

# Moral Hazard and Capital Structure Dynamics\*

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## Abstract

We base a contracting theory for a start-up firm on an agency model with observable but nonverifiable effort, and renegotiable contracts. Two essential restrictions on simple contracts are imposed: the entrepreneur must be given limited liability, and the investor's earnings must not decrease in the realized profit of the firm. All message game contracts with pure strategy equilibria (and no third parties) are considered. Within this class of contracts/equilibria, and regardless of who has the renegotiating bargaining power, debt and convertible debt maximize the entrepreneur's incentives to exert effort. These contracts are optimal if the entrepreneur has the bargaining power in renegotiation. If the investor has the bargaining power, the same is true unless debt induces excessive effort. In the latter case, a non-debt simple contract achieves efficiency – the non-contractibility of effort does not lower welfare. Thus, when the non-contractibility of effort matters, our results mirror typical capital structure dynamics: an early use of debt claims, followed by a switch to equity-like claims.

KEYWORDS: *moral hazard, renegotiation, convertible debt, capital structure*

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## 1. Introduction

We base a theory of optimal contracting for a start-up firm on an agency model with observable but nonverifiable effort. Contracts can be renegotiated after an entrepreneur has taken his effort, but before its consequence is realized. Both parties may be risk averse. Our goal is to determine the effects of two natural restrictions on contracts: limited liability for the entrepreneur, and monotonicity of the investor's income in the output of the firm. We show that within the set of contracts that satisfy these restrictions, under certain conditions debt and its renegotiation-proof equivalent, convertible debt, maximize incentives and are often optimal.

We thus obtain a simple theory of capital structure dynamics. It describes the case of an entrepreneur who first obtains debt finance from a bank, but then later adopts equity financing by going public. It also fits the case of an entrepreneur who issues convertible debt to a venture capitalist. Such dynamic patterns are well documented (e.g., Diamond, 1991; Sahlman, 1990). Real-world contracts are of course more complicated than the ones considered here; for example, they typically include provisions for dealing with projects that “go sour”.<sup>1</sup> Our model nonetheless generates a dynamic pattern of financial contracting that is fairly realistic, despite its relative simplicity.

As is well known, the classical agency model of, e.g., Mirrlees (1999) and Holmström (1979), does not yield optimal schemes that resemble standard instruments like debt or equity. The “security design” literature has therefore looked elsewhere to show that debt (or equity) are optimal. For example, Townsend (1979) and Gale and Hellwig (1985) consider “costly state verification” models in which output can be observed only at a cost. Bolton and Scharfstein (1990), Berglof and von Thadden (1994), and Hart and Moore (1994, 1998) consider “stealing models” in which output is entirely unverifiable, but debt holders can seize assets in some contingencies. In Aghion and Bolton (1992), and Dewatripont and Tirole (1994), output is costlessly verifiable, but actions that affect continuation values are not contractible.

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<sup>1</sup>Kaplan and Stromberg (2000) document the details of venture capital contracting. See also Cornelli and Yosha (1997) and Schmidt (2000).

Although our model is closer to the classical moral hazard paradigm, it departs in three ways. First, contracts can be renegotiated after the effort is chosen, but before the output is realized. This is an appropriate assumption for situations in which the entrepreneur (agent) is crucial to the initial stage of business, before the fruits of his labor are realized. This paper thus joins the literature on the renegotiation of moral hazard contracts, which is discussed below.

Second, although the entrepreneur's effort remains noncontractable, it is observed by the investor (principal). This is an appropriate assumption for situations like venture capital financing in which investors typically have expertise and engage in monitoring. Because of this assumption, and the previous one regarding renegotiation, the model resembles that of Hermalin and Katz (1991).

Third, feasible contracts must take account of the entrepreneur's limited resources, and give the investor a payoff that does not decrease in the firm's output. The former "limited liability" restriction holds naturally for an entrepreneur with little wealth. The latter "monotonicity" restriction can be derived as an equilibrium outcome from ex post moral hazard considerations. It arises, for example, if the investor can "burn output" in order to make the firm's performance appear lower than it really was. Alternatively, it arises if the entrepreneur can secretly borrow from an outside lender in order to make the firm's performance appear greater than it really was. Assuming such ex post moral hazards weakens the assumption that output is verifiable, but less so than in the costly state verification models, and much less so than in the stealing models.

Under these liability and monotonicity restrictions, Innes (1990) shows that debt is optimal if the parties are risk neutral. Debt gives the entrepreneur a return of zero – the minimal possible return when he has limited liability – if the firm's realized earnings are lower than the face value of the debt. This property of debt is useful for giving the entrepreneur incentives to choose an effort that lowers the probability of this low return. But it also makes debt a poor risk-sharing scheme if the entrepreneur is risk averse, in which case debt is not optimal in Innes' no-renegotiation model.

On the other hand, if the debt can be renegotiated after the effort is chosen but

before the output is realized, it may be possible to renegotiate it to a better risk-sharing contract without destroying incentives. A result of this type is due to Hermalin and Katz (1991), who examine a model like ours, with renegotiation, effort that is observable but not verifiable, and a risk averse entrepreneur (but risk neutral investor). They show that if the entrepreneur has the renegotiation bargaining power, then a riskless debt contract, i.e., a contract that pays the investor a fixed amount regardless of the realized output, achieves a first-best outcome.<sup>2</sup> The riskless debt provides appropriate incentives, and it is renegotiated to an efficient risk-sharing contract after the effort is chosen.

Riskless debt, however, will generally give the investor too low a return when the limited liability of the entrepreneur prevents him from paying back more than the firm earns. In this case, if the smallest possible output of the firm is less than the required start-up investment, and if the investor has no bargaining power in the renegotiation, any feasible riskless debt contract gives her a negative return on her investment. Our task, therefore, is to determine the nature of an optimal contract that gives the investor a higher payoff than would any feasible riskless debt contract.

We first restrict attention to “simple contracts”, which are contracts that specify a fixed rule for sharing the firm’s output. Within the class of simple contracts satisfying the liability and monotonicity restrictions, we find that any debt contract is optimal if the entrepreneur has all the bargaining power in the renegotiation. Thus, risky debt, instead of riskless debt, emerges when the latter gives the investor too low a payoff. The reason, roughly, is that within the set of simple contracts that give the investor some payoff, a debt contract elicits the greatest effort. Unless the debt is riskless, this effort is not high enough to be efficient, i.e., if effort were to be contractible, prescribing a higher effort would make both parties better off.

The “separation” theme that runs through these results is worth emphasizing. Renegotiation allows incentive provision to be separated from risk sharing, even given a

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<sup>2</sup>This describes both the proof and statement of Proposition 3 in Hermalin and Katz (1991). The result, unlike that of Innes (1990) and most of ours here, does not rely on the monotone likelihood ratio property. A similar result is obtained in Matthews (1995) for unobservable effort.

restriction to simple contracts. The initial contract provides only incentives, and the renegotiated contract provides only risk sharing. Accordingly, the optimal contract takes the same form as it does when the entrepreneur is risk neutral and renegotiation is not possible. Thus, it is well known that riskless debt is optimal when the agent is risk neutral, and Hermalin and Katz (1991) show that it is also optimal when he is risk averse but renegotiation is possible (and the agent has the bargaining power). In the presence of the liability and monotonicity constraints, Innes (1990) shows that debt is optimal when the agent is risk neutral, and we show that it is also optimal when he is risk averse but renegotiation is possible (and the agent has the bargaining power).

In addition to simple contracts, we consider “general contracts” that require the parties, after the effort is chosen, to send messages to the contract enforcer. The messages determine a (sometimes random) simple contract for sharing the output; the prescribed (random) simple contract can then be renegotiated. Such general contracts can be of value for implementation, as the literature following Maskin (1999) shows. For example, consider a contract in which both parties report the effort, and if they disagree, both are “shot” (made to pay big fines). Truthful reporting is then an equilibrium for any effort, which essentially makes it contractible. But of course, paying big fines violates our limited liability condition. More importantly, renegotiation prevents this scheme from working, since the parties will renegotiate the fines away if it ever becomes clear they are to be imposed.

An “investor-option contract” is a general contract in which only the investor sends a message. It can be viewed as a set of (random) simple contracts from which the investor will select after the effort is chosen. Our first result about general contracts is that investor-option contracts are optimal, given a restriction to pure strategy equilibria. (We show in Appendix B that both parties sending messages may be of value in a mixed strategy equilibrium.) Thus, subject to the pure strategy proviso, there is no need to consider contracts that require the entrepreneur to send a message. This result holds for any renegotiation bargaining procedure that achieves an ex post efficient outcome which is continuous in the disagreement outcome. Segal and Whinston (2002) prove

a similar result, but in a quasilinear framework and for a somewhat smaller class of bargaining rules.

Our focus returns to debt contracts when we consider in the general setting the case in which the entrepreneur has all the renegotiation bargaining power. Our main result here is that no investor-option contract, and hence no general contract, outperforms debt (again restricting attention to pure strategies). Of course, an investor-option contract that contains debt can be payoff-equivalent to debt. We view convertible debt as such a contract: it is an investor-option contract that consists of a debt contract and the simple contract to which, in equilibrium, it is renegotiated after the effort is chosen. In the equilibrium of a convertible debt contract, the entrepreneur takes the same effort as he would have given just the debt contract, and then the investor selects the alternative simple contract instead of the debt. The entrepreneur is deterred from shirking by the credible threat that the investor would then select the debt contract. This is like “converting” to equity some or all of the debt in a real convertible debt contract if the entrepreneur is observed to have performed well.

We then turn to the case in which the investor has the renegotiation bargaining power. We show that then debt still provides the strongest incentives. However, we also show that, if the entrepreneur is risk averse, the incentives provided by debt may be too strong. This is because the entrepreneur now does not gain from the renegotiation, and so the risk characteristics of the initial contract matter. Debt is very risky for him, since it gives him a zero return if the realized output is low. He may therefore over-exert himself in order to reduce the probability of low outputs. This possibility of excessive effort is surprising; one typically thinks effort should be too low when the entrepreneur chooses it without regard for the positive externality that increasing effort has on investors. Our excessive effort result is thus at odds with the conventional wisdom that external funding reduces managerial effort through the externality it induces on investors, the “debt overhang” problem that has been stressed in the corporate finance literature since Jensen and Meckling (1976) and Myers (1977).

Our final result is that under standard but strong concavity-like assumptions, either

a debt contract is optimal in the set of deterministic general contracts (again with the pure strategy proviso), or a simple contract that is not debt achieves an efficient allocation. In the latter case, the inability to contract on effort does not lower welfare. Debt is optimal when the non-contractibility of effort matters.

Our finding that a simple contract is optimal resembles the result of other models that an optimal contract has no messages, as in Hart and Moore (1988) and Segal and Whinston (2002), or is the null contract, as in Che and Hausch (1999), Segal (1999), Hart and Moore (1999), and Reiche (2001). It is renegotiation that causes simple contracts to arise in these models and ours, for two reasons. First, to some extent renegotiation “completes” an initial contract, since the renegotiated contract can depend on observable but non-contractible variables. Second, by ensuring ex post efficiency, renegotiation makes any message game strictly competitive, and hence of limited use. In our paper debt emerges as the simple contract because it maximizes incentives. In the other papers, either a simple profit sharing rule or the null contract are optimal because contracting is unable to strengthen incentives.<sup>3</sup>

Other related papers also consider renegotiation of incentive contracts, but under the assumption that the principal does not observe the agent’s effort. Fudenberg and Tirole (1990), Ma (1991, 1994), and Matthews (1995) study such models without liability or monotonicity restrictions. Matthews (2001) studies a model with these restrictions. Our environment differs from his only in so far as the investor observes the effort. Matthews (2001) shows that debt is optimal within the set of simple contracts, assuming the entrepreneur has the renegotiation bargaining power. The complications due to asymmetric information make his result more fragile than ours; e.g., multiple, non-equivalent equilibria may exist, non-debt simple contracts may also be optimal, and general contracts with pure strategy equilibria may outperform debt.

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<sup>3</sup>The inability to enforce trade ex post in Hart-Moore (1988) implies that the optimal contract is a simple profit-sharing rule. The null contract is optimal in the other papers; its optimality is due to either the presence of direct investment externalities (Che-Hausch, 1999), or to an inability to specify the nature of the good to be traded (Segal, 1999, Hart and Moore, 1999, and Reiche, 2001).

The paper is organized as follows. The environment is described in Section 2. The special case in which the investor is risk neutral, and the entrepreneur has the renegotiation bargaining power, is studied in Section 3. The case of general contracts and renegotiation is analyzed in Section 4. Sections 5 and 6 contain results for the general model in which the entrepreneur or, respectively, the investor has the bargaining power. Section 7 concludes. Appendix A contains proofs. Appendix B has two examples of non-debt contracts that outperform debt if third parties are introduced, or if joint lotteries controlled by mixed strategies can be implemented.

## 2. Preliminaries

An entrepreneur (agent) must contract with an investor (principal) to obtain the  $K$  dollars required to start a project. After contracting, the entrepreneur chooses an effort level  $e$  from an interval  $E = [\underline{e}, \bar{e}] \subset \mathbb{R}$ . His effort determines a probability distribution,  $g(e) = (g_1(e), \dots, g_n(e))$ , over the set of possible (monetary) *outputs*,  $\{\pi_1, \dots, \pi_n\}$ . We assume  $n > 1$  and  $\pi_i < \pi_{i+1}$ . Each  $g_i$  is twice continuously differentiable and positive on  $E$ . Output increases stochastically with effort in the sense of the strict monotone likelihood ratio property:

$$\text{(MLRP)} \quad \frac{g'_i(e)}{g_i(e)} \text{ increases in } i \text{ for any } e \in E.$$

The only contractible variable is output. Accordingly, a *simple contract* is a vector  $r = (r_1, \dots, r_n)$  specifying a payment from the entrepreneur to the investor for each possible output. An *allocation* is a pair  $(r, e)$ .

Given an allocation  $(r, e)$ , the entrepreneur's utility if  $\pi_i$  is realized is  $u(\pi_i - r_i, e)$ . His *payoff* (expected utility) from an allocation is

$$U(r, e) \equiv \sum g_i(e) u(\pi_i - r_i, e).^4$$

The function  $u$  is twice continuously differentiable. With respect to income, the entrepreneur's utility increases,  $u_1 > 0$ , and he is weakly risk averse:  $u_{11} \leq 0$ . His utility

<sup>4</sup>We omit the summation index if it is  $i = 1, \dots, n$ .



decreases with effort at all interior efforts:  $u_2(\cdot, e) < 0$  for  $e \in (\underline{e}, \bar{e})$ . Corner solutions are eliminated by assuming  $u_2(\cdot, \underline{e}) = 0$  and  $u_2(\cdot, \bar{e}) = -\infty$ .

The investor's net utility is  $v(y)$  if she makes the start-up investment and receives  $y$  dollars in return.<sup>5</sup> The function  $v$  has continuous derivatives  $v' > 0$  and  $v'' \leq 0$ . The investor's payoff from an allocation is

$$V(r, e) \equiv \sum g_i(e)v(r_i).$$

We assume at least one party is risk averse:  $u_{11} < 0$  or  $v'' < 0$ .

The timing and information structure of the game are as follows. After a contract is adopted, the entrepreneur chooses effort. The investor observes the effort immediately. Any messages that the contract may require the parties to send are then sent. As a function of the messages sent, the contract specifies a (possibly random) simple contract that, together with the chosen effort, determines a status quo allocation. The parties then renegotiate to another simple contract. Finally, output is realized and payments made according to the renegotiated contract.

