates is $C(s)$ = fi j s^i = 1g. Each citizen's expected payo® from this strategy pro¯le depends on the way he expects the polity to vote. We assume that all citizens have the same expectations and let ®(C) be the vector of voting decisions that they anticipate with candidate set C. The function ®(¢) represents individuals' beliefs about voters' behavior. These beliefs will be referred to as consistent if ®(C) 2 E(C) for all non-empty candidate sets C ½ N. If individuals have consistent beliefs then the voting decisions that they anticipate form a sincerely re¯ned voting equilibrium.

Given beliefs ®(¢), we use (3.3) to calculate the expected payo® to any citizen i from a particular pure strategy pro¯le s. This is given by:

$$U^i(s;®(¢)) = \begin{cases} \sum_{j2C} P^j(C(s);®(C(s)))v_{ij} \; ¡ \; ± & \text{if } i \; 2 \; C(s) \\ \sum_{j2C} P^j(C(s);®(C(s)))v_{ij} + P^0(C(s);®(C(s)))v_{i0} & \text{if } i \; \not 2 \; C(s); \end{cases} \quad (3.5)$$

where $P^0(C;\alpha(C))$ denotes the probability that the default outcome is selected. Thus, $P^0(C;\alpha(C))$ equals one if $C = \emptyset$ and zero otherwise. Citizen i's payoff is therefore the probability that each candidate j wins multiplied by i's payoff from j's preferred policy, less the entry cost if he chooses to enter.

To ensure the existence of an equilibrium, we need to allow the use of mixed strategies. Let γ^i be a mixed strategy for citizen i, with the interpretation that γ^i is the probability that i runs for office. The set of mixed strategies for each citizen is then the unit interval $[0,1]$. A mixed strategy profile is denoted by $\gamma = (\gamma^1,\ldots,\gamma^N)$. Citizen i's expected payoff from the mixed strategy profile γ is denoted by $u^i(\gamma;\alpha(\cdot))$.[19] A mixed strategy profile $(\hat{\gamma}^1,\ldots,\hat{\gamma}^N)$ is an equilibrium of the entry game if there are consistent beliefs $\alpha(\cdot)$ such that for all $i \in N$, $u^i(\hat{\gamma}^i,\hat{\gamma}_{-i};\alpha(\cdot)) \geq u^i(\gamma^i,\hat{\gamma}_{-i};\alpha(\cdot))$ for all $\gamma^i \in [0,1]$. Our next result is

Proposition 2. An equilibrium of the entry game exists.

As a theoretical matter, this result is quite straightforward. Nonetheless, viewed in the context of models of policy choice, it is of interest. The Downsian model of political competition is plagued by non-existence problems. Indeed, the central focus of the theoretical literature in the Downsian tradition has been on investigating the conditions under which equilibrium does or does not exist in the basic model and to developing extensions which might mitigate the existence problems. In this light, it is natural to wonder what features of our model permit the existence problem to be disposed of so compactly. Two key features can be identified. First, in the entry game, each citizen has only two alternatives: enter or not enter. Once he has entered his policy choice is given by (3.1). This means that the entry stage is a finite game (Fudenberg and Tirole (1991)). Second, we allow the use of mixed strategies. These two features allow us immediately to apply the standard existence result due to Nash (1950).

In the Downsian model, the competing parties choose policy platforms from an infinite set of alternatives and mixed strategies are not typically permitted. The assumption of an infinite set of alternatives (together with the properties of the parties' payoff functions) mean that mixed strategies are by no means an instant fix for the existence problem (see Kramer (1978)). Technical difficulties

[19] This is given by $u^i(\gamma;\alpha(\cdot)) = \prod_{j=1}^N \gamma^j U^i(1,\ldots,1;\alpha(\cdot)) + \prod_{j=2}^N \gamma^j (1-\gamma^1) U^i(0,1,\ldots,1;\alpha(\cdot)) + \ldots\ldots + \prod_{j=1}^N (1-\gamma^j) U^i(0,\ldots,0;\alpha(\cdot))$.

not withstanding, researchers also appear to have been reluctant to pursue the mixed strategy solution because of difficulties in interpretation.[20]

The reader may object that it seems no more sensible to assume that citizens randomize over the decision to run for office. Thus, to the extent that mixed strategies are necessary to get existence of equilibrium, it may be argued that our model entails no real advance over the Downsian model in this regard. One response is to point out that pure strategy equilibria of our entry game do exist in a broader class of models than do equilibria of the Downsian model (as we show in the next section). However, we would go further and argue that the mixed strategy equilibria of our model do have a natural interpretation.

Harsanyi (1973) demonstrated that mixed strategy equilibria of complete-information games can typically be interpreted as the limit of pure strategy equilibria of slightly perturbed games of incomplete information (see Fudenberg and Tirole (1991)). In our context, the slightly perturbed game is one in which each citizen i has a slightly different entry cost given by $\delta_i = \delta + \varepsilon \cdot \theta_i$: Here, $\varepsilon \in (0; 1)$ and θ_i is the realization of a random variable with range $(-\delta; \delta)$ and distribution function $G(\theta)$. In this game, θ_i, and hence citizen i's entry cost, is private information. A pure strategy for citizen i is then a mapping $\sigma^i : (-\delta; \delta) \to \{0; 1\}$, with the interpretation that $\sigma^i(\theta_i)$ denotes citizen i's entry decision when his "type" is θ_i. Mixed strategy equilibria of the entry game can then be interpreted as the limit of pure strategy equilibria of this extended game as ε goes to zero. The small amount of uncertainty needed here seems quite appealing. Our simplifying assumptions notwithstanding, individuals are likely to differ in the psychic costs of running for office, with this being private information. Thus we are comfortable treating the mixed strategy equilibria of our model as predictions about how the game might be played.[21] We therefore view Proposition 2 as a powerful result. It allows us to focus on discussing properties of equilibrium rather than worrying about making assumptions to guarantee existence. This is a major attraction of the approach taken here.

To understand the significance of Proposition 2, the generality of the set-

[20] Ordeshook (1986), for example, argues that "it seems silly to conceptualize candidates spinning spinners or rolling dice to choose policy platforms."

[21] It is natural to wonder whether Harsanyi's approach can be similarly utilitized to convince researchers of the value of studying mixed strategy equilibria in the Downsian model. On the face of it, this would seem problematic because, given the presumption that both parties just want to win, it seems unreasonable to postulate that the players have any private information about their payoffs. However, this merits further investigation.

up should be appreciated. Most models of policy making can be fitted into our framework. We conclude this section with a couple of examples from the literature. Bearing these in mind should help in interpreting some of the subsequent results.

Example 1: The standard public goods problem from Samuelson (1954) fits the model. Suppose that there are two goods, a private good with individual i's consumption denoted by y_i and a public good z. Each citizen is endowed with ω units of the private good and the economy has a technology that can transform one unit of private good into one unit of the public good. A policy vector is now $x = (y_1, \ldots, y_N, z) \in \mathbb{R}_+^{N+1}$. The feasible set of policy alternatives A (the same for all citizens) is the set of all policy vectors such that $\sum_{j=1}^{N} y_j + z \leq N\omega$. Suppose that $V^i(x) = u(y_i, z) + \sum_{k \neq i} \delta^{ik} u(y_k, z)$, where $u(\cdot)$ is a common consumption utility function over private and public goods and δ^{ik} is the weight that individual i attaches to individual k's consumption utility. It is readily shown that citizen i's policy choice will satisfy:

$$\begin{gathered} \frac{\partial u(y_i, z)}{\partial y_i} = \delta^{ik} \frac{\partial u(y_k, z)}{\partial y_k} \; \forall k \neq i \\ \text{and} \\ \sum_{j=1}^{N} \frac{\partial u(y_j, z)/\partial z}{\partial u(y_j, z)/\partial y_j} = 1. \end{gathered} \tag{3.6}$$

The second condition in (3.6) is the Samuelson condition for efficient public goods supply, while the first condition determines the distribution of the private good.