At the heart of our model is a set of restrictions on what makes a simple contract feasible. The first important one is a limited liability constraint for the entrepreneur:

$$(LE) \quad r_i \leq \pi_i \text{ for } i \leq n.$$

This constraint reflects the reality that entrepreneurs often have limited wealth, and so cannot pay back more than the project earns. If the start-up investment satisfies  $K > \pi_1$ , then LE rules out the riskless debt contract that requires the investor to be paid back  $K$  after any output realization.

The second important restriction is a monotonicity constraint for the investor that requires her income to weakly increase with the project's output:

$$(MI) \quad r_i \leq r_{i+1} \text{ for } i < n.$$

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<sup>5</sup>If the investor's utility function for income is  $\hat{v}$ , then  $v(y) \equiv \hat{v}(y) - \hat{v}(K)$ . Her utility gain is thus  $v(y)$  if she makes the investment and receives  $y$  in return.

This constraint has a different status than LE. It is the consequence of an ex post moral hazard that, for simplicity, we have chosen not to model explicitly. For example, it is easy to show that MI must be satisfied by any implementable contract if the investor can engage in sabotage that distorts the apparent  $\pi_i$  downwards. Alternatively, MI must be satisfied if the entrepreneur can borrow secretly from a lender after a contract has been signed, thereby distorting the apparent  $\pi_i$  upwards. Constraint MI, and the arguments for it, were first introduced by Innes (1990).<sup>6</sup>

We denote the set of feasible simple contracts as  $C$ , and assume it is defined by LE, MI, and one other constraint:

$$C \equiv \{r \in \mathbb{R}^n \mid r \text{ satisfies LE, MI, and LI}\}.$$

The additional constraint,

$$(LI) \quad r_i \geq \underline{r} \text{ for } i \leq n,$$

is a limited liability constraint for the investor that imposes a lower bound on how much she can be paid back. It simplifies the analysis by insuring that  $C$  is compact. We assume  $\underline{r} < \pi_1$ , so that  $C$  has an interior. It is also convex.

An *efficient risk-sharing contract* for a fixed effort  $e$  is a contract in  $C$  that solves the following program,

$$H(\hat{V}, e) \equiv \max_{r \in C} U(r, e) \text{ such that } V(r, e) \geq \hat{V}, \quad (1)$$

for some investor payoff  $\hat{V}$ . This is a “constrained efficiency” notion, taking as given the constraints that define  $C$ . (The modifier “first-best” will denote outcomes that are efficient in the full, unconstrained sense.) Any solution of (1) is unique, since both parties are weakly, and at least one is strictly, risk averse. The graph of  $H(\cdot, e)$  is the Pareto frontier of possible payoff pairs when the effort is fixed at  $e$ . The following lemma

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<sup>6</sup>It is equally likely that the entrepreneur can destroy output ex post, or that the investor can inject cash so as to inflate apparent profit. These ex post moral hazards lead to the constraint  $\pi_i - r_i \leq \pi_{i+1} - r_{i+1}$ . Most of our results would be unchanged if we were to include this constraint too.

confirms that it is downward sloping on its domain,  $[v(\underline{r}), V(\pi, e)]$ .<sup>7</sup> The proof, like all others missing from the text, is in Appendix A.

**Lemma 1.** *For all  $e \in E$ ,  $H(\cdot, e)$  is continuous and concave on  $[v(\underline{r}), V(\pi, e)]$ . For all  $e \in E$  and  $\hat{V} \in (v(\underline{r}), V(\pi, e))$ , the partial derivative  $H_1(\hat{V}, e)$  exists and is negative. If also  $e \in \text{int}(E)$ , then  $H(\cdot, \cdot)$  is differentiable, and has continuous partial derivatives, at  $(\hat{V}, e)$ .*

We refer to an allocation  $(r^*, e^*)$  as *efficient* if  $e^*$  maximizes  $H(\hat{V}, \cdot)$  for some  $\hat{V}$ , and  $r^*$  solves (1) when  $e = e^*$ . Such allocations set the welfare benchmark: they determine the Pareto frontier that would be achievable if effort as well as output were contractible, the parties could commit not to renegotiate, but the constraints MI, LE, and LI had to be respected.

Two types of contract will play a major role. The first is a *wage contract* that pays the entrepreneur a fixed wage regardless of the realized output. The wage contract with wage  $w$  is denoted by  $r^w$ , where

$$r_i^w \equiv \pi_i - w \text{ for } i \leq n.$$

Because of the liability constraints,  $r^w \in C$  if and only if  $w \in [0, \pi_1 - \underline{r}]$ .

Debt is the second important contract type. The *debt contract* with face value  $D$ , denoted as  $\delta(D)$ , is defined by

$$\delta_i(D) \equiv \min(D, \pi_i) \text{ for } i \leq n.$$

We sometimes denote  $\delta(D)$  merely as  $\delta$ . Note that  $\delta \in C$  if and only if  $D \in [\underline{r}, \pi_n]$ . The debt is *risky* if  $\delta_1 < \delta_n$ , which is equivalent to  $D > \pi_1$ .

The following proposition characterizes debt in terms of efficiency. It implies that if the entrepreneur is risk neutral, then for any effort, a contract in  $C$  is an efficient risk-sharing contract for that effort if and only if it is debt.

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<sup>7</sup>No feasible investor payoff is less than  $v(\underline{r})$ , or greater than  $V(\pi, e) \equiv \sum g_i(e)v(\pi_i)$ .

**Proposition 1.** *Suppose the investor is risk averse and the entrepreneur is risk neutral. Then, given any effort, a contract satisfying LE is not Pareto dominated by any other contract satisfying LE if and only if it is debt.*

### 3. Illustrative Case

In this section we examine, largely graphically, the canonical case in which the investor is risk neutral and the entrepreneur is risk averse. We further assume the entrepreneur has all the renegotiation bargaining power, so that he is able to offer a new contract on a take-it-or-leave-it-basis.<sup>8</sup>

Since the chosen effort is observed by both parties, in equilibrium the entrepreneur will propose to renegotiate to an efficient risk-sharing contract for that effort. Because now the investor is risk neutral, such a contract perfectly insures the entrepreneur: it is a wage contract. Thus, the renegotiation consists of the entrepreneur selling his entire stake in the firm to the investor.<sup>9</sup>

Suppose the parties have agreed to contract  $r \in C$ . Then, after he has chosen an effort  $e$ , the entrepreneur will offer the wage contract  $r^w$  that has the highest wage the investor will accept, i.e., the highest  $w$  satisfying

$$\sum g_i(e)\pi_i - w \geq \sum g_i(e)r_i.$$

This constraint binds – the investor receives no gain from the renegotiation. The resulting wage is given by the *wage function* defined by

$$w^*(r, e) \equiv \sum g_i(e)(\pi_i - r_i).^{10} \tag{2}$$

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<sup>8</sup>In order to simplify matters, we also assume the wage contracts that arise in this section are feasible. In particular, we assume they satisfy the investor’s liability constraint, LI.

<sup>9</sup>If the investor were to be risk averse, efficient risk sharing would require the entrepreneur to bear risk. Thus, in this case renegotiation would yield a contract that is more like equity sharing; both party’s earnings would increase in the realized output (linearly if they both had CARA utility).

<sup>10</sup>Because (2) implies  $w^*(r, e) \in [0, \pi_1 - \underline{r}]$ , we see that  $r^{w^*(r, e)} \in C$ .

When he chooses effort, the entrepreneur realizes that his ultimate wage will be determined by  $w^*(r, \cdot)$ . Thus, as is shown in Figure 1, an equilibrium outcome of  $r$  is a pair,  $(e^*, w^*)$ , that solves the following program:

$$\max_{w,e} u(w, e) \text{ subject to } w = w^*(r, e). \quad (3)$$

Observe that renegotiation separates incentive provision and risk sharing. The risk-sharing features of the initial contract, and the incentive provisions of the final contract, are both irrelevant. The initial contract provides incentives by its effect on the derivative  $w_e^*(r, e)$ , a measure of how responsive the wage is to changes in effort.

Only an efficient risk-sharing contract, i.e., a wage contract, will not be renegotiated. If  $r$  is a wage contract, the wage curve in Figure 1 is a horizontal line, and so the entrepreneur's optimal effort is the lowest possible,  $\underline{e}$ . Incentives are thus provided only by contracts that do not provide efficient risk sharing; any simple contract that provides incentives will be renegotiated.

A (first-best) efficient allocation can be represented now as a pair  $(e, w)$  at which the indifference curves of the entrepreneur and investor are tangent. The investor's indifference curve that gives her a payoff  $V$  is the graph of the equation

$$w = \sum g_i(e)\pi_i - V.$$

Thus, if this graph coincides with that of  $w^*(r, \cdot)$ , the equilibrium outcome of contract  $r$  is first-best efficient. The obvious candidate for such an  $r$  is the riskless debt contract that surely pays back  $V$ ,

$$\delta^V \equiv (V, \dots, V).$$

Since  $w^*(\delta^V, e) = \sum g_i(e)\pi_i - V$ , its graph is indeed an indifference curve of the investor. Riskless debt thus achieves first-best outcomes.<sup>11</sup>

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<sup>11</sup>The proof of Proposition 3 in Hermalin and Katz (1991) also shows that the renegotiation can achieve the first-best under the assumptions of this section. For a risk averse investor, we show in Section 5 that riskless debt still achieves efficient, but not necessarily first-best, allocations when the entrepreneur has the bargaining power.

The non-contractibility of effort may therefore be irrelevant. Even if any  $(e, w)$  could be directly enforced, both parties could not be made better off than they are when a riskless debt contract is adopted and renegotiated. However, any riskless debt contract that satisfies the entrepreneur's liability constraint gives the investor a payoff no greater than  $\pi_1$ . She will not accept such a contract if, for example,  $K > \pi_1$  and she must earn a nonnegative return on her investment.

The non-contractibility of debt does matter if the investor must be given a payoff greater than  $\pi_1$ . As we have seen, no riskless debt contract in  $C$  gives her this payoff, and no risky  $r \in C$  has an efficient equilibrium outcome. This is because the incentives provided by any risky contract satisfying the monotonicity constraint MI are too low. The argument is shown in Figure 2. Let  $(e^*, w^*)$  be an equilibrium outcome of the risky  $r \in C$ , and let  $V$  be the investor's payoff at this outcome. Her indifference curve at  $(e^*, w^*)$  is the graph of  $w_e^*(\delta^V, \cdot)$ . By MLRP, the fact that  $r$  satisfies MI and is not riskless implies that this indifference curve is everywhere steeper than the wage curve: for any  $e \in E$ ,

$$w_e^*(\delta^V, e) - w_e^*(r, e) = \sum g'_i(e)r_i > 0.$$

The efficient outcome,  $(e^F, w^F)$ , that gives the investor payoff  $V$  therefore satisfies  $e^F > e^*$ . Thus, the efficient effort that gives the investor a payoff of  $V$  is greater than the equilibrium effort achieved by any risky contract in  $C$ . Consequently, this efficient effort cannot be achieved if  $V > \pi_1$ .

Of all the contracts in  $C$ , debt provides the greatest incentives. To see why, suppose  $\delta$  is risky or riskless debt, and  $r \in C$  is not debt. Assume neither contract always pays back more than the other. Then, since  $r$  satisfies LE,  $r_i \leq \delta_i$  for low outputs  $\pi_i$ . But since  $r$  satisfies MI,  $r_i \geq \delta_i$  for high outputs. In other words, the debt contract pays less to the *entrepreneur* for low outputs, but more for high outputs. The debt contract thus gives the entrepreneur a greater incentive to shift probability from low to high outputs by increasing effort. This causes the wage curve generated by the debt to cross the wage curve generated  $r$  only from below. This single-crossing property is established by the following lemma.

**Lemma 2.** *Suppose  $\delta$  is debt and  $r \in C$  is not. If  $w^*(\delta, e^*) = w^*(r, e^*)$ , then*

- (i)  $w_e^*(\delta, e^*) > w_e^*(r, e^*)$ , and
- (ii)  $(e - e^*) [w^*(\delta, e) - w^*(r, e)] > 0$  for all  $e \neq e^*$ .

Lemma 2 implies that of all the feasible contracts that give the investor some equilibrium payoff, a debt contract achieves the largest equilibrium effort. But, as we have seen, no feasible contract has an equilibrium effort that is larger than the efficient effort that gives the investor this payoff. It follows that no feasible simple contract has an equilibrium that Pareto dominates that of a debt contract.

To see this more formally, refer to Figure 3. The outcome of a non-debt contract  $r \in C$  is  $(w^*, e^*)$ , and it gives the investor payoff  $V$ . The indicated  $\delta$  is the debt contract satisfying  $w^*(\delta, e^*) = w^*$ . By Lemma 2,  $w^*(\delta, \cdot)$  lies above  $w^*(r, \cdot)$  to the right of  $e^*$ . As shown above, since  $\delta$  satisfies MI, the investor's indifference curve  $w^*(\delta^V, \cdot)$  is even higher than  $w^*(\delta, \cdot)$  to the right of  $e^*$ , and lower to the left. Any equilibrium outcome of  $\delta$  must be on the thick portion of  $w^*(\delta, \cdot)$ , which is in the lens between the parties' indifference curves. The equilibrium of  $\delta$  thus Pareto dominates  $(w^*, e^*)$ . This yields the following proposition (which is a special case of the upcoming Theorems 2 and 3.)

**Proposition 2.** *Given entrepreneur-offer renegotiation and a risk neutral investor, any equilibrium of a non-debt  $r \in C$  is Pareto dominated by an equilibrium of a debt contract.*

We now turn to more complicated contracts that require messages to be sent. Convertible debt, a standard way of financing venture capital, is an example. It is a debt security that the investor has the option of converting to equity in the future. Convertible preferred equity is similar; it prescribes a schedule of dividends like debt does, and it can be converted at the investor's option to common stock. These are examples of *investor-option contracts* in which the implemented simple contract is determined by the investor's message only.

In this section we restrict attention to investor-option contracts. This will be nearly without loss of generality, as we show in Section 4. We further restrict attention here to

contracts that have only a finite number of options. Such an investor-option contract can be represented as a finite set  $R \subset C$ . After the contract has been agreed and the entrepreneur has chosen his effort, the investor selects a simple contract from  $R$ . Then renegotiation may occur; the entrepreneur offers a new simple contract on a take-it-or-leave-it basis to supplant the one the investor selected from  $R$ .<sup>12</sup>

Renegotiation is still efficient, and so the entrepreneur offers a wage contract. It will be the wage contract that gives the investor the same payoff as does the simple contract she had selected from  $R$ . Foreseeing this, when she observes that the entrepreneur has chosen an effort  $e$ , the investor selects an  $r \in R$  to maximize  $\sum g_i(e)r_i$ . Renegotiation thus yields a wage contract with wage

$$\begin{aligned} w^*(R, e) &\equiv \sum g_i(e)\pi_i - \max_{r \in R} \sum g_i(e)r_i \\ &= \min_{r \in R} w^*(r, e). \end{aligned}$$

So, the wage curve generated by an investor-option contract  $R$  is the lower envelope of the wage curves generated by the simple contracts in  $R$ . An equilibrium outcome of  $R$  is a pair  $(e, w)$  that maximizes  $u(w, e)$  subject to  $w = w^*(R, e)$ .