Example 2: This example illustrates how incentive constraints can be incorporated into the definition of the feasible set, along the lines of Mirrlees (1971). There are two goods: a private consumption good c and labor l. Each citizen is endowed with an identical amount of labor but citizens differ in their productivity. High ability citizens produce a_H units of the private good with one unit of labor, while low ability citizens produce only a_L units. Without loss of generality, let citizens $i = 1, \ldots, m$ be of high ability and citizens $i = m+1, \ldots, N$ be of low ability. Policy alternatives are represented by a duple of consumption-income pairs $\{(c_H, y_H), (c_L, y_L)\}$. Here, c_H (c_L) is a high (low) ability individual's consumption and y_H (y_L) his income. The difference $y_H - c_H$ ($c_L - y_L$) represents taxes (transfers). Citizens have a common utility function defined over their own consumption and labor supply, but they differ in their concern for individuals in the other group. Formally, for all $i = 1, \ldots, m$, $V^i = u(c_H, y_H/a_H) + \delta^i u(c_L, y_L/a_L)$

and for all $i = m+1, \ldots, n$, $V^i = u(c_L, y_L/a_L) + \delta^i u(c_H, y_H/a_H)$ where $\delta^i \in (0, 1)$. Thus both groups care most about their own well-being and differ in their desire to make transfers to the other group. The policy maker is assumed to be unable to distinguish high and low ability individuals. He must therefore choose a duple of consumption-income pairs that is incentive compatible. The set of feasible policy alternatives A is therefore the set of all $\{(c_H, y_H), (c_L, y_L)\}$ which satisfy the resource constraint $m(y_H - c_H) = (N - m)(c_L - y_L)$ and the incentive compatibility constraints

$$u(c_H, y_H/a_H) \geq u(c_L, y_L/a_H) \text{ and } u(c_L, y_L/a_L) \geq u(c_H, y_H/a_L). \tag{3.7}$$

The policy choice and utility imputation associated with any citizen i is straightforwardly determined. The presence of the incentive constraints restrict the amount of redistribution that can be achieved.

4. Pure Strategy Equilibria

This section studies pure strategy equilibria, providing a characterization of them via a series of Propositions. As well as giving a fairly complete picture of such equilibria, the results comprise a tool kit for applying the model in specific contexts.

A pure strategy profile $\hat{s}$ is an equilibrium of the entry game if there are consistent beliefs $\alpha(\cdot)$ such that for all $i \in N$, $U^i(\hat{s}^i, \hat{s}_{-i}, \alpha(\cdot)) \geq U^i(s^i, \hat{s}_{-i}, \alpha(\cdot))$ for $s^i \in \{0, 1\}$. More usefully, it can be shown that s is a pure strategy equilibrium if and only if there exist consistent beliefs $\alpha(\cdot)$ such that the following two conditions are satisfied. First, for all $i \in C(s)$

$$\begin{aligned} &\sum_{j \in C(s)} P^j(C(s), \alpha(C(s)))v_{ij} - \delta \geq \\ &\qquad \sum_{j \in C(s)/\{i\}} P^j(C(s)/\{i\}, \alpha(C(s)/\{i\}))v_{ij} + P^0(C(s)/\{i\})v_{i0}, \end{aligned} \tag{4.1}$$

where $C/\{i\}$ is the candidate set with individual i removed. This says that each candidate be willing to run, given who else is in the race. For (4.1) to hold, each candidate's withdrawal must affect the outcome. Second, for all $i \notin C(s)$

$$\begin{aligned} &\sum_{j \in C(s)} P^j(C(s), \alpha(C(s)))v_{ij} + P^0(C(s))v_{i0} \geq \\ &\sum_{j \in C(s) \cup \{i\}} P^j(C(s) \cup \{i\}, \alpha(C(s) \cup \{i\}))v_{ij} - \delta. \end{aligned} \tag{4.2}$$

This says that the equilibrium is entry proof, i.e., there is no individual not in the race who would like to enter. Our characterization results basically involve a more detailed appreciation of what conditions (4.1) and (4.2) imply. We begin by investigating the possibility of one candidate pure strategy equilibria.

4.1. One Candidate Equilibria

In some situations, there is an equilibrium in which only one citizen runs and is elected unopposed. The following proposition develops the necessary and sufficient conditions for this to arise.

Proposition 1. Citizen i running unopposed is a pure strategy equilibrium if and only if

(i) $v_{ii} - v_{i0} \geq \delta$

and

(ii) for all $k \in N \backslash \{i\}$ such that $\#N_k \geq \#N_i$ for all sincere partitions $(N_i; N_k; N_0)$, then $\frac{1}{2}(v_{kk} - v_{ik}) \leq \delta$ if there exists a sincere partition such that $\#N_i = \#N_k$ and $v_{kk} - v_{ik} \leq \delta$ otherwise.

This result is easily understood. The first condition guarantees that the hypothesized candidate's gain from running is sufficient to compensate him for the entry cost. The second condition guarantees the existence of consistent beliefs which give no other citizen an incentive to enter the race. Finding an individual for whom the first condition is satisfied is not a problem if the default option is poor enough and the costs of running are small. The second condition, however, is much more difficult to satisfy. It requires that citizen i's policy alternative be preferred by a majority to the policy alternative of any other citizen with significantly different policy preferences. As the following result shows, if entry costs are small, this amounts to citizen i's policy choice being a Condorcet winner[22] in the set of preferred policy alternatives of the N citizens.

[22] Suppose that $V^j(x;1) = V^j(x;0) = V^j(x)$ for all $j \in N$. Then an alternative $x \in S \subset A$ is a Condorcet winner in S if for all $z \in S \backslash \{x\}$

$$\#\left\{j \mid V^j(x) \geq V^j(z)\right\} \geq \#\left\{j \mid V^j(x) < V^j(z)\right\}.$$

Corollary 1. Suppose that $A^j = A$ and $V^j(x;1) = V^i(x;0)$ for all citizens $j \in N$. Then if citizen i running unopposed is a pure strategy equilibrium for all $\delta \in (0, v_{ii} - v_{i0}]$, x_i^* is a Condorcet winner in the set of alternatives $\{x_j^* : j \in N\}$.

The conditions for the existence of a Condorcet winner are well-known to be extremely restrictive, making it unlikely that one candidate pure strategy equilibria exist in most environments. Nonetheless, since the Downsian model of political competition only produces a prediction in such cases, such equilibria will exist in most cases where that model is used (see section 5.1 for an example).[23]

The weak dominance refinement on voting equilibria is important for Proposition 3. Without requiring that voters do not employ weakly dominated voting strategies, it would be possible to construct pure strategy equilibria with any citizen who is willing to enter against the default option as the sole candidate! Such equilibria would be supported by beliefs that no entrant would garner any support against this candidate. These beliefs could be consistent if we only required that $\alpha(C)$ was a voting equilibrium. This is the sense in which voting equilibrium by itself is extremely permissive.

4.2. Two-Candidate Equilibria

Political scientists have long taken seriously Duverger's empirically motivated "law" that two party competition is the democratic norm. We do not, as yet, have parties in our model. Nonetheless, the study of two candidate equilibria is a central case of interest. The following result gives necessary and sufficient conditions for them to arise.

Proposition 2. Suppose that citizens i and j running against each other is a pure strategy equilibrium, then

(i) $\frac{1}{2}(v_{ii} - v_{ij}) \geq \delta$; $\frac{1}{2}(v_{jj} - v_{ji}) \geq \delta$; and there exists a sincere partition $(N_i; N_j; N_0)$ such that $\#N_i = \#N_j$;

and

(ii) for $k \neq i; j$, if k is dominant in the set of candidates $\{i; j; k\}$ then $v_{kk} - \delta \leq \frac{1}{2}(v_{ki} + v_{kj})$; if i is dominant then $\frac{1}{2}(v_{ki} - v_{kj}) \leq \delta$; and if j is dominant then $\frac{1}{2}(v_{kj} - v_{ki}) \leq \delta$.

[23] It is actually more likely that a one candidate equilibrium exists in our model, since we only need to find a Condorcet winner in the set of policies that would be chosen by some citizen if elected, rather than in the set of all feasible policies.

Furthermore, if $N_0 = \{\ell \in N \mid v_{\ell i} = v_{\ell j}\}$ and $\#N_0 + 1 < \#N_i = \#N_j$, then these conditions are sufficient for i and j running against each other to be a pure strategy equilibrium.

Part (i) guarantees that both candidates want to be in the race. For this to hold, they must significantly prefer their own policy choice to that of the other candidate and both must have some chance of winning. The first condition in part (ii) guarantees that no citizen who would be dominant if they joined i and j in the race wishes to enter. The sincerity refinement implies that such a citizen must win (and hence obtain a payoff $v_{kk} - \delta$) if he were to enter. The second two conditions in part (ii) refer to cases where k's entry makes i or j dominant. In such circumstances, citizen k may be tempted to enter as a strategic candidate — a candidate whose presence guarantees the victory of another. The second two conditions in part (ii) guarantee that no citizen has an incentive to enter as a strategic candidate.

The sincere refinement has power in this characterization of two candidate equilibria. Its real bite is in the second part of the Proposition, ruling out the following scenario. Suppose that there are two candidates who satisfy condition (i) of the Proposition, i.e. are willing to run against each other and receive half of the votes, and a third "consensus" candidate who is preferred by, let us say, 70% of the voters. Then will this latter candidate win if he enters the race? While continuing to vote for the original candidates remains a voting equilibrium, the sincere refinement picks the voting equilibrium in which the consensus candidate wins. Thus, if beliefs are consistent, the consensus candidate enters if the cost of doing so is small enough.

4.3. Equilibria with three or more Candidates

Casual empiricism suggests that equilibria with three or more candidates are quite possible in representative democracies. Turning to the conditions for pure strategy equilibria of this form, we begin by arguing that they will be one of two types. Either the election outcome is close between all candidates (in fact in our set-up all candidates are exactly tying for victory) or there is single winner. Key to our argument is the presumption that citizens' beliefs about voters' decisions are likely to have the following property.

Independence of Irrelevant Candidates: The beliefs $\alpha(\cdot)$ satisfy Independence of Irrelevant Candidates (IIC) if whenever $F^i(C; \alpha(C)) = 0$, then $\alpha(C) = \alpha(C/\{i\})$.

If beliefs have this property, then if a particular citizen is in the race and receives no votes, citizens believe that his withdrawal will not affect individuals' voting decisions.

Proposition 3. *Let s be a pure strategy equilibrium such that $\#C(s) \geq 3$ and let $\alpha(\cdot)$ be the supporting beliefs. If these beliefs satisfy IIC and if no citizen is indifferent between any two candidates then either the winning set contains all of the candidates ($W(C(s); \alpha(C(s))) = C(s)$) or it contains only one ($\#W(C(s); \alpha(C(s))) = 1$).*

The logic behind this Proposition is easily seen in the case of exactly three candidates. Suppose that there are two in the winning set. Then unless those voting for the losing candidate are indifferent between the two winners, they would be better off switching their votes. This would leave the losing candidate with no support. If beliefs satisfy IIC, he would drop out because he must believe his presence in the race to have no effect.[24]

We now study each of the two types of equilibria described in Proposition 5, giving conditions for each kind to arise. We begin by developing a necessary condition for the case where all candidates are in the winning set.

Proposition 4. *Let s be a pure strategy equilibrium such that $\#C(s) \geq 3$. Let $\alpha(\cdot)$ be the supporting beliefs and suppose that $W(C(s); \alpha(C(s))) = C(s)$. Then there must exist a sincere partition $(N_i)_{i \in C(s) \cup \{0\}}$ such that $\#N_i = \#N_j$ for all $i, j \in C(s)$ and for all $i \in C(s)$*

[24] The assumption that no citizen be indifferent between any two candidates precludes the existence of multi-candidate equilibria in which, say, two candidates are close and a third losing candidate gets some positive support. In such equilibria, the voters for the loser are indifferent between the two winners. The losing candidate, however, has a strict preference for one winning candidate over the other and believes that his supporters would be more likely to vote for his least preferred candidate if he withdrew. While possible, such equilibria seem somewhat unlikely, since there is no good reason to expect an indifferent voter to be more likely to vote for one candidate than another.

$$\sum_{j \in C(s)} \left(\frac{1}{\#C(s)} \right) v_{\ell j} \geq \max\{v_{\ell j} \mid j \in C(s) \setminus \{i\}\} \quad \text{for all } \ell \in N_i.$$

To understand this result observe that, in a multi-candidate election where all candidates are tying, each voter is decisive. This implies that each voter is voting sincerely, or else he could switch to his most preferred candidate and ensure his election (see also Lemma 1 of Feddersen, Sened and Wright (1992)). The stated inequality should also hold; each citizen must prefer the lottery over all the candidates to the certain victory of any candidate other than his most preferred. In many applications this condition cannot be satisfied. For example, in a large economy in which voters' preferences vary continuously, then for any set of three or more candidates, there will be some set of citizens nearly indifferent between two candidates.[25] The inequality in Proposition 6 then fails. While multi-candidate equilibria in which all candidates are in the winning set may be unusual, they are not ruled out by our framework. Our next proposition develops a set of sufficient conditions for such an equilibrium.