The possible value of an investor-option contract can be seen in Figure 4. The outcome of contract  $r^a$  is the point  $\hat{a}$ , which has a lower effort. But if the investor is given the option of selecting  $r^b$  instead of  $r^a$ , i.e. if the parties adopt the investor-option contract  $R = \{r^a, r^b\}$ , the equilibrium outcome is  $a$ , which has a relatively high effort. Contract  $r^b$  results in a very low wage if the entrepreneur chooses a low effort, and so the entrepreneur is induced to work hard in order to prevent the investor from selecting  $r^b$ . The investor's option of choosing  $r^b$  increases the incentives that  $r^a$  alone can provide.

However, an investor-option contract cannot improve upon a debt contract. The argument is essentially the same as before, and is again based on the single-crossing property of Lemma 2. Let the outcome of an investor-option contract  $R$  be  $(w^*, e^*)$ , and denote by  $V$  the payoff it gives the investor. Let  $\delta$  be the debt contract satisfying  $w^*(\delta, e^*) = w^*$ . Because it is a lower envelope, the wage curve  $w^*(R, \cdot)$  never has a

<sup>12</sup>The same results obtain if renegotiation instead occurs before the investor selects from  $R$ .



greater slope, as given by either its right or left derivatives, than any of the supporting curves  $w^*(r, \cdot)$ ,  $r \in R$ . Lemma 2 thus implies that  $w^*(\delta, \cdot)$  lies above  $w^*(R, \cdot)$  to the right of  $e^*$ , and below  $w^*(R, \cdot)$  to the left of  $e^*$ . Hence,  $\delta$  yields an effort no lower than  $e^*$ , and strictly higher unless some contract in  $R$  is debt (or  $e = \underline{e}$ ). By the same logic as before, the investor also prefers this higher effort to  $e^*$ , since her payoff is as though the debt contract is not renegotiated. These arguments, made more generally in the upcoming Theorems 2 and 3, yield the following proposition.

**Proposition 3.** *Assume entrepreneur-offer renegotiation, and that the investor is risk neutral. Then any equilibrium of a finite investor-option contract is weakly Pareto dominated by that of some debt contract. At least one party strictly prefers the latter if the investor-option contract does not contain debt.*

We can now see that a debt contract is equivalent to a type of convertible debt. The following argument is shown in Figure 5. Let  $\delta$  be debt, and let  $r^* = r^{w^*}$  be the wage contract to which it is renegotiated. It specifies the wage  $w^* = w^*(\delta, e^*)$ , where  $e^*$  is the equilibrium effort. The investor-option contract  $R^* = \{\delta, r^*\}$  is convertible debt: it can be viewed as a security that will implement the debt contract  $\delta$  unless the investor exercises her option of converting it to  $r^*$ . The wage curve  $w^*(r^*, \cdot)$  is horizontal at  $w^*$ , but  $w^*(\delta, \cdot)$  is upward sloping.<sup>13</sup> Hence, if the entrepreneur chooses an effort less than  $e^*$ , the investor will select  $\delta$  because it results in a lower wage. If the entrepreneur chooses an effort greater than  $e^*$ , the investor will instead select  $r^*$  because it now results in the lower wage. Since  $e^*$  is the entrepreneur's optimal effort from  $w^*(\delta, \cdot)$ , it also his optimal choice from the lower envelope of the two curves. The convertible debt contract thus gives rise to an equilibrium in which the entrepreneur chooses  $e^*$  and the investor exercises her "converting" option of selecting  $r^*$ . This yields the same outcome as would the adoption and renegotiation of  $\delta$ .

On the equilibrium path, the convertible debt contract  $R^*$  is renegotiation proof. When the entrepreneur chooses  $e^*$  or any greater effort, the investor selects  $r^*$ . Since

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<sup>13</sup>Assume the face value of  $\delta$  is less than  $\pi_n$ .

it shares risk efficiently,  $r^*$  will not be renegotiated. In this sense the convertible debt contract is the renegotiation-proof equivalent of the debt contract. (However, off the equilibrium path, at efforts lower than  $e^*$ ,  $R^*$  would be renegotiated because the investor then selects  $\delta$ , a contract that shares risk inefficiently.)

#### 4. The General Model

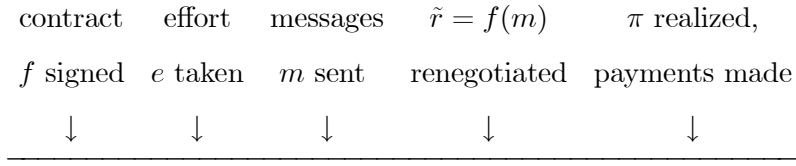
We now consider general “message game” contracts, in the general model in which both parties may be risk averse. We make no assumptions here about the distribution of bargaining power. Furthermore, the results of this section do not depend on our specific definition of a feasible simple contract: they hold for any feasible set  $C \subset \mathbb{R}$  that is non-empty and compact, and leads to a downward sloping Pareto function  $H(\cdot, e)$ . The main result is that any pure strategy equilibrium outcome of a general contract is also an equilibrium outcome of an investor-option contract.

A *general contract (game form, mechanism)* is a function

$$f : M_E \times M_I \rightarrow \Delta C,$$

where  $M_E$  and  $M_I$  are sets of messages that the entrepreneur and investor can respectively send, and  $\Delta C$  is the space of probability distributions on  $C$ .<sup>14</sup> Let  $M = M_E \times M_I$ , and denote a message pair as  $m = (m_E, m_I)$ . When  $m$  is sent, the contract prescribes a random simple contract,  $\tilde{r} = f(m) \in \Delta C$  that would, if it were not renegotiated, determine the entrepreneur’s payment to the investor.

Bargaining and renegotiation occur according to the following time line:




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<sup>14</sup>Endow  $\Delta C$  with the topology of weak convergence. Note that it is compact, since  $C$  is compact in  $\mathbb{R}^n$ .

Two features are noteworthy. First, renegotiation takes place *ex post*, after the messages are sent. This, however, is assumed only for simplicity. So long as the parties cannot commit to not renegotiate at this *ex post* date, our results still hold if renegotiation is also possible at the *interim* date that occurs after the effort is chosen but before the messages are sent. This is made clear below. Second, renegotiation occurs before the randomness in the mechanism’s prescribed outcome  $\tilde{r}$  is realized. This is the same convention as in Segal and Whinston (2002), but differs from that in Maskin and Moore (1999).<sup>15</sup>

We let  $\hat{V}(\tilde{r}, e)$  and  $\hat{U}(\tilde{r}, e)$  denote the post-renegotiation payoffs of the investor and entrepreneur, respectively, when effort  $e$  has been taken and messages  $m$  have been sent, where  $\tilde{r} = f(m)$ . We assume that renegotiation is efficient,

$$\hat{U}(\tilde{r}, e) = H(\hat{V}(\tilde{r}, e), e) \text{ for all } (\tilde{r}, e) \in \Delta C \times E, \quad (4)$$

and that the post-renegotiation payoffs are continuous in the prescribed outcome:

$$\hat{V}(\cdot, \cdot) \text{ and } \hat{U}(\cdot, \cdot) \text{ are continuous on } \Delta C \times E. \quad (5)$$

The efficient renegotiation assumption (4) implies that  $\tilde{r}$  is renegotiated to an efficient risk-sharing contract; any randomness in  $\tilde{r}$  has no efficiency consequence. The continuity assumption (5) is weaker than the continuity and differentiability assumed in Segal and Whinston (2002), and it holds fairly generally. It requires the bargaining powers of the parties in the renegotiation game not to shift discontinuously in  $(\tilde{r}, e)$ , the allocation that determines their disagreement payoffs.

Given a contract  $f$ , the *message game following effort  $e$*  is the game in which the strategies are messages, and the payoff functions are  $\hat{U}(f(\cdot), e)$  and  $\hat{V}(f(\cdot), e)$ . This game is “strictly competitive”, which means that the two players have opposing preferences on the set of message pairs. This is because renegotiation is efficient, and so any message profile results in a post-renegotiation payoff pair on the downward-sloping Pareto

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<sup>15</sup>In Maskin and Moore (1999), the parties can commit not to renegotiate during the time interval between the sending of messages and the realization of the contract’s random outcome.

frontier for the given effort. In particular, since

$$\hat{U}(f(m), e) = H(\hat{V}(f(m), e), e),$$

the entrepreneur's best reply to any  $m_I$  minimizes the investor's payoff  $\hat{V}(f(\cdot, m_I), e)$ .

Unless explicitly stated otherwise, the term “equilibrium” denotes a pure strategy subgame perfect equilibrium. (Mixed strategy equilibria are considered in Appendix B.)

Consider an equilibrium  $m^*(e)$  of the message game. Denote the corresponding equilibrium payoffs as  $V^*(e)$  and  $U^*(e) = H(V^*(e), e)$ . Because the message game is strictly competitive,  $m^*(e)$  is also an equilibrium of the zero-sum game in which the investor's payoff is  $\hat{V}(f(m), e)$  and the entrepreneur's is  $-\hat{V}(f(m), e)$ . (This is not true of mixed strategy equilibria, as we discuss below.) Therefore, by a standard “maxmin” argument,

$$V^*(e) = \sup_{m_I} \inf_{m_E} \hat{V}(f(m_I, m_E), e).^{16} \quad (6)$$

An *equilibrium of (the game generated by) contract  $f$*  is a pair  $(e^*, m^*(\cdot))$ , where  $e^*$  is an effort that maximizes the entrepreneur's equilibrium continuation payoff in the message game:

$$e^* \in \arg \max_{e \in E} U^*(e). \quad (7)$$

We now prove that an investor-option contract performs as well as any general contract. For a quasilinear model the result is Proposition 9 in Segal and Whinston (2002). The heuristic argument is the following. Consider an equilibrium  $(e^*, m^*(\cdot))$  of a contract  $f$ . Define an investor-option contract  $f^I : M_I \rightarrow \Delta C$  by holding the entrepreneur's message fixed at  $m_E^*(e^*)$ :

$$f^I(m_I) = f(m_E^*(e^*), m_I).$$

Given this option contract, after any effort the investor can obtain a payoff at least as large as she would get from the equilibrium of the message game determined by  $f$ . This is because, as we discussed above, the entrepreneur chooses a message to minimize the investor's payoff when the contract is  $f$ . But  $f^I$  does not allow him to choose a message

<sup>16</sup>Since  $m^*(e)$  is an equilibrium, the “sup” in (6) can be replaced by “max”.

to harm the investor in this way. Hence, if  $f^I$  generates equilibrium payoffs  $V^I(e)$  and  $U^I(e)$ , we have  $V^I(\cdot) \geq V^*(\cdot)$ , with equality at  $e^*$  because  $m_I^*(e^*)$  is a best reply to  $m_E^*(e^*)$ . Efficient renegotiation then implies  $U^I(\cdot) \leq U^*(\cdot)$ , with equality at  $e^*$ . Thus, since it maximizes  $U^*(\cdot)$ ,  $e^*$  indeed maximizes  $U^I(\cdot)$ .

The unwarranted assumption in this heuristic proof is that  $f^I$  has an equilibrium. A correct proof is given in Appendix A.

**Theorem 1.** *Given any equilibrium of any contract, an investor-option contract exists that has the same equilibrium payoffs and effort.*

Theorem 1 also holds if the parties can renegotiate at the interim stage, after the effort is chosen but before any messages are sent, so long as they are also able to renegotiate ex post. This is because the theorem refers to equilibria that are in pure strategies, and hence yield, after any effort choice  $e$ , continuation payoffs  $(V^*(e), U^*(e))$  that are on the Pareto frontier given  $e$ . Knowing that these payoffs will obtain when  $e$  is chosen and the contract is not renegotiated, every interim renegotiation proposal by one party will be rejected by the other. It is consequently irrelevant whether the parties can commit not to renegotiate at the interim date.

## 5. Entrepreneur-Offer Renegotiation

We now show in the general model that if the entrepreneur has all the bargaining power in the renegotiation stage, then any general contract is weakly Pareto dominated by debt. The general contract is Pareto dominated by debt if it does not resemble debt in a particular sense.

Since the entrepreneur has the bargaining power, the investor receives the same payoff regardless of whether she agrees to renegotiate. Thus, after an effort  $e$  is taken and a message pair  $m$  is sent, renegotiation of the prescribed  $\tilde{r} = f(m)$  yields an efficient risk-sharing contract for  $e$  that gives the investor the same payoff as does  $\tilde{r}$ . Her post-renegotiation payoff is

$$\hat{V}(\tilde{r}, e) = V(\tilde{r}, e) = \mathcal{E}_{\tilde{r}} \left\{ \sum g_i(e) v(\tilde{r}_i) \right\}, \quad (8)$$

and the entrepreneur's is

$$\hat{U}(\tilde{r}, e) = H(V(\tilde{r}, e), e). \quad (9)$$

The two assumptions made in Section 4 are satisfied: renegotiation is efficient, and the post-renegotiation payoffs are continuous in  $\tilde{r}$ .

We first dispense with random contracts. The investor's certainty equivalent for  $\tilde{r} \in \Delta C$  is the  $r^{ci} \in \mathbb{R}^n$  defined by  $v(r_i^{ci}) \equiv \mathcal{E}_{\tilde{r}} v(\tilde{r}_i)$ . Since  $V(r^{ci}, \cdot) = V(\tilde{r}, \cdot)$ , we see from (8) and (9) that for any effort,  $r^{ci}$  and  $\tilde{r}$  yield the same post-renegotiation payoffs. Thus, for any contract  $f$ , an equivalent deterministic contract  $\bar{f}$  is defined by letting  $\bar{f}(m)$  be the investor's certainty equivalent for  $f(m)$ . The contracts  $f$  and  $\bar{f}$  have the same equilibrium efforts and payoffs. Since the certainty equivalent of any  $\tilde{r} \in \Delta C$  is in  $C$ ,<sup>17</sup> we have proved the following.

**Lemma 3.** *The equilibrium efforts and payoffs of any contract  $f : M \rightarrow \Delta C$  are the same as those of a contract  $\bar{f} : M \rightarrow C$  defined by letting  $\bar{f}(m)$  be the investor's certainty equivalent for  $f(m)$ .*

In light of Theorem 1 and Lemma 3, we can restrict attention to deterministic investor-option contracts. By the revelation principle, we can further restrict attention to revelation mechanisms for the investor,  $r^* : E \rightarrow C$ , that are incentive compatible. Given such an  $r^*$ , its truthful equilibrium yields post-renegotiation payoffs

$$V^*(e) = V(r^*(e), e) \text{ and } U^*(e) = H(V^*(e), e). \quad (10)$$

Any maximizer of  $U^*(\cdot)$  is an equilibrium effort.

It is now easy to see that when the entrepreneur has the bargaining power, an equilibrium of a riskless debt contract is efficient. Suppose that for all possible reports,  $r^*(\cdot)$  specifies a riskless debt contract,  $\delta^D \equiv (D, \dots, D)$ . By (10), the investor's post-renegotiation payoff is then  $V(\delta^D, e) = v(D)$ , which is independent of  $e$ . The equilibrium effort maximizes  $U^*(\cdot) = H(v(D), \cdot)$ , and is hence the effort component of the efficient

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<sup>17</sup>In particular,  $r^{ci}$  satisfies MI because  $v(r_{i+1}^{ci}) - v(r_i^{ci}) = \mathcal{E}_{\tilde{r}} [v(\tilde{r}_{i+1}) - v(\tilde{r}_i)] \geq 0$ , since any realization of  $\tilde{r}$  satisfies MI because it is in  $C$ .

allocation that gives the investor payoff  $v(D)$ . This efficient allocation is thus the equilibrium outcome, since renegotiation is efficient and does not benefit the investor.

Of course, as we observed in Section 3, a riskless debt contract that is acceptable to the investor may not be feasible. We accordingly turn to debt contracts that may be risky. The following lemma, a generalization of Lemma 2, shows that debt provides the greatest incentives of all contracts in  $C$ .

**Lemma 4.** *For any  $(r, e) \in C \times E$  such that  $r$  is not debt, a unique debt contract  $\delta \in C$  exists for which  $V(r, e) = V(\delta, e)$ . Furthermore,*

- (i)  $V_e(r, e) > V_e(\delta, e)$ , and
- (ii)  $(e - e') [V(r, e') - V(\delta, e')] < 0$  for all  $e' \neq e$ .

We now prove the first main result of this section: any equilibrium of a general contract is weakly Pareto dominated by an equilibrium of a debt contract. Again considering the investor-option incentive-compatible revelation mechanism  $r^*(\cdot)$  and its equilibrium effort  $e^*$ , the desired debt contract is defined by

$$V(\delta, e^*) = V(r^*(e^*), e^*). \quad (11)$$

If this  $\delta$  is adopted and effort  $e$  is taken, the equilibrium post-renegotiation payoffs are

$$V^\delta(e) = V(\delta, e) \text{ and } U^\delta(e) = H(V(\delta, e), e). \quad (12)$$

It follows from (10) – (12) that when  $\delta$  is adopted,  $e^*$  yields the same payoffs as it does when  $r^*(\cdot)$  is adopted:

$$V^\delta(e^*) = V^*(e^*) \text{ and } U^\delta(e^*) = U^*(e^*). \quad (13)$$

The entrepreneur therefore weakly prefers any equilibrium of  $\delta$  to the given one of  $r^*(\cdot)$ , since any equilibrium effort of  $\delta$  maximizes  $U^\delta(\cdot)$ . The investor has the same preference, provided that the equilibrium effort of  $\delta$ , say  $e^\delta$ , is not less than  $e^*$ . This is because

$$V^\delta(e^\delta) = V(\delta, e^\delta) \geq V(\delta, e^*) = V^*(e^*),$$

where the inequality follows from the monotonicity of  $\delta$ , MLRP, and  $e^\delta \geq e^*$ . The proof is once complete once  $e^\delta \geq e^*$  is proved; this is done in Appendix A using Lemma 4.

**Theorem 2.** *Assume entrepreneur-offer renegotiation. Then, given any equilibrium of any general contract, a debt contract exists that has an equilibrium with a weakly greater effort, and which both parties weakly prefer.*

**Remark.** Recall that when renegotiation is impossible, Innes (1990) showed that debt contracts are optimal simple contracts if both parties are risk neutral, given MLRP and the contract restrictions LE and MI. We can now see that his result extends to a risk averse investor (as well as to general contracts). Theorem 2 establishes that debt is optimal when renegotiation is possible, and Proposition 2 shows that debt will not be renegotiated if the entrepreneur is risk neutral and the investor is risk averse. Thus, in this case, a debt contract is optimal even when renegotiation is impossible.

Since it proves only the weak Pareto dominance of debt, Theorem 2 leaves open the possibility that a contract totally unlike debt has an equilibrium with a Pareto optimal outcome. The following theorem shows this is not true. Its result is that in an equilibrium of an optimal contract, if the entrepreneur chooses an effort that is almost as large as the equilibrium effort, then the ensuing equilibrium messages result in the prescription of a simple contract which is approximately debt. The theorem implies, for example, that any optimal investor-option contract takes the form of a generalized convertible debt contract. If the contract specifies only a finite number of simple contracts, in equilibrium it must prescribe a debt contract following the choice of any effort in some interval that has the equilibrium effort as its upper endpoint. If the contract is simple, it must be debt.

**Theorem 3.** *Assume entrepreneur-offer renegotiation. Suppose an equilibrium  $(e^*, m^*(\cdot))$  of a general contract  $f$  is not Pareto dominated by an equilibrium of a debt contract, and  $e^* \in \text{int}(E)$ . Then the left hand limit,*

$$\tilde{r}^* = \lim_{e \rightarrow e^* -} f(m^*(e)),$$

*exists in  $\Delta C$ , and it puts all probability either on a debt contract  $\delta$ , or on a set of riskless debt contracts.*



## 6. Investor-Offer Renegotiation

In this section the investor has the renegotiation bargaining power. We show that in this case, debt still provides the greatest incentives, but it may induce too much effort if the entrepreneur is risk averse. The reason for this is that now the risk characteristics of the initial contract matter, since it determines the entrepreneur's payoff because he does not gain from the renegotiation. Debt is very risky for him, as it gives him no return at all if the realized output is low. Debt can thus induce him to work too hard to lower the probability of this event. Our final result is that under standard but strong curvature assumptions, either debt is optimal, or a non-debt simple contract is optimal and actually achieves an efficient allocation, i.e., the inability to contract on effort does not restrict efficiency.

We now simplify the analysis in two ways. First, the following separability assumption ensures that the entrepreneur's attitude towards risk does not depend on his effort:

$$\begin{aligned} \text{(SEP)} \quad & \text{real-valued functions } a(\cdot) > 0 \text{ and } c(\cdot) \text{ on } E, \text{ and } \bar{u}(\cdot) \\ & \text{on } \mathbb{R}, \text{ exist such that } u(w, e) = a(e)\bar{u}(w) - c(e). \end{aligned}$$

Second, we consider only deterministic contracts, those that assign to each pair of messages a simple contract rather than a random simple contract.<sup>18</sup>

It is now convenient to describe the Pareto frontier for a given effort  $e$  by the inverse function  $J(\cdot, e) \equiv H^{-1}(\cdot, e)$ . It is well-defined, since  $H(\cdot, e)$  is continuous and decreasing, and it has the same properties as  $H$ . The Pareto frontier given  $e$  is the graph of  $J(\cdot, e)$ , and  $(r^*, e^*)$  is an efficient allocation if and only if  $e^*$  maximizes  $J(U(r^*, e^*), \cdot)$ .

Given a deterministic contract  $f : M_E \times M_I \rightarrow C$ , suppose messages  $m$  are sent. The contract prescribes the simple contract  $r = f(m)$ . Because the investor has all the bargaining power, the entrepreneur's equilibrium post-renegotiation payoff is as though

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<sup>18</sup>Random contracts when the investor has the bargaining power are problematic. They cannot be eliminated by appeal to the entrepreneur's certainty equivalent contract, as we did in Lemma 3 with respect to the investor, because it can violate MI. See Lemma 6 in Matthews (2001).

renegotiation does not occur. Thus, given effort  $e$ , the post-renegotiation payoffs are

$$\hat{U}(r, e) = U(r, e) \text{ and } \hat{V}(r, e) = J(U(r, e), e).$$

The following analog of Lemma 4 will imply that no contract provides greater incentives than debt. (The implication is immediate for simple contracts,  $f(\cdot) \equiv r \in C$ .)

**Lemma 5.** *For any  $(r, e) \in C \times E$  such that  $r$  is not debt, a unique debt contract  $\delta \in C$  exists for which  $U(\delta, e) = U(r, e)$ . If SEP holds, then*

- (i)  $U_e(\delta, e) > U_e(r, e)$ , and
- (ii)  $(e - e') [U(\delta, e') - U(r, e')] < 0$  for all  $e' \neq e$ .

**Proposition 4.** *Assume investor-offer renegotiation and SEP. Then, given any equilibrium of any deterministic general contract, a debt contract exists that has an equilibrium with a weakly greater effort, and it gives the same payoff to the entrepreneur.*

The debt contract in Proposition 4 is Pareto superior to the general contract if and only if it is preferred by the investor. Let  $e^*$  and  $\hat{e} \geq e^*$  be, respectively, the equilibrium efforts of the general contract and the debt contract. Each equilibrium gives the same payoff to the entrepreneur, say  $U^*$ . The switch to the debt contract therefore increases the investor's payoff by  $J(U^*, \hat{e}) - J(U^*, e^*)$ . As we show with an example below, this gain need not be positive. It can be negative if the debt contract provides incentives that are too strong, so that  $\hat{e}$  exceeds the efficient effort that maximizes  $J(U^*, \cdot)$ .

Surprisingly, however, debt can cause the entrepreneur to exert too much effort only if he is risk averse. Recall that if he is risk neutral, then by Proposition 1, debt is an efficient risk-sharing contract given any effort. A debt contract is then not renegotiated, and so it will determine the investor's as well as the entrepreneur's post-renegotiation payoff. The investor's payoff from a debt contract increases with effort, since debt satisfies MI. This implies that she too is better off, in the setting of Proposition 4, when the debt contract is adopted instead of the original contract, and the entrepreneur increases his effort.

**Theorem 4.** *Assume investor-offer renegotiation, and that the entrepreneur is risk neutral. Then, given any equilibrium of any deterministic general contract, a debt contract exists that has an equilibrium which both parties weakly prefer.*

If the entrepreneur is risk averse, two standard (albeit strong) curvature assumptions imply that either an efficient allocation is attainable, or debt is optimal. The first assumption is that the so-called “first-order approach” is valid:

(FOA) For any  $r \in C$ , the effort maximizing  $U(r, \cdot)$  is unique.

Various properties of the primitives imply FOA (Rogerson, 1985; Jewitt, 1988). The second assumption requires  $J(\hat{U}, \cdot)$  to be pseudoconcave (“single peaked”):

(SP) For any feasible  $U^*$ , maximizer  $e^{**}$  of  $J(U^*, \cdot)$ , and  $e \in E$ ,  $(e - e^{**})J_2(U^*, e) \leq 0$ .

In addition, the theorem below applies only to equilibria that give the entrepreneur a payoff  $U^*$  that satisfies:

(WA)  $U^* \leq u(\pi_1 - \underline{r}, \underline{e})$ .

Recalling that a wage contract  $r^w$  is in  $C$  if and only if  $w \in [0, \pi_1 - \underline{r}]$ , we see that WA must hold if a feasible wage contract exists that gives the entrepreneur the payoff  $U^*$  when he chooses the minimal effort  $\underline{e}$ . The lower is the liability bound  $\underline{r}$ , the larger is the interval of payoffs satisfying WA. It holds for all equilibrium payoffs  $U^*$  if  $\underline{r} = -\infty$ , so that the investor has no liability bound.

Our final theorem shows that under these assumptions, any optimal contract is either payoff equivalent to a debt contract, or it achieves an efficient allocation, i.e., the inability to contract on effort does not restrict efficiency. Furthermore, in the latter case efficiency can also be achieved by a simple contract.

**Theorem 5.** *Assume investor-offer renegotiation, SEP, FOA, SP. and  $\underline{r} = -\infty$ . Suppose an equilibrium of a deterministic general contract is not Pareto dominated by an equilibrium of any other deterministic contract, and it gives the entrepreneur a payoff  $U^*$  satisfying WA. Then, either (i) this equilibrium achieves an efficient allocation,*

which is also attained by a simple contract, or (ii) a debt contract exists that gives rise to the same payoffs.

The following example illustrates some of the results of this section. In particular, it shows that debt can elicit too much effort when the entrepreneur is risk averse, and that in this case a non-debt simple contract achieves an efficient allocation.

**Example.** The investor is risk neutral, with a liability bound  $\underline{r}$  so low that it never binds for the contracts we consider. The entrepreneur's utility function is  $u(w, e) = \bar{u}(w) - .5e^2$ , where

$$\bar{u}(w) = \min(w, 11w - 1).$$

The entrepreneur is thus risk averse, but risk neutral with respect to any gamble for which all the payments to him are on one side of .1. The possible outputs are  $(\pi_1, \pi_2, \pi_3) = (0, .5, 1)$ . The interval of possible efforts is  $[0, 1]$ . The probability that  $\pi_i$  occurs given effort  $e$  is

$$g_i(e) = \begin{cases} .5 - .5e & \text{for } i = 1 \\ .5 & \text{for } i = 2 \\ .5e & \text{for } i = 3. \end{cases}$$

Given a simple contract  $r$ , let  $u_i = \bar{u}(\pi_i - r_i)$ . The entrepreneur's expected payoff given  $r$  and an effort  $e$  can be written as

$$U(r, e) = .5 [(u_3 - u_1)e - e^2 + u_1 + u_2]. \quad (14)$$

Consider the allocations  $(r, e)$  that, for some  $\bar{U}$ , satisfy three conditions: (i)  $U(r, e) = \bar{U}$ ; (ii) each payment  $\pi_i - r_i$  to the entrepreneur exceeds .1, so that he is effectively risk neutral; and (iii)  $e = .5$ , the effort that maximizes expected output net of effort cost:

$$\sum g_i(e)\pi_i - .5e^2 = .5(e - e^2 + .5).$$

Such allocations exist for any  $\bar{U} \geq -.025$ , and they are the first-best efficient allocations that give the entrepreneur payoff  $\bar{U}$ . The obvious efficient allocation of this type is

$(r^w, .5)$ , where  $r^w$  is the wage contract with wage  $w = \bar{U} + .125$ . (Note that  $w \geq .1$ , since  $\bar{U} \geq -.025$ .)

In our second-best world, when the investor has all the renegotiation bargaining power,  $U(r, e)$  is the entrepreneur's post-renegotiation utility if  $r$  is adopted and he takes effort  $e$ . His optimal effort is thus  $e = .5(u_3 - u_1)$ , provided this effort is in  $[0, 1]$ .

Consider a debt contract  $\delta$  for which the face value is  $D \in [0, .28516]$ . For this contract,  $(u_1, u_2, u_3) = (-1, .5 - D, 1 - D)$ . The entrepreneur's best effort is  $e^\delta = .5(2 - D)$ . His equilibrium utility satisfies  $U(\delta, e^\delta) \geq -.025$ , as is easily shown.<sup>19</sup> But we have just seen that any first-best allocation that gives the entrepreneur a utility in this range must have effort equal to  $.5$ . Since  $e^\delta > .5$ , these debt contracts provide incentives that are too strong.

We pause here to discuss this excessive effort result. The entrepreneur exerts too much effort because, if the debt claim is not renegotiated, it threatens to leave the entrepreneur with no monetary return. The debt thus imposes a severe risk on the entrepreneur, since his marginal utility of income is high for  $w < .1$ . He therefore chooses a socially excessive effort in order to reduce the probability that his income will be so low. Of course, his actual income will never be so low because the debt claim will be renegotiated. But the entrepreneur obtains no gain from the renegotiation because the investor has all the bargaining power, and so the entrepreneur still cares about the riskiness of the debt.

We now show that each of these contracts  $\delta$  is Pareto dominated by a non-debt contract, and the latter actually achieves the first best. Simplify by setting  $U^\delta = U(\delta, e^\delta)$ , and recall that  $U^\delta \geq -.025$ . The desired contract  $r$  is defined implicitly by

$$u_i = \begin{cases} U^\delta - .375 & \text{for } i = 1 \\ U^\delta + .125 & \text{for } i = 2 \\ U^\delta + .625 & \text{for } i = 3. \end{cases} \quad (15)$$

The  $r$  so defined satisfies the LE and MI constraints, and is not debt.<sup>20</sup> Given this  $r$ ,

<sup>19</sup> $U(\delta, e^\delta) = (.5) [-.5 - D + (.25)(2 - D)^2]$  decreases in  $D$ , and is equal to  $-.025$  at  $D = 0.28516$ .