Proposition 5. Let s be a pure strategy profile with $\#C(s) \geq 3$, and suppose that $A^i = A^j$ for all $i, j \in C(s)$ and $V^i(x;1) \geq V^i(x;0)$. Suppose that there is a sincere partition $(N_i)_{i \in C(s) \cup \{0\}}$ such that

(i) $\#N_i = \#N_j > \#N_0 + 1$ for all $i, j \in C(s)$

(ii) for all $i \in C(s)$

$$\sum_{j \in C(s)} \left(\frac{1}{\#C(s)} \right) v_{\ell j} > \max\{v_{\ell j} \mid j \in C(s) \setminus \{i\}\} \quad \text{for all } \ell \in N_i.$$

(iii) for all $i \in C(s) \cup \{0\}$ and for all $k \in N_i$ neither i nor k is dominant in the candidate set $C(s) \cup \{k\}$:

Then, for sufficiently small δ, s is a pure strategy equilibrium supported by beliefs $\alpha(\cdot)$ such that $W(C(s); \alpha(C(s))) = C(s)$:

[25] Feddersen (1992) exploits this fact in a related model. In his set-up, voters may cast their vote for one of an infinite number of policy alternatives. The alternative which gets the most votes is implemented. Voting is costly and voters vote strategically. His main result, which exploits an inequality similar to that in Proposition 6, is that only two alternatives receive support in equilibrium.

The first condition of the Proposition guarantees that the set of indifferent voters is not so large that if all of them switched to an entrant they could change the outcome of the election. The second condition guarantees both that each voter votes sincerely and that all candidates wish to run (for sufficiently small δ). The third condition guarantees that no citizen not in the race wishes to enter.

We now consider equilibria with three or more candidates with a single winner.

Proposition 6. Let s be a pure strategy equilibrium such that $\#C(s) \geq 3$. Let $\alpha(\cdot)$ be the supporting beliefs and suppose that $\#W(C(s);\alpha(C(s))) = \{i\}$. Then for all $j \in C(s)/\{i\}$

(i) $W(C(s)/\{j\};\alpha(C(s)/\{j\})) \neq \{i\}$,

and

(ii) there exists $k \in C(s)$ such that $v_{ji} - \delta \geq v_{jk}$.

These two facts follow directly from considering the incentives for losing candidates to run in this type of equilibrium. Losing candidates remain in the race because they prefer the current winner's policy to that of the candidate who would win if they dropped out. All but the winning candidate are being strategic in this type of equilibrium. However, each is decisive to the final outcome. Stating sufficient conditions for the existence of this type of equilibrium is not particularly enlightening. Nonetheless, it is not difficult to produce examples of such equilibria in particular applications.

5. The Model at Work

This section considers some examples to see what our model predicts for them. This will give an idea of how the approach can be used in practice and what kinds of predictions it gives about equilibrium policy choices. In our first two examples we focus on pure strategy equilibria and use the propositions of the previous section. The third example exhibits a mixed strategy equilibrium.

5.1. A One-Dimensional Model with Euclidean Preferences

Our first example is the standard one-dimensional issue space model which is widely used in the formal political science literature and is basically the model used by Downs. The set of policy alternatives is the unit interval $[0;1]$, which crudely captures the idea of ideological disagreement from left to right. Each citizen has Euclidean preferences over these alternatives with ideal point ω_i. We suppose that $V^i(x;1) = V^i(x;0) = -\|\omega_i - x\|$ and a default policy of $x_0 = 0$. For simplicity, we assume that the number of citizens is odd. Let m denote the citizen with the median ideal point. We also assume that the distribution of ideal points is symmetric in the sense that $\omega_i = 1 - \omega_{N+1-i}$ for all $i \in \{1, \ldots, m\}$. This, together with our other assumptions, implies that $\omega_m = \frac{1}{2}$. Recall that in this environment, the Downsian model predicts that both parties will offer the platform preferred by the median voter; i.e. $x = 1/2$.

We will calculate the pure strategy equilibria of this model.[26] We begin with one candidate equilibria. Using Proposition 3, we obtain the following result.

Claim 1. Citizen i running unopposed is a pure strategy equilibrium if and only if

(i) $\omega_i \geq \delta$

and

(ii) $\{\ell \mid \omega_\ell \in (1 - \omega_i, \omega_i - \delta)\} = \{\ell \mid \omega_\ell \in (\omega_i + \delta, 1 - \omega_i)\} = \emptyset$.

The first condition guarantees that citizen i wishes to run against the default outcome. The second condition guarantees that no other citizen wishes to enter. Essentially, condition (ii) implies that citizen i's ideal point is not too far away from the median. A sufficient condition for (ii) to be satisfied is that $\omega_i \in (1/2 - \delta/2, 1/2 + \delta/2)$. A necessary condition is that $\omega_i \in (1/2 - \delta, 1/2 + \delta)$. Thus in the one candidate equilibrium, the policy prediction is much the same as in the Downsian model. However, unlike that model, this median outcome does not emerge from two party competition, but it is a monopoly phenomenon. Since candidates care about policy rather than winning, there is no reason for a citizen to run against a candidate who would implement the same policy as him.

Turning to two candidate equilibria, we apply Proposition 4 to obtain:

Claim 2. Citizens i and j, with $\omega_i < \omega_j$, form a two candidate pure strategy equilibrium if and only if

(i) $\omega_i + \omega_j = 1$ and $\omega_j - \omega_i \geq 2\delta$

and

[26] This is essentially the model analyzed by Osborne and Slivinski (1994). They, however, assume a continuum of citizens and allow citizens to receive some independent benefit from holding office; that is, $V^i(x;1) = \beta_i - \|\omega_i - x\|$. As noted in the introduction, their treatment also differs from ours in assuming that citizens vote sincerely.

(ii) if $\omega_j - \omega_i > 2\delta$; then for all k such that $\omega_k \in (\omega_i, 1/2]$,

$$\#\left\{\ell \mid \omega_\ell \in \left[\frac{(\omega_i + \omega_k)}{2}, \frac{(\omega_j + \omega_k)}{2}\right]\right\} \leq \#\left\{\ell \mid \omega_\ell \in \left[\frac{(\omega_j + \omega_k)}{2}, 1\right]\right\} + 1.$$

These conditions have straightforward interpretations. The first condition in part (i) says that the ideal points of the two candidates be on opposite sides and equidistant from the median. This ensures that the two candidates split the electorate and the race is close. The second condition says that the candidates must be far enough apart so that each finds it worthwhile to compete against the other. This prevents policy convergence in our two candidate equilibrium. Part (ii) refers to entry proofness and is tantamount to the requirement that the candidates be not too far apart. It guarantees that the two candidate's ideal points are sufficiently close that no citizen with an intermediate ideal point would be dominant if he entered. The left hand side of the inequality represents the number of citizens who would prefer a candidate with ideal point ω_k to candidates i and j. The right hand side is the number who would support candidate j. The symmetry of the problem implies that candidate j would attract more supporters than candidate i in a three way race and thus he is the candidate citizen k has to beat. Symmetry also implies that if the inequality holds for k such that $\omega_k \in (\omega_i, 1/2]$ then it must also hold for k such that $\omega_k \in [1/2, \omega_j)$. By taking $\omega_k = 1/2$, a necessary condition for part (ii) is that the number of citizens with ideal points in the interval $[1/4 + \omega_j/2, 1]$ plus one, must exceed the number with ideal points in the interval $[3/4 - \omega_j/2, 1/4 + \omega_j/2]$. If there is a large number of citizens with uniformly distributed ideal points, this requires that $\omega_j \leq 5/6$.

These two candidate equilibria are at variance with the predictions of the Downsian model. While our model predicts that two candidate elections will typically be close (as they are in the Downsian model because both parties offer the same platforms), the policies associated with the two candidates may be quite different. The model predicts a see-saw across the political spectrum by candidates whose ideologies counter-balance each other. In particular, the model predicts that more extreme conservatives should be pitted against more extreme liberals. Otherwise, the less extreme candidate would be bound to win and the extremist would not wish to enter the race.

Finally, we turn to races with more than two candidates. Our main finding is

Claim 3. There are no pure strategy equilibria involving three or more candidates in which all the candidates tie.

The proof of this result draws on Proposition 6. It is first shown that the inequality in Proposition 6 implies that there can be only three candidates in such an equilibrium. It is then demonstrated that if there are three candidates, at least one would be better off not entering. The possibility of multi-candidate equilibria in which only one candidate wins does, however, remain. We have not, as yet, been able to rule out this possibility.

5.2. A Simple Two-Dimensional Model

Our next example is a two-dimensional model of policy choice in which our approach yields a family of two candidate pure strategy equilibria, even though the Downsian approach produces no pure strategy equilibrium. Citizens differ in their preferences over two issues: a discrete policy, denoted by $\chi \in \{0;1\}$, where 1 denotes the policy being implemented, and a continuous policy variable, denoted by $p \in [0;1]$. The set of policy alternatives is therefore $[0;1] \times \{0;1\}$: While this set-up is special, one can think of many sensible interpretations. The discrete policy might represent an issue like the death penalty or abortion, or an economic issue like the passing of NAFTA or the building of a bridge. The continuous policy might be defense spending or foreign aid. There are no costs and benefits associated with being policy maker and each citizen's preferences over the set of policy alternatives are of the form:

$$V^i(p;\chi) = \theta^i \chi - \| p - p^i \| :$$

The willingness to pay for the discrete policy (θ^i) takes on one of two values $\{-\theta_L;\theta_H\}$, where $0 < \theta_L < \theta_H$. We will describe those with willingness to pay θ_H as in favor of the discrete policy and those with preference parameter $-\theta_L$ as opposed to it.

To keep the analysis clean, we adopt the fiction that the polity consists of a continuum of citizens. We assume that the distribution of ideal points of the continuous policy variable (p^i) is uniform on $[0;1]$ and denote by λ the fraction of the population who are in favor of the discrete policy. Throughout we assume (i) $\lambda \in (\frac{1}{3};\frac{1}{2})$; (ii) $\theta_L \in (0;\frac{}{4(1-\lambda)})$; (iii) $\theta_H > 1$ and (iv) $\theta_L > \delta$. The first assumption says that a minority of the population favor the discrete policy. The second and third assumptions imply that those in favor are willing to pay more for the policy than those against are willing to pay to avoid it. Assumption (iii) also implies that in comparing two policy alternatives $(p_A;1)$ and $(p_B;0)$, an individual in favor of the discrete policy will always prefer $(p_A; 1)$. Thus citizens in favor of the policy are close to being single issue voters. Assumption (iv) says that the cost of running for office is relatively small.

We establish two facts about the model. First, we show that there is no Condorcet winner.

Claim 4. For any policy alternative $(p; \chi)$; there exists another policy $(p'; \chi')$ which is preferred by a majority of the citizens.

Figure 1 illustrates this result graphically. The median position $(\frac{1}{2}; 0)$ is defeated by the alternative $(\frac{1}{2} - \theta_L; 1)$. The latter alternative is preferred by a coalition of those in favor of the discrete policy and those opposed whose ideal points for the continuous policy lie to the left of $\frac{1}{2} - \theta_L$. It should be clear that this result depends on the fact that θ_L is small relative to θ_H. This result tells us two things — that the Downsian approach would produce no pure strategy equilibria and that there are no one candidate equilibria of our model.

Our second result demonstrates the possibility of two candidate pure strategy equilibria in our model.

Claim 5. There exists a family of two candidate pure strategy equilibria in which both candidates are in favor of the discrete policy.

As in the one-dimensional model, these equilibria involve the two candidates' views on the continuous policy being on opposite sides and equidistant from the median. It is interesting that both candidates are in favor of the discrete policy despite this being preferred by a minority of the polity!

5.3. A One-Dimensional Model with Non-Single Peaked Preferences

We now return to a one-dimensional model but consider a famous example where the median voter theorem fails: non-single peaked preferences. To be concrete, we take an application from public economics: public provision of private goods when individuals can opt out and consume in the private sector (see, for example, Stiglitz (1974) and Besley and Coate (1991)).

The polity is divided into three groups; rich, middle class and poor. Their sizes are N_R, N_M, and N_P. We assume that $\frac{N}{2} > N_M > Max\{N_R; N_P\} + 1$ and also that $N_i \neq N_j$ for $i; j \in \{P; M; R\}$. Society must choose the level of public provision of a private good, such as public health care or education. Each citizen also has the option of buying the good in the market, making no public provision a policy option. We assume that there is a unit demand for the publicly provided good. However, quality may di®er. We allow quality provided in the public sector to be at one of two levels, q_L and q_H; with L standing for low and H for high. Thus the set of social alternatives is $f0; q_L; q_H g$. We assume that the status quo point is zero provision.

Citizens in each group have identical tastes and order policy choices as follows:

$$v_R(0) > v_R(q_L) > v_R(q_H)$$
$$v_M(q_H) > v_M(0) > v_M(q_L)$$
$$v_P(q_L) > v_P(q_H) > v_P(0)$$

These preferences can be justi¯ed by the fact that the rich always prefer to use the private sector and are forced to pay taxes for the poor and middle classes to consume in the public sector. The middle class use the public sector only if quality is high and would rather have no public sector than one that they did not use. Finally, the poor prefer low quality provision to high because they have to ¯nance some of the tax burden associated with the public sector and quality is a normal good. That preferences can have this property is shown by Stiglitz (1974) for the case of public education.

It is straightforward to verify that there is no Condorcet winner in this environment. Low quality would lose to zero provision; zero provision would lose to high quality; and high quality would lose to low quality. Thus the Downsian approach again produces no pure strategy equilibrium. It is also true that our approach yields no pure strategy equilibria (for su±ciently small ±).[27] However, there are interesting mixed strategy equilibria.[28]

We focus on mixed strategy equilibria involving one citizen from each of the three groups entering with positive probability. We label the representatives from each of the groups as M, P and R. The normal form of the game between these three citizens is in Figure 2. There are two payo® matrices, where M choose the column, P chooses the row and R chooses the payo® matrix. We show in the Appendix that, for su±ciently small ±, there is a unique mixed strategy

[27] This is proven in the Appendix.

[28] In this example, because of the discrete set of policy alternatives, it is very easy to calculate mixed strategy equilibria for the Downsian model. There is a unique equilibrium of this form which involves each party choosing each alternative with probability 1=3.

equilibrium of this three person game given by:

$$\gamma_P = 1; \gamma_M = \frac{v_R(0) - v_R(q_L) - \delta}{v_R(0) - v_R(q_H)} \text{ and } \gamma_R = \frac{\delta}{v_M(q_H) - v_M(0)}.$$

It can also be verified that, given the three representatives of each group are entering with these probabilities, no other citizen has an incentive to enter. Thus, the three representatives M, P and R entering with probabilities γ_M; γ_P and γ_R and every other citizen entering with probability zero is a mixed strategy equilibrium of the entry game. In this equilibrium, as δ gets small, the probability of the poor individual being selected to choose policy goes to one. Thus the policy outcome is low quality public provision with the rich and the middle class consuming in the private sector. This is interesting since the biggest group (the middle class) almost always get their least preferred policy. In effect, the equilibrium involves the poor and rich ganging up on the middle class to keep them out of power.