<sup>20</sup>From the previous footnote, we see that  $U^\delta \in [-.025, .25]$  for  $D \in [0, .28516]$ . This and (15) imply

the entrepreneur chooses  $e = .5(u_3 - u_1) = .5$  and obtains utility  $U(r, .5) = U^\delta$ . The renegotiation of  $r$  when  $e = .5$  yields a first-best allocation that gives the entrepreneur utility  $U^\delta$ , and is therefore strictly preferred by the investor to the allocation obtained when  $\delta$  is renegotiated and the effort is  $e^\delta > .5$ .

## 7. Conclusions

We have analyzed a dynamic entrepreneurial incentive problem with: (i) observable but nonverifiable effort; (ii) renegotiation after effort has been chosen but before output is realized; and (iii) two contractual constraints: limited entrepreneurial liability, and monotonicity of the investor's return with the firm's performance. We have shown in this setting the optimality of debt-cum-renegotiation or equivalently, convertible debt. The optimality of these contracts stems from their leading to maximum effort by the entrepreneur, as in Innes' (1990) problem. Debt was optimal in his no-renegotiation setup only because both parties were risk neutral. Renegotiation allows here the parties to separate the issue of effort provision from that of optimal insurance. Our results are therefore similar to those of Matthews (2001), who considers privately observed entrepreneurial effort.

These results provide a simple theory of the dynamics of capital structure: debt is signed as an initial effort-maximizing contract in order to generate maximum effort from the entrepreneur. Once effort has been chosen, the parties move to optimal insurance. The investor takes on more risk, that is, she transforms her claim into something closer to equity. As is well known, standard equity is the optimal risk-sharing contract if both parties have CARA preferences. In this case the model delivers a very simple prediction: the firm starts as an all-debt firm, and then becomes an all-equity firm by, for example, going public.

The results of Section 6 qualify the conclusion that debt is optimal. Specifically, when the investor has bargaining power in renegotiation, and the entrepreneur is risk averse, debt may sometimes induce excessive effort because it threatens the entrepreneur

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that  $r_1 = -(U^\delta + .625) < 0$  and  $r_2 = r_3 = .375 - U^\delta \in [.125, .4]$ . So  $r$  satisfies LE and MI, is not debt.

with very harsh consequences when output is low. In this case an efficient allocation can be achieved by a simple contract that is not debt, so that the non-contractibility of effort does not lower welfare. It therefore remains true, under the assumptions of Theorem 5, that debt is optimal whenever the non-contractibility of effort matters.

We end on a more technical note. Our result that debt is optimal in the set of message-contingent contracts relies on a restriction to pure strategy equilibria, and on the non-involvement of third parties. In Appendix B we show that debt may not be optimal without these restrictions. However, the contracts that are then optimal do not seem very realistic. We welcome further theoretical research that would help us understand why such contracts do not emerge in the real world.

## A. Appendix A: Proofs Missing from the Text

**Proof of Lemma 1.** Fix  $e \in E$  and  $\hat{V} \in [v(\underline{r}), V(\pi, e)]$ . Then some convex combination of  $(\underline{r}, \dots, \underline{r})$  and  $\pi$ , say  $r$ , satisfies  $\hat{V}(r, e) = \hat{V}$ . Since  $C$  is convex and contains  $\pi$  and  $\underline{r}$ ,  $r \in C$ . So the constraint set of (1) is nonempty. As it is also compact, and the objective is continuous, the program has a solution. As stated in the text, its solution is unique; denote it as  $r^*$ . We thus see that  $H(\cdot, e)$  is well-defined on  $[v(\underline{r}), V(\pi, e)]$ . It is continuous on this interval by the maximum theorem. It is concave on this interval by a direct argument using Jensen's inequality, the concavity of  $U(\cdot, e)$  and  $V(\cdot, e)$ , and the convexity of  $C$ .

Now let  $\hat{V} \in (v(\underline{r}), V(\pi, e))$ . Then some convex combination of  $(\underline{r}, \dots, \underline{r})$  and  $\pi$ , say  $r$ , satisfies  $\hat{V}(r, e) > \hat{V}$ . So Slater's condition holds. Thus,  $\lambda^* \geq 0$  exists such that  $(r^*, \lambda^*)$  is a saddle point in  $C \times \mathbb{R}_+$  of the Lagrangian

$$\mathcal{L}(r, \lambda, e, \hat{V}) \equiv U(r, e) + \lambda [V(r, e) - \hat{V}].$$

We claim  $(r^*, \lambda^*)$  is the only saddle point of  $\mathcal{L}(\cdot, \cdot, e, \hat{V})$ . Since  $r^*$  is the unique solution of (1), any other saddle point takes the form  $(r^*, \lambda)$ , with  $\lambda \neq \lambda^*$ . The following argument shows that  $\lambda^*$  is determined by  $r^*$ , and so  $(r^*, \lambda^*)$  is unique.

Note first that  $\lambda^* > 0$ . If  $\lambda^* = 0$ , the saddle point property would imply that  $r^*$  maximizes  $U(\cdot, e) = \mathcal{L}(\cdot, 0, e, \hat{V})$  on  $C$ . But then  $r^* = \underline{r}$ , contrary to  $\hat{V} > v(\underline{r})$ .

Instead of maximizing  $\mathcal{L}(\cdot, \lambda^*, e, \hat{V})$  on  $C$ , consider the relaxed problem obtained by deleting MI. A solution  $r$  to this relaxed program satisfies the Kuhn-Tucker condition

$$-u_1(\pi_i - r_i, e) + \lambda^* v'(r_i) = (\beta_i - \alpha_i) / g_i(e) \quad (\text{A1})$$

for each  $i = 1, \dots, n$ , where  $\alpha_i \geq 0$  and  $\beta_i \geq 0$  are the multipliers for (LI)  $r_i \geq \underline{r}$  and (LE)  $r_i \leq \pi_i$ , respectively. If  $r_i > r_{i+1}$  for some  $i < n$ , then  $r_i > \underline{r}$  and  $r_{i+1} < \pi_i$ , and in turn  $\alpha_i = 0$  and  $\beta_{i+1} = 0$  by complementary slackness. Hence, (A1) would imply

$$\begin{aligned} \beta_i / g_i(e) &= -u_1(\pi_i - r_i, e) + \lambda^* v'(r_i) \\ &< -u_1(\pi_{i+1} - r_{i+1}, e) + \lambda^* v'(r_{i+1}) = -\alpha_{i+1} / g_{i+1}(e), \end{aligned}$$



where the inequality follows from  $u_{11} \leq 0$  and  $v'' \leq 0$ , with one strict,  $\lambda^* > 0$ ,  $r_i > r_{i+1}$ , and  $\pi_i - r_i < \pi_{i+1} - r_{i+1}$ . But this is contrary to  $\alpha_{i+1} \geq 0$  and  $\beta_i \geq 0$ . We conclude that any solution of the relaxed problem satisfies the neglected constraint MI. Hence, the solution of the relaxed problem is the unique solution  $r^*$  of the unrelaxed problem. So  $r^*$  satisfies (A1). Now, suppose there is no  $i \leq n$  such that  $\underline{r} < r_i^* < \pi_i$ . Then by MI and  $\hat{V} \in (v(\underline{r}), V(\pi, e))$ ,  $1 \leq k < n$  exists such that  $r_k^* = \underline{r}$  and  $r_{k+1}^* = \pi_{k+1}$ . Hence,  $\beta_k = 0$  and  $\alpha_{k+1} = 0$ , and (A1) implies

$$\begin{aligned} -\alpha_k/g_k(e) &= -u_1(\pi_k - \underline{r}, e) + \lambda^* v'(\underline{r}) \\ &> -u_1(0, e) + \lambda^* v'(\pi_{k+1}) = \beta_{k+1}/g_{k+1}(e). \end{aligned}$$

This is contrary to  $\alpha_k \geq 0$  and  $\beta_{k+1} \geq 0$ . We conclude that  $\underline{r} < r_i^* < \pi_i$  for some  $i \leq n$ . For this  $i$  we have  $\alpha_i = \beta_i = 0$ , and (A1) implies

$$\lambda^* = \frac{u_1(\pi_i - r_i^*, e)}{v'(r_i^*)}. \quad (\text{A2})$$

This proves that  $(r^*, \lambda^*)$  is the unique saddle point of  $\mathcal{L}(\cdot, \cdot, e, \hat{V})$ .

This uniqueness implies that a general envelope theorem, Corollary 5 of Milgrom and Segal (2002), now applies. The derivative  $H_1(\hat{V}, e)$  therefore exists, with  $H_1(\hat{V}, e) = -\lambda^* < 0$ . If also  $e \in \text{int}(E)$ , then  $H_2(\hat{V}, e)$  exists and is given by

$$H_2(\hat{V}, e) = \mathcal{L}_e(r^*, \lambda^*, e, \hat{V}) = U_e(r^*, e) + \lambda^* V_e(r, e).$$

Now, since the solution  $r^*$  of (1) is unique, the maximum theorem implies that it is a continuous function of  $(\hat{V}, e)$ . In turn, (A2) implies that  $\lambda^*$  is a continuous function of  $(\hat{V}, e)$ . Thus, both  $H_1$  and  $H_2$  are continuous at any interior point, i.e., at any  $(\hat{V}, e)$  satisfying  $e \in \text{int}(E)$  and  $\hat{V} \in (v(\underline{r}), V(\pi, e))$ . So  $H$  is differentiable at such points. ■

**Proof of Proposition 1.** Fix  $e \in E$ , and for simplicity write  $g_i = g_i(e)$ . Since the entrepreneur is risk neutral, we can denote the derivative  $u_1(w, e)$  as a constant  $z > 0$ , independent of  $w$ . The efficient contracts within those that satisfy LE are the solutions,

as the reservation payoff  $u^0$  varies, of the following program:

$$\begin{aligned} & \max_r \sum g_i v(r_i) \text{ such that} \\ & \text{(IR)} \quad \sum g_i u(\pi_i - r_i, e) \geq u^0, \text{ and} \\ & \text{(LE)} \quad \pi_i - r_i \geq 0 \text{ for } i = 1, \dots, n. \end{aligned}$$

If  $u^0 \leq u(0, e)$ , the solution is  $r = \pi$ , which is a debt contract with a face value greater than  $\pi_n$ . So assume  $u^0 > u(0, e)$ . Then any solution satisfies  $r_i < \pi_i$  for some  $i$ , and IR must bind. The program is concave and has a constraint set with nonempty interior. Thus,  $r$  is a solution if and only if it satisfies the constraints, IR with equality, and nonnegative multipliers  $\lambda, \alpha_1, \dots, \alpha_n$ , exist such that the following hold for each  $i = 1, \dots, n$ :

$$\begin{aligned} v'(r_i) - \lambda z &= \alpha_i / g_i, \\ \alpha_i (\pi_i - r_i) &= 0. \end{aligned}$$

Hence, if  $r_i < \pi_i$ , then  $r_i = v'^{-1}(\lambda z)$ . If  $r_i = \pi_i$  and  $i > 1$ , then  $r_{i-1} \leq \pi_{i-1} < r_i$ , and so  $v'' < 0$  implies

$$\begin{aligned} \alpha_{i-1} / g_{i-1} &= v'(r_{i-1}) - \lambda z \\ &> v'(r_i) - \lambda z = \alpha_i / g_i. \end{aligned}$$

Therefore  $\alpha_{i-1} > 0$ , and so  $r_{i-1} = \pi_{i-1}$ . We have shown that a solution satisfies, for some  $k \leq n$ ,

$$r_i = \begin{cases} \pi_i & \text{for } i < k \\ D & \text{for } i \geq k, \end{cases} \quad (\text{A3})$$

where  $D = v'^{-1}(\lambda z)$ . For any  $i$ ,

$$v'(r_i) - v'(D) = v'(r_i) - \lambda z = \alpha_i / g_i \geq 0.$$

Hence,  $D \geq r_i$  for all  $i$ . This shows that (A3) is indeed a debt contract with face value  $D$ . We conclude that every efficient contract within the set of simple contracts satisfying LE is a debt contract. The converse can be shown by reversing the argument, using the fact that the Kuhn-Tucker conditions are sufficient as well as necessary for a solution

to a concave problem. Hence, every debt contract is efficient within the set of contracts satisfying LE. ■

We use an important technical result to prove Lemma 2. Following Karamardian and Schaible (1990), define a vector  $x \in \mathbb{R}^n$  to be *quasi-monotone* if and only if  $k \in \{1, \dots, n\}$  exists such that  $x_i \leq 0$  for  $i < k$  and  $x_i \geq 0$  for  $i > k$ . Equivalently,  $x$  is quasi-monotone if and only if  $x_i > 0$  implies  $x_j \geq 0$  for all  $j > i$ . The crucial property of a quasi-monotone vector is that by MLRP, its expectation is a quasi-monotone function of effort.<sup>21</sup>

**Lemma A1.** *For any  $(x, e) \in \mathbb{R}^n \times E$ , if  $x \neq 0$  is quasi-monotone and  $\sum g_i(e)x_i = 0$ , then (i)  $\sum g'_i(e)x_i > 0$ , and (ii)  $(e - \hat{e}) \sum g_i(\hat{e})x_i < 0$  for all  $\hat{e} \neq e$ .*

**Proof.** Routine calculus proves (ii), if (i) holds for all  $e$  such that  $\sum g_i(e)x_i = 0$ . To prove (i), note that

$$\begin{aligned} \sum g'_i(e)x_i &= \sum \left( \frac{g'_i(e)}{g_i(e)} \right) g_i(e)x_i \\ &> \sum_{i < k} \left( \frac{g'_k(e)}{g_k(e)} \right) g_i(e)x_i + \sum_{i \geq k} \left( \frac{g'_k(e)}{g_k(e)} \right) g_i(e)x_i \\ &= \left( \frac{g'_k(e)}{g_k(e)} \right) \sum g_i(e)x_i = 0. \end{aligned}$$

The inequality follows from MLRP; it is strict because  $x \neq 0$  implies  $k < n$ , and  $x_i > 0$  for some  $i > k$ . ■

**Proof of Lemma 2.** Let  $x = r - \delta$ . It is easily verified that since  $\delta$  is debt and  $r$  satisfies LE and MI,  $x$  is quasi-monotone. By assumption,  $x \neq 0$  and  $\sum g_i(e)x_i = w^*(\delta, e) - w^*(r, e) = 0$ . Thus, (i) and (ii) following directly from the corresponding parts of Lemma A1. ■

**Proof of Theorem 1.** Let  $f$  be a contract with an equilibrium  $(e^*, m^*(\cdot))$ . Simplify notation by denoting  $m_E^*(e^*)$  as  $m_E^*$ . For all  $e \in E$ , define

$$V^I(e) \equiv \sup_{m_I \in M_I} \hat{V}(f(m_E^*, m_I), e). \tag{A4}$$

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<sup>21</sup>Versions of this lemma are proved by, e.g., Innes (1990), Matthews (2001), and most generally Athey (1998, Theorem 3).