6. Normative Analysis of Representative Democracy

This section investigates the performance of representative democracy. The social choice problem faced by the polity can be framed as selecting two things, a citizen to govern and a policy to be implemented. We consider in what sense, if any, the particular selection produced through representative democracy has desirable features. This is answered under two headings: efficiency and equity, the latter referring to the relative altruism of the individuals who are elected to govern.

6.1. Efficiency

We begin with some terminology. A selection is a pair $\{i; x\} \in N \times A$, with the interpretation that citizen i is selected to implement a policy alternative x. A selection $\{i; x\}$ is feasible if the policy selected can be implemented by citizen i; that is, if $x \in A^i$. A selection is efficient if it is feasible and there exists no alternative feasible selection $\{j; x'\}$ such that $V^i(x'; 0) > V^i(x; 1)$, $V^j(x'; 1) > V^j(x; 0)$ and $V^k(x'; 0) > V^k(x; 0)$ for all $k \in N/\{i; j\}$.[29] Our first question concerns the

[29]We are using a slightly weaker notion of efficiency than is standard. We require only that there exists no feasible selection such that every citizen is better off. This definition leads to simpler results and avoids some odd special cases.

ability of representative democracy to produce an e±cient selection.[30]

We begin with the simplest case where the choice of policy maker does not a®ect the feasible set of social alternatives, i:e:, $A^i = A$ for all $i \in N$ and where being the policy maker is not actually costly, i:e:; $V^i(x;1) \geq V^i(x;0)$ for all $i \in N$. In this case, from an e±ciency perspective, the identity of the citizen who implements policy is irrelevant; the only issue is whether the policy choice is e±cient. The following result shows that e±ciency is guaranteed if at least one individual runs for o±ce.

Proposition 1. Suppose that $A^i = A$ and $V^i(x;1) \geq V^i(x;0)$ for all $i \in N$. Then, provided that the equilibrium set of candidates C is not empty, representative democracy produces an e±cient selection.

The logic behind this result is straightforward. The equilibrium policy choice maximizes the utility of the citizen who wins the election.[31] Thus, there can be no alternative policy that makes all citizens (including the policy maker) better o®. A common reaction is to suggest that the preferences of the policy maker should not count. This is understandable given the tradition of modeling policy choices by mythical planners in normative models or memberless political parties in positive models.[32] However, policies are chosen and implemented by citizens

[30] Our analysis ignores two other possible costs of democratic selection. First, there is some randomness in the selection if the winning set contains more than one candidate or individuals use mixed strategies. This may reduce citizens' ex ante expected utilities. Second, resources are used up in the process of generating the selection; a candidate set C costs society $\#C \cdot \delta$. Even if representative democracy produces an e±cient selection, there may be a method of selecting policy which is both ex post e±cient and uses fewer "campaign" resources. Further discussion of these issues can be found in our companion paper (Besley and Coate (1995)).

[31] Our model of representative democracy can be related to a study of implementation in Nash equilibrium by Hurwicz and Schmeidler (1978). They investigate the existence of a non-dictatorial mechanism for selecting a social outcome such that (i) for every preference pro¯le there exists a Nash equilibrium and (ii) such equilibria are e±cient. They prove by construction that there exists such a mechanism which they call the kingmaker outcome function. This involves one individual, or a group of individuals, selecting another to make social decisions. Our model of representative democracy can be thought of as a particular kingmaker outcome function. Propositions 2 and 9 con¯rm its desirable properties.

[32] Ignoring the utility of the policy maker is well established in the public choice literature. The rent seeking literature, beginning with Tullock (1967), typically takes no account of the utility derived by the policy maker from the rent seeking activities. Expenditures on bribes, expensive dinners, etc. are viewed as waste rather than as transfers. Similarly, the literature on

and the notion of Pareto efficiency properly demands that we take the policy maker's preferences into account. To do otherwise would be to make an implicit distributional judgment about the social value of different individuals' utilities.

Proposition 9 requires at least one candidate, which may not be a trivial requirement. With high entry costs, this is clear. However, even with small entry costs, we may have non-entry if citizens have very similar tastes. Suppose, for example, that there are two citizens with identical tastes and a status quo that is Pareto dominated by either being in power. Let $\bar{v}$ be the utility if either individual is in power and let $\underline{v}$ be the utility in the status quo. Each citizen would then prefer that the other run for office if there is any entry cost. For $\delta < \bar{v} - \underline{v}$, the unique symmetric Nash equilibrium in entry decisions involves each individual running with probability $\frac{\bar{v} - \delta - \underline{v}}{\bar{v} - \underline{v}}$. Hence with probability $\left(\frac{\delta}{\bar{v} - \underline{v}}\right)^2$ nobody is elected and a Pareto inferior outcome obtains. This kind of inefficiency is typical of private supply of discrete public goods.[33]

The assumptions that $A^i = A$ and $V^i(x; 1) \geq V^i(x; 0)$ are strong. When the task of the policy maker includes the implementation of policy, different competence levels seem reasonable. Moreover, it is natural to postulate costs associated with governing. The question of whether representative democracy produces an efficient selection then becomes much more subtle. In particular, it is no longer true that citizen i's utility as a policy maker is as great as it would be under any other feasible selection, i.e. $v_{ii} \geq V^i(x; 0)$ for all $(j; x) \in N \times A^j$. Efficiency is therefore not guaranteed — the identity of the policy maker matters. The question is now whether representative democracy will pick the right citizen.

The answer, in general, is no. Consider first an example where individuals have different feasible sets. There are two individuals and a single transferable good. A policy alternative, denoted $(x_1; x_2)$, is an allocation of this good between the two individuals. Both individuals are purely selfish, so that $V^1(x_1; x_2) = x_1$ and $V^2(x_1; x_2) = x_2$. Individual 1 is more competent than 2, in the sense of being able to generate strictly more of the good when he is in power. This is illustrated

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the interaction between politicians (modelled, uncharacteristically, as perfect agents of the people!) and bureaucrats as in Niskanen (1971) typically ignores the well-being of the bureaucrats. Efficiency is defined with reference to the output level which maximizes the politician's utility, with expenditures in excess of this level being viewed as entirely wasteful.

[33] In this case and in the situation where entry costs are high there remains the possibility of citizens contributing to the campaigns of others. We will discuss this possibility further in the conclusion.

in Figure 3. The allocation at A will prevail if 1 is selected to be policy maker, while B will prevail if 2 is selected. For su±ciently small ±, the equilibrium has both individuals entering the race and each winning with probability 1=2. Clearly, the selection f2; Bg is not e±cient; nonetheless, it arises with probability 1=2.

The same logic applies if it is costly to take on the role of policy maker. Consider the same example as above, but suppose that individual 2 is just as competent as 1. Assume that individual 1 likes being the policy maker, while individual 2 dislikes it. The allocation at A will prevail if 1 is selected to be policy maker and C if 2 is. Again, for su±ciently small ±, both individuals will enter the race provided that 2's dislike of being the policy maker is not too large. In this case, the selection f2; Cg is ine±cient. Both individuals are better o® under the selection f1; Cg.