Let  $\{m_I^k(e)\}_{k=1}^\infty$  be a sequence in  $M_I$  such that  $\hat{V}(\tilde{r}^k(e), e) \rightarrow V^I(e)$  as  $k \rightarrow \infty$ , where  $\tilde{r}^k(e) = f(m_E^*, m_I^k(e))$ . Since  $\Delta C$  is compact, there is a subsequence, which for simplicity we take to be  $\{\tilde{r}^k(e)\}$  itself, that converges to some  $\tilde{r}(e) \in \Delta C$ . The continuity of  $\hat{V}(\cdot, e)$  implies

$$V^I(e) = \hat{V}(\tilde{r}(e), e). \quad (\text{A5})$$

For any  $e' \neq e$ , (A4) implies

$$V^I(e) \geq \hat{V}(f(m_E^*, m_I^k(e')), e) = \hat{V}(\tilde{r}^k(e'), e).$$

Taking limits,  $\tilde{r}^k(e') \rightarrow \tilde{r}(e')$  and the continuity of  $\hat{V}(\cdot, e)$  imply

$$V^I(e) \geq \hat{V}(\tilde{r}(e'), e). \quad (\text{A6})$$

By (A5) and (A6),  $\tilde{r}(\cdot) : E \rightarrow \Delta C$  is an incentive compatible revelation mechanism for the investor. Thus,  $f^I(\cdot) \equiv \tilde{r}(\cdot)$  defines an investor-option contract with message set  $E$ , and an equilibrium of it following any  $e \in E$  is given by the identity function,  $\iota(e) \equiv e$ . We now show that  $(e^*, \iota(\cdot))$  is an equilibrium of  $f^I$ .

Given  $f^I$  and  $e \in E$ , the equilibrium  $\iota(e)$  gives the investor payoff  $V^I(e)$ , and it gives the entrepreneur payoff  $U^I(e) \equiv H(V^I(e), e)$ . From (A4),

$$\begin{aligned} V^I(e) &\equiv \sup_{m_I} \hat{V}(f(m_E^*, m_I), e) \\ &\geq \sup_{m_I} \inf_{m_E} \hat{V}(f(m_E, m_I), e) = V^*(e), \end{aligned}$$

using (6). Hence, (4) and the presumption that each  $H(\cdot, e)$  is a decreasing function imply that  $U^I(e) \leq U^*(e)$  for all  $e \in E$ , with equality at  $e = e^*$  because  $V^I(e^*) = V^*(e^*)$ . Thus,  $e^*$  maximizes  $U^I(\cdot)$  because it maximizes  $U^*(\cdot)$ . This proves that  $(e^*, \iota(\cdot))$  is an equilibrium of  $f^I$ . The equality of the equilibrium payoffs,  $(V^I(e^*), U^I(e^*)) = (V^*(e^*), U^*(e^*))$ , is obvious. ■

The following lemma is used to prove Lemmas 4 and 5.

**Lemma A2.** *For any  $(r, e) \in C \times E$ , unique debt contracts  $\delta, \hat{\delta} \in C$  exist such that  $V(\delta, e) = V(r, e)$  and  $U(\hat{\delta}, e) = U(r, e)$ .*

**Proof.** The first claim is that the following equation in  $D$  has a unique solution:<sup>22</sup>

$$\sum g_i(e)v(\min(\pi_i, D)) = \sum g_i(e)v(r_i). \quad (\text{A7})$$

The RHS of this equation is not less than  $v(\underline{r})$  by LI, and not more than  $\sum g_i(e)v(\pi_i)$  by LE. Since  $\underline{r} < \pi_n$ , the interval  $[\underline{r}, \pi_n]$  is not empty. Viewed as a function of  $D$  on this interval, the LHS of (A7) is continuous and strictly increasing, and it takes on all values in  $[v(\underline{r}), \sum g_i(e)v(\pi_i)]$ . So (A7) has a unique solution  $D$  in  $[\underline{r}, \pi_n]$ . The debt contract  $\delta$  with face value  $D$  is the desired debt contract. As it obviously satisfies MI, LE, and LI,  $\delta \in C$ .

The second claim is that the following equation in  $\hat{D}$  has a unique solution:

$$\sum g_i(e)u(\pi_i - \min(\pi_i, \hat{D}), e) = \sum g_i(e)u(\pi_i - r_i, e).$$

The RHS of this equation is not more than  $\sum g_i(e)u(\pi_i - \underline{r}, e)$  by LI, and not less than  $u(0, e)$  by LE. On  $[\underline{r}, \pi_n]$ , the LHS is continuous and strictly decreasing in  $\hat{D}$ , and it takes on all values in  $[\sum g_i(e)u(\pi_i - \underline{r}, e), u(0, e)]$ . So again a unique  $\hat{D} \in [\underline{r}, \pi_n]$  exists that yields the desired debt contract  $\hat{\delta} \in C$ . ■

**Proof of Lemma 4.** Lemma A2 shows the existence of the debt contract  $\delta$  satisfying  $V(r, e) = V(\delta, e)$ . Define  $x \in \mathbb{R}^n$  by  $x_i \equiv v(r_i) - v(\delta_i)$ . Then,  $\sum g_i(e)x_i = 0$ , and  $x \neq 0$  since  $r$  is not debt. Because LE implies  $v(r_i) \leq v(\delta_i)$  for low  $i$ , and MI implies  $v(r_i) \geq v(\delta_i)$  for high  $i$ ,  $x$  is quasi-monotone. Lemma A1 now implies the rest of the proposition, since  $V(r, e') - V(\delta, e') = \sum g_i(e')x_i$ . ■

**Proof of Theorem 2.** Continuing from the text, now let  $e^\delta$  be the largest equilibrium effort of  $\delta$ , i.e., the largest maximizer of  $U^\delta(\cdot)$ .<sup>23</sup> In the text we proved that  $U^\delta(e^\delta) \geq U^*(e^*)$ , and that the theorem is proved once we show that  $e^\delta \geq e^*$ . Consider any  $e < e^*$ .

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<sup>22</sup>The one exception is when (A7) is solved by  $D = \pi_n$ , for then it is solved by all  $D > \pi_n$  as well. But these all yield the same debt contract,  $\delta_i = \pi_i$  for all  $i$ .

<sup>23</sup>This  $e^\delta$  exists: since  $U^\delta(\cdot)$  is continuous on the compact set  $E = [\underline{e}, \bar{e}]$ , its set of maximizers is nonempty and compact.

Let  $\delta'$  be the debt contract determined by

$$V(r^*(e), e) = V(\delta', e). \quad (\text{A8})$$

Then

$$V(\delta', e^*) \leq V(r^*(e), e^*) \leq V(r^*(e^*), e^*) = V(\delta, e^*),$$

where the first inequality follows from Lemma 4 (ii) and  $e < e^*$ ; the second from the incentive compatibility of  $r^*(\cdot)$  for the investor; and the third is (11). The face value of  $\delta'$  is thus no more than that of  $\delta$ . Hence,  $V(\delta', \cdot) \leq V(\delta, \cdot)$ . This and (A8) imply  $V(r^*(e), e) \leq V(\delta, e)$ . Therefore, since  $V^*(e) = V(r^*(e), e)$ , we have proved that

$$V^*(e) \leq V(\delta, e) \text{ for all } e < e^*. \quad (\text{A9})$$

Now, if  $e^\delta < e^*$ , then

$$U^*(e^*) \geq U^*(e^\delta) = H(V^*(e^\delta), e^\delta) \geq H(V(\delta, e^\delta), e^\delta) = U^\delta(e^\delta),$$

where the second inequality comes from (A9) and  $H_1 < 0$ . But then  $U^\delta(e^*) = U^\delta(e^\delta)$ , contrary to  $e^\delta$  being the largest maximizer of  $U^\delta(\cdot)$ . This proves that  $e^\delta \geq e^*$ . ■

The following lemma is used to prove Theorem 3. The background assumptions are those of the general model in Section 4; in particular, the lemma does not assume entrepreneur-offer renegotiation.

**Lemma A3.** *Given a contract  $f$  and any  $e \in E$ , let  $m^*(e)$  be an equilibrium of the message game following  $e$ . Denote the corresponding equilibrium payoff of the investor as  $V^*(e)$ . Then, for any sequence  $\{e^k\}$  converging to some  $e^*$ , and any limit point  $\tilde{r}$  of the sequence  $\{\tilde{r}^k\} = \{f(m^*(e^k))\}$ , we have  $\hat{V}(\tilde{r}, e^*) = V^*(e^*)$ .*

**Proof.** To simplify notation, let  $m^* = m^*(e^*)$  and  $\tilde{r}^* = f(m^*)$ . Message  $m_I^*(e^k)$  is a best reply to  $m_E^*$  in the message game following  $e^k$ . Hence,

$$\hat{V}(\tilde{r}^k, e^k) \geq \hat{V}(\tilde{r}^*, e^k),$$

where  $\bar{r}^k = f(m_E^*(e^k), m_I^*)$ . Similarly,  $m_E^*$  is a best reply to  $m_I^*$  in the game following  $e^*$ . Since the entrepreneur wishes to minimize  $\hat{V}$ , this implies

$$\hat{V}(\tilde{r}^*, e^*) \leq \hat{V}(\bar{r}^k, e^*).$$

Reversing the “ $k$ ” and “ $*$ ” in this argument yields two more inequalities:

$$\hat{V}(\tilde{r}^*, e^*) \geq \hat{V}(\hat{r}^k, e^*) \text{ and } \hat{V}(\tilde{r}^k, e^k) \leq \hat{V}(\hat{r}^k, e^k),$$

where  $\hat{r}^k = f(m_E^*, m_I^*(e^k))$ . Combine these four inequalities to obtain

$$\hat{V}(\bar{r}^k, e^k) - \hat{V}(\bar{r}^k, e^*) \leq \hat{V}(\hat{r}^k, e^k) - \hat{V}(\tilde{r}^*, e^*) \leq \hat{V}(\hat{r}^k, e^k) - \hat{V}(\hat{r}^k, e^*). \quad (\text{A10})$$

Since  $C$  is compact, there exists a subsequence  $\{\tilde{r}^{k_j}\}$  that converges to  $\tilde{r}$ , and for which  $\{\bar{r}^{k_j}\}$  and  $\{\hat{r}^{k_j}\}$  both converge. Taking limits in (A10) along this subsequence, and using the continuity of  $\hat{V}(\cdot, \cdot)$ , we conclude that  $\hat{V}(\tilde{r}, e^*) = \hat{V}(\tilde{r}^*, e^*) = V^*(e^*)$ . ■

**Proof of Theorem 3.** Consider the certainty equivalent contract  $\bar{f}$  that has the same equilibrium  $(e^*, m^*(\cdot))$  and corresponding payoffs as  $f$ . We need only show that  $\lim_{e \rightarrow e^* -} \bar{f}(m^*(e))$  exists, and that it is a debt contract  $\delta$ . This is because the certainty equivalent of any  $\tilde{r} \in \Delta C$  is debt if and only if either the support of  $\tilde{r}$  is that debt contract alone (when the face value of  $\delta$  exceeds  $\pi_1$ ), or the support of  $\tilde{r}$  contains only riskless debt contracts. To simplify notation, we henceforth assume  $f = \bar{f}$ , i.e.,  $f$  specifies only non-random simple contracts.

To simplify notation more, let  $m^* = m^*(e^*)$  and  $r^* = f(m^*)$ . The equilibrium payoffs are  $V^*(e^*) = V(r^*, e^*)$  and  $U^*(e^*) = H(V^*(e^*), e^*)$ . Let  $\delta$  be the debt contract determined by (11). Given  $\delta$  the entrepreneur’s post-renegotiation payoff following any  $e \in E$  is  $U^\delta(e) = H(V(\delta, e), e)$ . Hence, using (11),

$$U^\delta(e^*) = H(V^*(e^*), e^*) = U^*(e^*). \quad (\text{A11})$$

Let  $e^\delta$  be the maximizer of  $U^\delta(e) = H(V(\delta, e), e)$  for which Theorem 2 implies  $V(\delta, e^\delta) \geq V^*(e^*)$  and  $U^\delta(e^\delta) \geq U^*(e^*)$ . By assumption, neither of these inequalities is strict, and so

$$U^\delta(e^\delta) = U^*(e^*). \quad (\text{A12})$$

Since  $e^* \in \text{int}(E)$ , the derivative  $U^{\delta'}(e^*)$  exists and is given by

$$U^{\delta'}(e^*) = H_1(V(\delta, e^*), e^*)V_e(\delta, e^*) + H_2(V(\delta, e^*), e^*). \quad (\text{A13})$$

Now, let  $\{e^k\}$  be a sequence converging from below to  $e^*$ , and set  $r^k = f(m^*(e^k))$ . Let  $r$  be a limit point of  $\{r^k\}$ . We shall show that  $r = \delta$ . Since  $C$  is compact, this will imply  $r^k \rightarrow \delta$ , proving the theorem.

Note first that Lemma A3 implies  $V(r, e^*) = V(r^*, e^*)$ . Thus, (11) implies  $V(\delta, e^*) = V(r, e^*)$ . From (A13), therefore, we have

$$U^{\delta'}(e^*) = H_1(V(r, e^*), e^*)V_e(\delta, e^*) + H_2(V(r, e^*), e^*). \quad (\text{A14})$$

Now, observe that since  $e^*$  maximizes  $U^*(\cdot)$ ,

$$0 \leq U^I(e^*) - U^I(e^k) = H(V(r^*, e^*), e^*) - H(V(r^k, e^k), e^k). \quad (\text{A15})$$

We also have

$$\begin{aligned} V(r^*, e^*) &= V(f(m^*), e^*) \geq V(f(m_E^*, m_I^*(e^k)), e^*) \\ &\geq V(f(m_E^*, m_I^*(e^k)), e^k) \\ &\geq V(f(m_E^*(e^k), m_I^*(e^k)), e^k) = V(r^k, e^k), \end{aligned}$$

where the first inequality comes from  $m_I^*$  being a best reply to  $m_E^*$  following  $e^*$ ; the second inequality comes from MLRP,  $e^* > e^k$ , and the simple contract  $f(m_E^*, m_I^*(e^k))$  satisfying MI; and the third inequality comes from  $m_E^*(e^k)$  being a best reply to  $m_I^*(e^k)$  following  $e^k$ , and the fact that this best reply minimizes  $V(f(\cdot, m_I^*(e^k)), e^k)$ . Thus, from (A15) and  $H_1 < 0$  we obtain

$$0 \leq H(V(r^k, e^*), e^*) - H(V(r^k, e^k), e^k).$$

Since  $H$  and  $V(r^k, \cdot)$  are differentiable, the mean value theorem applied twice yields

$$0 \leq \left\{ H_1(V(r^k, \bar{e}^k), \bar{e}^k)V_e(r^k, \hat{e}^k) + H_2(V(r^k, \bar{e}^k), \bar{e}^k) \right\} (e^* - e^k), \quad (\text{A16})$$

where both  $\bar{e}^k$  and  $\hat{e}^k$  are in  $(e^k, e^*)$ . Since  $H_1$ ,  $H_2$ , and  $V$  are continuous functions, dividing (A16) by  $e^* - e^k > 0$ , and taking the limit along the subsequence for which



$r^k \rightarrow r$ , yields

$$0 \leq H_1(V(r, e^*), e^*)V_e(r, e^*) + H_2(V(r, e^*), e^*). \quad (\text{A17})$$

Assume now that  $r \neq \delta$ . Then, since  $V(\delta, e^*) = V(r, e^*)$ , Lemma 4 (i) implies  $V_e(r, e^*) > V_e(\delta, e^*)$ . This,  $H_1 < 0$ , and (A17) imply

$$0 < H_1(V(r, e^*), e^*)V_e(\delta, e^*) + H_2(V(r, e^*), e^*) = U^{\delta'}(e^*),$$

using (A13) for the equality. But  $U^{\delta'}(e^*) > 0$ , contradicts (A11) and (A12). So  $r = \delta$ , and the proof is finished. ■

**Proof of Lemma 5.** Lemma A2 shows the existence of the debt contract  $\delta$  satisfying  $U(r, e) = U(\delta, e)$ . Now assume SEP, and define  $x \in \mathbb{R}^n$  by  $x_i \equiv \bar{u}(\pi_i - \delta_i) - \bar{u}(\pi_i - r_i)$ . Then

$$\sum g_i(e)x_i = a(e)^{-1} [U(\delta, e) - U(r, e)] = 0. \quad (\text{A18})$$

Since  $r$  is not debt,  $x \neq 0$ . Suppose  $x_i > 0$ . Then  $\delta_i < r_i$ . Since  $\delta_i = \min(\pi_i, D)$  and  $r_i \leq \pi_i$ , we have  $\delta_i = D < \pi_i$ . Hence,  $\delta_{i+1} = D < r_i \leq r_{i+1}$ , using MI for the last equality. This implies  $x_{i+1} > 0$ . Continuing in this fashion proves that  $x_j > 0$  for all  $j > i$ , and so  $x$  is quasi-monotone. Now Lemma A1 (i) and SEP imply

$$U_e(\delta, e) - U_e(r, e) = a'(e) \sum g_i(e)x_i + a(e) \sum g'_i(e)x_i > 0,$$

using (A18) and  $a(e) > 0$ . This proves (i). For  $e' \neq e$ , Lemma A1 (ii) implies  $(e - e') \sum g_i(e')x_i < 0$ , and so

$$(e - e') [U(r, e') - U(\delta, e')] = -a(e')(e - e') \sum g_i(e')x_i > 0.$$

This proves (ii). ■

**Proof of Proposition 4.** By the argument of Theorem 1 and the revelation principle, we can restrict attention to deterministic revelation mechanisms for the investor that are incentive compatible. Let  $r^* : E \rightarrow C$  be such a mechanism. Its truthful equilibrium gives the entrepreneur a post-renegotiation payoff of  $U(r^*(e), e)$ . Let  $e^*$  maximize this.

The entrepreneur's equilibrium payoff is then  $U^* \equiv U(r^*(e^*), e^*)$ . By Lemma 5, a debt contract  $\delta^* = \delta(D^*)$  exists such that

$$U(\delta^*, e^*) = U^*. \quad (\text{A19})$$

By the maximum theorem, the function defined by  $\bar{U}(D) \equiv \max_{e \in E} U(\delta(D), e)$  is continuous. By (A19),  $\bar{U}(D^*) \geq U^*$ . Since  $r^*(e^*)$  satisfies LE,

$$\bar{U}(\pi_n) = u(0, \underline{e}) \leq U(r^*(e^*), \underline{e}) \leq U^*.$$

Hence,  $\hat{D} \in [D^*, \pi_n]$  exists such that  $\bar{U}(\hat{D}) = U^*$ . The desired debt contract is  $\hat{\delta} \equiv \delta(\hat{D})$ . Letting  $\hat{e}$  be the largest maximizer of  $U(\hat{\delta}, \cdot)$ , we have

$$U(\hat{\delta}, \hat{e}) = U^*. \quad (\text{A20})$$

The entrepreneur's equilibrium payoff is thus the same from  $\hat{\delta}$  as from  $r^*(\cdot)$ . We now need only to prove that  $\hat{e} \geq e^*$ .

Assume otherwise, so  $\hat{e} < e^*$ . For any  $e$ , let  $\gamma(e) \equiv U(\hat{\delta}, e) - U(r^*(\hat{e}), e)$ . Note that

$$\begin{aligned} \gamma(\hat{e}) &= U(\hat{\delta}, \hat{e}) - U(r^*(\hat{e}), \hat{e}) \\ &= U^* - U(r^*(\hat{e}), \hat{e}) \\ &= U(r^*(e^*), e^*) - U(r^*(\hat{e}), \hat{e}) \geq 0, \end{aligned}$$

using (A20) and the fact that  $e^*$  maximizes  $U(r^*(\cdot), \cdot)$ . Furthermore,

$$\begin{aligned} \gamma(e^*) &= U(\hat{\delta}, e^*) - U(r^*(\hat{e}), e^*) \\ &< U^* - U(r^*(\hat{e}), e^*) \\ &\leq U^* - U(r^*(e^*), e^*) = 0. \end{aligned}$$

(The first inequality is due to  $\hat{e} < e^*$  being the largest maximizer of  $U(\hat{\delta}, \cdot)$ . The second is due to  $r^*(\cdot)$  being incentive compatible for the investor and the renegotiation game being strictly competitive, so that  $U(r^*(\cdot), e^*)$  is minimized by  $e^*$ .) Hence, since  $\gamma(\cdot)$  is continuous,  $e \in [\hat{e}, e^*)$  exists such that  $\gamma(e) = 0$  and  $\gamma(e') < 0$  for all  $e' \in (e, e^*]$ . But by Lemma 5, this is impossible. ■

**Proof of Theorem 4.** This proof builds on that of Proposition 4, which applies because SEP holds when the entrepreneur is risk neutral.<sup>24</sup> By Proposition 1, any debt contract is a first-best risk-sharing contract (in  $C$ ) for any effort. Thus, for any debt contract  $\delta$  and effort  $e$ ,

$$J(U(\delta, e), e) = V(\delta, e). \quad (\text{A21})$$

Therefore, in the notation of the proof of Proposition 4, the switch from  $r^*(\cdot)$  to the debt contract  $\hat{\delta}$ , given that the equilibria with the efforts  $e^*$  and  $\hat{e}$  are played (respectively), causes the investor's payoff to increase by

$$\begin{aligned} J(U^*, \hat{e}) - J(U^*, e^*) &= J(U(\hat{\delta}, \hat{e}), \hat{e}) - J(U(\delta^*, e^*), e^*) \\ &= V(\hat{\delta}, \hat{e}) - V(\delta^*, e^*) \\ &\geq V(\hat{\delta}, e^*) - V(\delta^*, e^*) \geq 0, \end{aligned}$$

where the first equality comes from (A20) and (A19); the comes from (A21); the first inequality holds because  $\hat{e} \geq e^*$ , MLRP, and the fact that  $\hat{\delta}$  satisfies MI; and the final inequality holds because the face values satisfy  $\hat{D} \geq D^*$ . The investor thus agrees with the entrepreneur that the equilibrium of  $\hat{\delta}$  in which effort  $\hat{e}$  is taken is at least weakly preferred to the equilibrium of  $r^*(\cdot)$  in which  $e^*$  is taken. ■

**Proof of Theorem 5.** Let  $e^*$  be the effort taken in the general contract's equilibrium. Let  $\hat{\delta} = \delta(\hat{D})$  be the debt contract of Proposition 4, so that it has an equilibrium in which the effort is some  $\hat{e} \geq e^*$ , and the entrepreneur's payoff is  $U(\hat{\delta}, \hat{e}) = U^*$ . The investor's payoffs in the two equilibria are  $J(U^*, e^*)$  and  $J(U^*, \hat{e})$ . Since the equilibrium of the debt does not Pareto dominate the initial equilibrium,  $J(U^*, e^*) \geq J(U^*, \hat{e})$ . If this is an equality, case (ii) holds. So assume  $J(U^*, e^*) > J(U^*, \hat{e})$ . Then  $e^* < \hat{e}$ . These two inequalities, together with SP, imply  $e^{**} < \hat{e}$ . We now show that (i) holds.

Let  $e^{**} \in E$  be a maximizer of  $J(U^*, \cdot)$ . We present a simple contract  $r^* \in C$  that has an equilibrium in which the entrepreneur chooses  $e^{**}$ , and  $U(r^*, e^{**}) = U^*$ . This

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<sup>24</sup>Integrating  $u_{11}(w, e) \equiv 0$  twice shows that the entrepreneur is risk neutral if and only if functions  $c(\cdot)$  and  $a(\cdot) > 0$  exist such that  $u(w, e) = a(e)w - c(e)$ .

equilibrium achieves an efficient allocation that gives the entrepreneur payoff  $U^*$ , since the renegotiation provides first-best risk-sharing given  $e^{**}$ , and the entrepreneur does not gain from the renegotiation. Since the equilibrium of the general contract that has effort  $e^*$  is not Pareto dominated by an equilibrium of  $r^*$ , and the entrepreneur has the same payoff in both equilibria, so must the investor. Thus, the equilibrium of the general contract also achieves an efficient allocation.

It remains to prove this  $r^*$  exists. First, note that a wage  $w \in [0, \pi_n - \hat{D}]$  is defined by  $u(w, \underline{e}) = U^*$ .<sup>25</sup> Since  $U^*$  satisfies WA,  $w \leq \pi_1 - \underline{r}$ , and so  $r^w \in C$ . Now define, for any  $b \in B \equiv [0, w]$  and  $t \in T \equiv [0, \pi_n - \hat{D} - w]$ , a simple contract  $r(b, t)$  by

$$r_i(b, t) \equiv \begin{cases} \pi_i - b & \text{for } \pi_i \leq b + \hat{D} \\ \hat{D} & \text{for } b + \hat{D} < \pi_i \leq \pi_n - t \\ \pi_i + \hat{D} + t - \pi_n & \text{for } \pi_n - t < \pi_i. \end{cases}$$

Then  $r(0, 0) = \hat{\delta}$  and  $r(w, \pi_n - \hat{D} - w) = r^w$ . In general  $r(b, t) \in C$ : it satisfies MI by construction, LE because  $b \geq 0$ , and LI because  $b \leq w \leq \pi_1 - \underline{r}$ . Figure 6 shows the entrepreneur's income as a function of output under the contracts  $\hat{\delta}$ ,  $r(b, t)$ , and  $r^w$ .

For each  $(b, t) \in B \times T$ , let

$$e(b, t) \equiv \arg \max_{e \in E} U(r(b, t), e).$$

As  $E$  is compact, the maximum theorem and FOA imply that  $e(\cdot, \cdot)$  is a well-defined continuous function on  $B \times T$ , as is the maximized utility,

$$\bar{U}(b, t) \equiv U(r(b, t), e(b, t)).$$

Now, as is easy to show,<sup>26</sup> for each  $t \in (0, \pi_n - \hat{D} - w]$  there exists a unique  $b(t) \in B$  such that  $\bar{U}(b(t), t) = U^*$ . Since  $\bar{U}(\cdot, \cdot)$  is continuous,  $b(\cdot)$  is continuous on  $(0, \pi_n - \hat{D} -$

<sup>25</sup>As  $0 \leq \pi_i - \hat{\delta}_i$  for each  $i$ , we have  $u(0, \underline{e}) \leq U(\hat{\delta}, \underline{e}) \leq U(\hat{\delta}, \hat{e}) = U^*$ . Since  $\pi_n - \hat{D} \geq \pi_i - \hat{\delta}_i$ ,  $u(\pi_n - \hat{D}, \underline{e}) \geq u(\pi_n - \hat{D}, \hat{e}) \geq U(\hat{\delta}, \hat{e}) = U^*$ . So a unique  $w \in [0, \pi_n - \hat{D}]$  satisfying  $u(w, \underline{e}) = U^*$  exists.

<sup>26</sup>For example, consider the case  $\hat{D} \geq \pi_1$ . Then  $r_1(\cdot, t)$  strictly decreases on  $B$ , and  $r_i(\cdot, t)$  is non-increasing on  $B$  for each  $i > 1$ . It follows that  $\bar{U}(\cdot, t)$  strictly increases on  $B$ . Since  $\bar{U}(\cdot, t)$  is also continuous, and  $r_i(0, t) \geq \hat{\delta}_i$  implies  $\bar{U}(0, t) \leq U(\hat{\delta}, e(0, t)) \leq U(\hat{\delta}, \hat{e}) = U^*$ , and  $r_i(w, t) \leq r_i^w$  implies  $\bar{U}(w, t) \geq U(r(w, t), \underline{e}) \geq U(r^w, \underline{e}) = U^*$ , for each  $t \in T$  there exists one and only one  $b \in B$  such that  $\bar{U}(b, t) = U^*$ . The argument is similar for the case  $\hat{D} < \pi_1$ , although then  $b(t)$  is unique only for  $t > 0$ .

$w]$ . Define  $b(0)$  so that  $b(\cdot)$  is continuous on  $T$ . Then  $\bar{U}(b(t), t) = U^*$  for all  $t \in T$ . Furthermore,  $e(b(t), t)$  is continuous on  $T$ . Because  $r(b(0), 0) = \hat{\delta}$ , we have  $e(b(0), 0) = \hat{e} > e^{**}$ . Because  $r(b(\pi_n - \hat{D} - w), \pi_n - \hat{D} - w) = r^w$ , we have  $e(b(\pi_n - \hat{D} - w), \pi_n - \hat{D} - w) = \underline{e} \leq e^{**}$ . Thus,  $t^* \in T$  exists such that  $e(b(t^*), t^*) = e^{**}$ . The desired simple contract is  $r^* \equiv r(b(t^*), t^*)$ . ■

## B. Appendix B: Third Parties and Mixed Strategies

Our results in the text apply to situations in which there are no active third parties, and the players do not play mixed message strategy equilibria. In this appendix we give two examples in which non-debt contracts perform better than debt when either restriction is relaxed, even when the entrepreneur has the renegotiation bargaining power.

### The Value of Third Parties

As is common in contract theory, introducing a risk neutral third party can be beneficial – provided she is not susceptible to collusion. The third party does not even need to be able to observe the effort. Consider the following example.

The investor is risk neutral. Let  $(r^{w^*}, e^*)$  be an efficient allocation, where  $r^{w^*}$  is the wage contract that pays the entrepreneur a wage  $w^*$ . Let  $R = \{s^E, s^T\}$  be an investor-option contract according to which the investor, after the effort has been chosen, selects either scheme  $s^E$  or scheme  $s^T$  to determine the incomes of the three parties as a function of output. According to scheme  $s^E$ , the entrepreneur is paid the fixed wage  $w^*$  for any output, and the third party is paid nothing. The resulting incomes when output  $\pi_i$  is realized are then  $w^*$ ,  $\pi_i - w^*$ , and 0 for the entrepreneur, investor, and third party, respectively. According to scheme  $s^T$ , the entrepreneur is paid nothing and the third party is paid an amount  $r_i^T$  if output  $\pi_i$  is realized. So  $s^T$  yields, when  $\pi_i$  is realized, incomes of 0,  $\pi_i - r_i^T$ , and  $r_i^T$  for the entrepreneur, investor, and third party, respectively. The payment  $r_i^T$  is, for each  $i \leq n$ , given by

$$r_i^T = a + b\pi_i, \tag{B1}$$

where  $a$  and  $b$  are chosen so that  $0 < b < 1$ , and

$$a + b \sum g_i(e^*)\pi_i = w^*. \quad (\text{B2})$$

Note that under these schemes, each party's income is nondecreasing in output, and satisfies the entrepreneur's liability constraint LE. Furthermore, LI is satisfied if  $\underline{r} \leq (1 - b)\pi_1 - a$ . Any liability constraint for the third party is satisfied if her liability limit is less than  $a + b\pi_1$ .

Because of MLRP, we see that the expected payment to the third party under  $s^T$ ,

$$\sum g_i(e)r_i^T = a + b \sum g_i(e)\pi_i,$$

increases with the entrepreneur's effort  $e$ . It exceeds the wage  $w^*$  the investor must pay the entrepreneur under  $s^E$  if and only if  $e > e^*$ . The investor, since she wishes to minimize the expected payment she must make, will select the scheme  $s^T$  that pays the entrepreneur nothing if she observes that he chose an effort less than  $e^*$ . She will select the scheme  $s^E$  that pays him  $w^*$  if he chooses any  $e > e^*$ . Consequently, so long as  $u(w^*, e^*)$  is not less than the entrepreneur's minimal possible utility of  $u(0, \underline{e})$ , this investor-option contract has an equilibrium in which the entrepreneur takes effort  $e^*$  and the investor selects scheme  $s^E$ . This equilibrium achieves the efficient allocation  $(w^*, e^*)$ . The contract is also renegotiation proof: regardless of which effort the entrepreneur chooses or scheme the investor selects, there is no scheme that will make all three parties better off.