These ine±ciencies are symptomatic of a lack of commitment. In the ¯rst example, if individual 1 could commit to implement a policy at, or to the right of D with probability 1=2, then individual 2 would be willing to vote for 1. However, the incompetent individual 2 will continue to stand and run for o±ce even though there exists a feasible Pareto superior alternative to his policy choice. In the second example, the problems would be resolved if individual 1 could commit to implement the policy C with probability 1=2. Thus the lack of binding promises to make feasible transfers imply that ine±cient candidates can persist. Of course, in repeated settings reputation formation could reduce this ine±ciency. However, it is unlikely to eliminate it altogether.

These examples suggest investigating a slightly less stringent notion of e±ciency. First, de¯ne a selection fi; xg as being incentive compatible if x is the social alternative that maximizes citizen i's payo® when he holds o±ce; that is, if $x = x_i^¤$ (see (3.1)). With an incentive compatible selection the choice of the social alternative can be delegated to the individual selected to implement policy, without there being a tension between the policy maker's preferences and the social choice. Clearly, the selection produced by representative democracy is incentive compatible. Thus, we consider whether it is e±cient in this restricted class of selections. De¯ne an incentive compatible selection fi; $x_i^¤$g to be incentive constrained e±cient (IC e±cient) if there exists no other incentive compatible selection n j; $x_j^¤$ o such that $v_{kj} > v_{ki}$ for all k 2 N. The examples above do not show that representative democracy is IC ine±cient. In the ¯rst example, there are two incentive compatible selections f1; Ag and f2; Bg. The selection f1; Ag does not Pareto dominate f2; Bg. Similarly, in the second example, the selection

$\{1, A\}$ does not Pareto dominate $\{2, C\}$. The idea of lack of commitment that we discussed above is tantamount to the need to respect incentive compatibility.

Does representative democracy produce IC efficient selections? We begin with a positive result.

Proposition 2. Let s be a pure strategy equilibrium in which a single citizen (say, citizen i) runs unopposed. Then, if δ is sufficiently small, $(i; x_i^*)$ must be an IC efficient selection.

An appealing logic underlies this proposition. If an IC inefficient citizen were running unopposed, then by definition there would exist some other citizen whom, if elected, would produce a Pareto superior outcome. Since voting sincerely is the only weakly undominated strategy in two candidate races, then if such a citizen entered, he would win. Thus he will enter if the entry cost is small enough. Inefficient candidates are thus driven out by the forces of political competition, which plays a similar role to market competition in ensuring efficiency.

Unfortunately, this logic does not cleanly generalize to elections with two candidates. Consider the following example. There are four individuals, labeled $1; 2; 3; 4$, with the following preferences:

$$
\begin{aligned}
v_{12} &> v_{11} > v_{13} > v_{14} \\
v_{21} &> v_{22} > v_{24} > v_{23} \\
v_{32} &> v_{34} > v_{31} > v_{33} \\
v_{42} &> v_{44} > v_{43} > v_{41}
\end{aligned}
$$

Using Proposition 4, it is readily verified that, for sufficiently small δ, individuals 1 and 4 entering against each other is a pure strategy equilibrium if $v_{21} - v_{22} > v_{22} - v_{24}$. However, the selection $\{4; x_4^*\}$ is IC inefficient: it is dominated by the selection $\{2; x_2^*\}$. The above logic breaks down because, while individual 2 would prefer that he was in power rather than individual 4, his entry would ensure the defeat of his preferred candidate - individual 1.

The essential problem here, is that individual 2 prefers someone else other than himself to be in power. This is not unnatural if being the policy maker is costly, since individuals would prefer an equally competent citizen who shared (even approximately) their policy preferences.[34] Nonetheless, in many situations, the following assumption will hold.

[34] This will depend on what rewards the constitution assigns to policy makers. There is no logical reason why society cannot pay the policy maker a large amount. The design of incentive schemes for policy makers merits further investigation in this framework.

Assumption 1: For all $j \in N$ such that $(j; x_j^*)$ is an IC inefficient selection, there exists an $i \in N$ such that $v_{ki} > v_{kj}$ for all $k \in N$ and $v_{ii} \geq v_{ik}$ for all $k \in N$.

If a citizen is IC inefficient, then, by definition, there must exist some Pareto dominant citizen. Thus the content of Assumption 1 is that there is a dominant citizen whose utility is maximized when he holds office. This rules out the configuration of preferences in the previous example.

Assumption 1 is not quite sufficient to ensure that two candidate equilibria are IC efficient. It guarantees only that there is a citizen willing to enter to displace an IC inefficient candidate if he believes that he would be in the winning set if he entered. Our requirement that beliefs be consistent, imply that this must be so if the entrant would be dominant. This does not preclude the possibility that all those citizens supporting the IC inefficient candidate's original opponent continue to prefer him to the new entrant. If the new entrant is not dominant, our equilibrium refinements do not ensure that he will be in the winning set and he may attract none of the inefficient candidate's supporters. This, however, seems rather unlikely and can be ruled out if beliefs have the following property:

Irrelevance of Inefficient Candidates: The beliefs $\alpha(\cdot)$ satisfy Irrelevance of Inefficient Candidates (IRIC) if whenever $v_{ki} > v_{kj}$ for all $k \in N$ for $i; j \in C$, $F^j(C; \alpha(C)) = 0$.

This says that candidates who are Pareto dominated receive no votes. Now we can prove:

Proposition 3. Let s be a pure strategy equilibrium in which two candidates (say, citizens i and j) run against each other and let $\alpha(\cdot)$ be the supporting beliefs. Then, if δ is sufficiently small, if Assumption 1 is satisfied and if the beliefs satisfy IRIC, $(i; x_i^*)$ and $(j; x_j^*)$ must be IC efficient selections.

Again, there are difficulties in generalizing this result to multi-candidate elections. In a three candidate race in which all candidates are in the winning set, there is no guarantee that an efficient entrant will be in the winning set, even if IRIC is satisfied. Suppose, for example, that the efficient entrant is preferred by all the supporters of the inefficient candidate (say, candidate 1) together with a small number of another candidate's (say, candidate 2). The remaining supporters of candidate 2 may switch their votes to candidate 3 causing the efficient entrant to lose! This logic suggests little hope of obtaining a general efficiency result for multi-candidate elections in which all candidates are in the winning set.

In multi-candidate elections in which only one candidate is in the winning set, the following result can be readily established.

Proposition 4. Let s be a pure strategy equilibrium in which three or more candidates run against each other. Let $\alpha(\cdot)$ be the supporting beliefs and suppose that $W(C(s); \alpha(C(s))) = \{i\}$. Then, if δ is sufficiently small, if Assumption 1 is satisfied and if i is dominant in the set of candidates $C(s)$, then $(i; x_i^*)$ must be an IC efficient selection.

The logic of this result is as follows: if citizen i were IC inefficient then, by Assumption 1, there would exist a Pareto superior citizen who would enter if he could win. Since citizen i is dominant, the Pareto superior citizen must be dominant and hence will win.

Overall, our results provide guarded support for the view that representative democracy produces efficient results. The key positive result is Proposition 9. In most models of policy choice, the feasible set of policies is independent of the characteristics of the policy maker who is selecting them. The question of efficiency then boils down to whether the policy selected is efficient. Proposition 9 establishes that representative democracy will produce efficient policy choices, suggesting that many ideas that are normally discussed under the heading of normative economics might actually deserve a place in discussions of actual policy choices. The remaining Propositions explore the sorting role of political competition to select "efficient" policy makers. Here we found a number of caveats to claims about efficiency, reinforcing the importance of developing a formal theoretical framework to explore these issues.

6.2. Equity

Our model supposes that elected candidates choose policy to maximize their payoff. If the latter cared solely about their own consumption, it is unlikely that the outcome would be equitable. However, policy preferences need not be purely self-regarding, with casual empiricism suggesting that many candidates have a broader agenda than maximizing their own consumption. Here, we argue that representative democracy may have a tendency to select individuals who are more altruistic over venal candidates, creating a tendency towards relatively equitable outcomes.

Perhaps the best known political economy model with a focus on self-interested behavior, is Brennan and Buchanan (1980)'s Leviathan model. Like us, they model the incumbent as a monopolist once in office. However, they postulate purely self-interested behavior, justified by a view that electoral competition will not act as an effective disciplining mechanism. Even accepting this, there is still a possible sorting role of elections to find less self-interested individuals. The Leviathan view implicitly assumes that all individuals are selfish. In our model, a universe of selfish individuals would imply that every citizen had an incentive to run for office. In the limit, everyone would stand, and founding a democratic government would seem likely to replace Hobbesian anarchy with electoral chaos, mitigated only by barriers to entry. Solace might then be found in constitutional constraints which reduce the wealth extracting abilities of elected officials, as argued for by Brennan and Buchanan.

Our approach permits the sorting role of elections to be modeled explicitly. Since altruistic candidates would attract support from self-interested ones, and hence fair better in electoral competition, just a few altruists might be able to keep Leviathan out. To explore this logic further we consider a pure distribution game in which the incumbent's task is to distribute a stock of wealth, W. Thus $A = \left\{ x \in \Re^N_+ \mid \sum_{i \in N} x_i = W \right\}$. There are two types of citizens. Selfish citizens with preferences $V^i(x;1) = V^i(x;0) = x_i$, and altruistic citizens with preferences: $V^i(x;1) = V^i(x;0) = \frac{1}{N}\sum_{j \in N} u(x_j)$ where $u(\cdot)$ is increasing and strictly concave. The latter care about something akin to social welfare and, if elected, divide the wealth equally. By contrast, selfish individuals consume everything themselves. We assume that if nobody runs, then the wealth is lost.[35] Applying our model yields

Proposition 5. Suppose that there are at least two altruists in the polity and that $u(\frac{W}{N}) - \left[\frac{N-1}{N}u(0) + \frac{1}{N}u(W)\right] > \delta$. Then, the only pure strategy equilibria involve a single altruist running uncontested.

Thus for small enough costs, the only pure strategy equilibria involve government by an altruist. Only a few altruists are needed for representative democracy

[35] Thus the wealth is best interpreted as the bounty of government, which is distributed among the citizens.

to avoid Leviathan. While our example allowed representative democracy to produce a completely equitable outcome, this is not a general conclusion. It casts doubt on the reasonableness of the pure Leviathan model, rather than suggesting a rosy picture where equity always prevails. Factionalism where leaders favor certain sub-groups in society seems perfectly possible in our model, and the electoral success of fascism in the twentieth century makes it hard to be sanguine that democracy can avoid the tyranny of ideologies that advocate extreme forms of repression against certain populations. Understanding when such extremism can arise in our model is an important issue for investigation.

7. Concluding Remarks

This paper o®ers a stylized representation of policy selection in a representative democracy. It provides a tractable alternative to the Downsian paradigm which has dominated the literature on political competition for almost forty years. A key innovation is government by the people, rather than by mythical planners or memberless political parties. Whether the model is useful depends upon the range of issues to which it can fruitfully be applied. Incorporating political parties is a natural extension. Here, we assumed that candidates could only ¯nance their own campaigns. However, individuals would have an incentive to contribute to others' campaigns in order to have their preferred candidate run. Parties might then arise to solidify the fund raising process | to facilitate the Coasian bargain between interested individuals. Other important extensions include incorporating pressure groups and considering the role of legislatures. More generally, we see the model developed here as an ideal vehicle for modeling the formation of political institutions endogenously, rather than assuming them deus ex machina. The model could also be used to compare di®erent constitutional rules as in Osborne and Slivinski (1994).

Apart from laying out a framework, the main results developed here concern e±ciency. This speaks to the positive relevance of normative models which study e±cient policy choice. While only a benchmark, we hope that by taking the presumption of government e±ciency to heart, we will gain a better understand of government behavior and its possible failings. This may help, in turn, to bridge the gap between positive and normative economics which has traditionally been large. In Besley and Coate (1995) we consider a two period version of this model. This makes clearer that, even though outcomes can be Pareto e±cient, there may be some significant distortions in policy choice when representative democracy is used to assign control rights to policies. For example, a government may turn down surplus maximizing investments in political equilibrium. This analysis speaks to the strength of our framework in making the meaning of government failure precise, which is an essential pre-requisite to understanding where the economic borders of the state really should lie.

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8. Appendix A: Proof of Main Results

Proof of Proposition 1: It will be convenient to first introduce some new notation. For all $C \subset N$ such that $C \neq \emptyset$ and for all $\alpha \in [C \cup \{0\}]^N$ let

$$W^*(C;\alpha) = \{k \in C : F^k(C;\alpha) + 1 \geq F^i(C;\alpha)\ \forall\, i \in C\}.$$

The set $W^*(C;\alpha)$ consists of those candidates who are winning or are within one vote of the winners.

We now begin the proof. Let $C \subset N$ be such that $C \neq \emptyset$. If $\#C = 1$, the sole candidate is automatically elected, so that any vector of voting decisions is a sincerely refined voting equilibrium. If $\#C = 2$, then any vector of voting decisions in which individuals vote sincerely is a sincerely refined voting equilibrium.

For $\#C \geq 3$, let $(N_i)_{i \in \{C \cup \{0\}\}}$ be a sincere partition with N_0 containing all the voters who are indifferent between all of the candidates, i.e., $N_0 = \{\ell \in N \mid v_{\ell i} = v_{\ell j}$ for all $i, j \in C\}$. Consider the vector of voting decisions α generated by this partition; i.e.,

$$\alpha_\ell = i \Leftrightarrow \ell \in N_i; \text{ for all } \ell \in N; \text{ for all } i \in C \cup \{0\}.$$

If there exists a dominant candidate, then α will be a voting equilibrium and will be sincerely refined.

In the absence of a dominant candidate, there are two possibilities. First, α could be a voting equilibrium. In this case, it will be sincerely refined because all voters are voting for their preferred candidates and hence are not employing weakly dominated strategies. Second, α is not a voting equilibrium. In this case there must exist some citizen ℓ and candidates i, j, k such that $\ell \in N_i$; $j \in W^*(C;\alpha)$; $k \in W(C;\alpha)$ and $v_{\ell j} > v_{\ell k}$. We will use this information to construct a further candidate vector of voting decisions which we will call $\alpha(1)$. If this is a voting equilibrium the proof is complete, otherwise we will use the same procedure to construct a further candidate. We will demonstrate that this procedure must eventually produce a sincerely refined voting equilibrium.

The procedure for constructing the candidate vector of voting decisions depends on whether or not $j \in W(C;\alpha)$. Suppose first that $j \in W(C;\alpha)$. Then we will transfer supporters of candid