Let us not overemphasize this example. It is well known that adding a third party can improve on renegotiation-proof schemes between two parties, since making a third party a contingent claimant can eliminate ex-post inefficiencies. However, if the entrepreneur and the investor have an informational advantage over the third party about the effort, they may be able to collude so as to mis-report the effort – in this case we are back in the two-party case. The three-person contract also breaks down if the entrepreneur and third party can collude, whereby the entrepreneur shirks in return for an under-the-table compensation from the third party. Moreover, third parties may have costs of their own. such as the cost of acquiring information about the environment, etc. In

any case, the problem of third parties is not specific to this paper, but applies to the contract literature generally.

### **The Value of Mixed Message Strategies**

Mixed strategy equilibria of two-sided message game contracts may also perform better than debt. This is because at least one party is risk averse, and so the Pareto frontier defined by  $H(\cdot, e)$  is strictly concave. Hence, even though the message game is strictly competitive because all its payoff pairs are on this frontier, it is not constant-sum. A mixed strategy equilibrium of the message game generates a convex combination of frontier payoff pairs that lies below the frontier. This allows the mechanism to punish both players if one of them misreports the effort. A central difficulty in two-person implementation theory is thereby avoided, namely, how to punish one player for deviating without rewarding the other player so much that he will want to deviate.

One might think that if the players foresaw that they would be playing a mixed strategy equilibrium of the upcoming message game, they would want to renegotiate to a contract with a Pareto superior equilibrium beforehand. But this is not necessarily the case. The message game in the example below has, for any effort, a mixed strategy equilibrium that achieves an efficient contract for that effort, and so it will not be renegotiated at the interim stage.

Nonetheless, for two reasons we think mixed strategy equilibria are problematic. First, their implementation requires the contract-enforcing entity to take an unusually active role. Because there are situations in which the players will have “ex post regret” after their mixed message strategies are realized, the messages they send will need to be certified to prevent them from claiming that they had actually sent different messages. (In the example below, ex post regret occurs with positive probability when the effort reports differ.) Such certification is not a typical court function. Second, the investor may need to make large payments to the entrepreneur, thereby violating her liability limit  $\underline{r}$  unless it is very small.

The example is for the following case of our general model. The bargaining power in

the renegotiation stage is shared in any way. The investor is risk neutral. Her liability limit is so small it will not bind (e.g.,  $\underline{r} = -\infty$ .) The entrepreneur's utility function is  $u(w) - c(e)$ , where  $u' > 0$ ,  $u'' < 0$ , and

$$u'(w) \rightarrow 0 \text{ as } w \rightarrow \infty. \quad (\text{B3})$$

Two necessary conditions for  $(r^*, e^*)$  to be an equilibrium allocation is for  $r^*$  to be a wage contract, and for it to give the entrepreneur a payoff no less than if he were paid nothing and chose the minimal effort,  $u(0) - c(\underline{e})$ . We show that these conditions together are also sufficient. Any wage contract itself implements the minimal effort, and so we restrict attention to non-minimal efforts. Accordingly, let  $e^* \in (0, \bar{e}]$  and  $w^* \in (0, \infty)$  be an effort and wage that satisfy

$$u(w^*) - c(e^*) > u(0) - c(\underline{e}). \quad (\text{B4})$$

We construct a two-sided message game contract that achieves  $(w^*, e^*)$  via a mixed strategy equilibrium.

The message of player  $i = E, I$  in this contract is a two-tuple,  $(e_i, x_i) \in E \times [0, 1] \equiv M_i$ . The player's effort report is  $e_i$ . If the effort reports agree, say  $e_E = e_I = e$ , the contract requires the investor to pay the entrepreneur the wage

$$w_a(e) \equiv \begin{cases} w^* & \text{if } e = e^* \\ \underline{w} & \text{if } e \neq e^*, \end{cases} \quad (\text{B5})$$

where  $\underline{w} \in (0, w^*)$  and

$$u(w^*) - c(e^*) \geq u(\underline{w}) - c(\underline{e}). \quad (\text{B6})$$

(By (B4), this  $\underline{w}$  exists.) The numbers  $x_E$  and  $x_I$  form a "jointly controlled lottery" (Aumann et al., 1968) that comes into play if the reported efforts are not the same. Let  $y \equiv x_E + x_I - \lfloor x_E + x_I \rfloor$  be the fractional part of  $x_E + x_I$ . Then, if  $e_E \neq e_I$ , the investor pays the wage

$$w_d(x_E, x_I) \equiv \begin{cases} 0 & \text{if } y \leq p \\ \hat{w} & \text{if } y > p, \end{cases} \quad (\text{B7})$$

where  $\hat{w}$  and  $p \in (0, 1)$  are numbers to be determined.



This defines a deterministic contract,  $f : M_E \times M_I \rightarrow C$ . It prescribes a wage contract for any message pair. Thus, since here a wage contract shares risk efficiently for any effort, renegotiation will not occur after the messages are sent, on or off the equilibrium path, regardless of how the bargaining power in the renegotiation stage is shared. The prescribed wage contract determines both parties' payoffs.

We now find numbers  $\hat{w}$  and  $p$  such that an equilibrium of the message game played after the choice of any effort  $e$  is for each player to report truthfully  $e_i = e$ , and to choose  $x_i$  randomly from a uniform distribution on  $[0, 1]$ . By (B5) and (B6), it will then follow that  $e^*$  is an equilibrium effort, and the equilibrium contract is the wage contract with wage  $w^*$ .

First, we verify that the strategies of choosing  $x_E$  and  $x_I$  from uniform distributions are mutual best replies. Note that when  $x_j$  is uniformly distributed,  $y$  will also be uniformly distributed regardless of how  $x_i$  is chosen. Thus, neither player can affect the distribution of  $y$  so long as the other player chooses  $x_j$  uniformly. The strategies that choose  $x_E$  and  $x_I$  uniformly are therefore mutual best replies.

**Remark.** Since the players report the true effort, their randomizations do not come into play on the equilibrium path. Their equilibrium payoffs are deterministic, given by the efficient wage contract  $r^{w_a(e)}$ . Thus, as we claimed above, renegotiation will not occur at the interim stage before messages are sent, regardless of which effort was chosen.

It remains to find  $(\hat{w}, p)$  such that each player will report any effort truthfully, given the equilibrium strategy of the other player. The investor is willing to report any effort truthfully if

$$(1 - p)\hat{w} \geq w^*. \tag{B8}$$

Similarly, the entrepreneur is willing to report any effort truthfully if

$$u(\underline{w}) \geq pu(0) + (1 - p)u(\hat{w}). \tag{B9}$$

We thus need only to find some  $(\hat{w}, p)$  satisfying (B8) and (B9). For any  $\hat{w}$ , let (B8) determine  $p$  by setting

$$p = 1 - \frac{w^*}{\hat{w}}.$$

It remains to show the existence of  $\hat{w} > w^*$  (so that  $0 < p < 1$ ) such that

$$u(\underline{w}) \geq \left(1 - \frac{w^*}{\hat{w}}\right) u(0) + \left(\frac{w^*}{\hat{w}}\right) u(\hat{w}). \quad (\text{B10})$$

As  $\hat{w} \rightarrow \infty$ , the right side of this inequality converges to

$$u(0) + w^* \lim_{\hat{w} \rightarrow \infty} \frac{u(\hat{w})}{\hat{w}} = u(0),$$

since  $u(\hat{w})/\hat{w} \rightarrow 0$  (obviously if  $u(\cdot)$  is bounded above, and otherwise by (B3) and L'Hôpital's rule). Thus, since  $\underline{w} > 0$ , (B10) holds for any sufficiently large  $\hat{w} > w^*$ . ■

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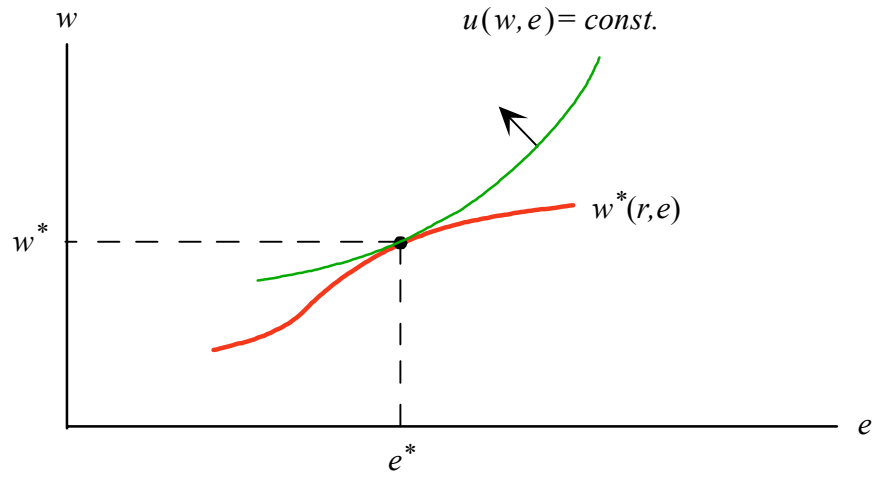


Figure 1: The outcome of contract  $r$  is  $(e^*, w^*)$ .

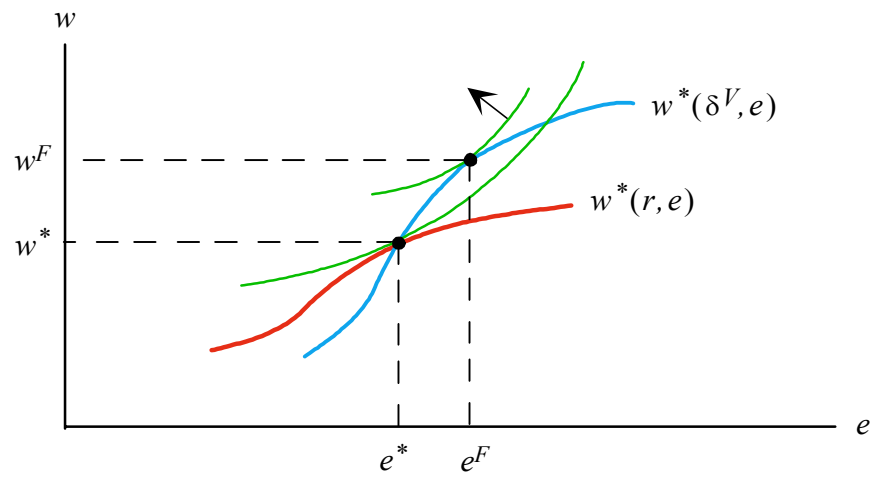


Figure 2: Contracts that are not riskless debt induce too little effort, when the entrepreneur has the bargaining power during renegotiation.

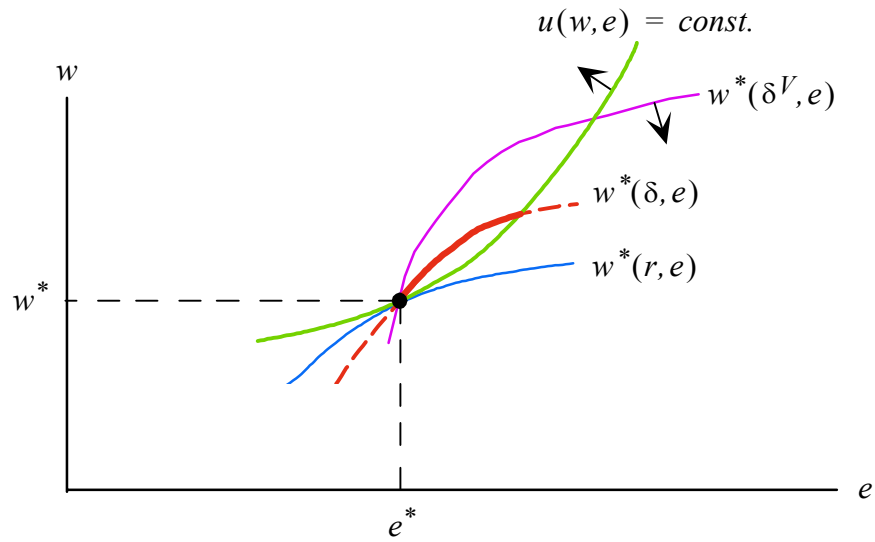


Figure 3: Proof of Proposition 2.

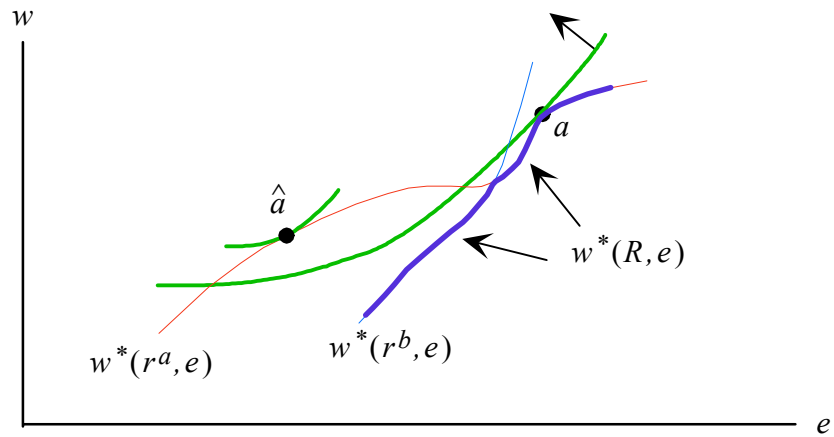


Figure 4: The outcome of the simple contract  $r^a$  is the low-effort  $\hat{a}$ . But the outcome of the investor-option contract  $R = \{r^a, r^b\}$  is the high-effort  $a$ .



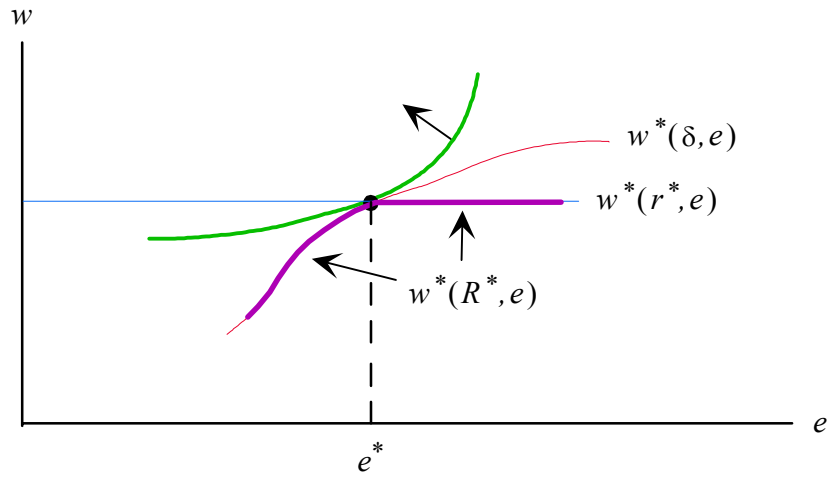


Figure 5: Adopting and renegotiating the debt contract  $\delta$  is equivalent to adopting the convertible debt contract  $R^*$ .

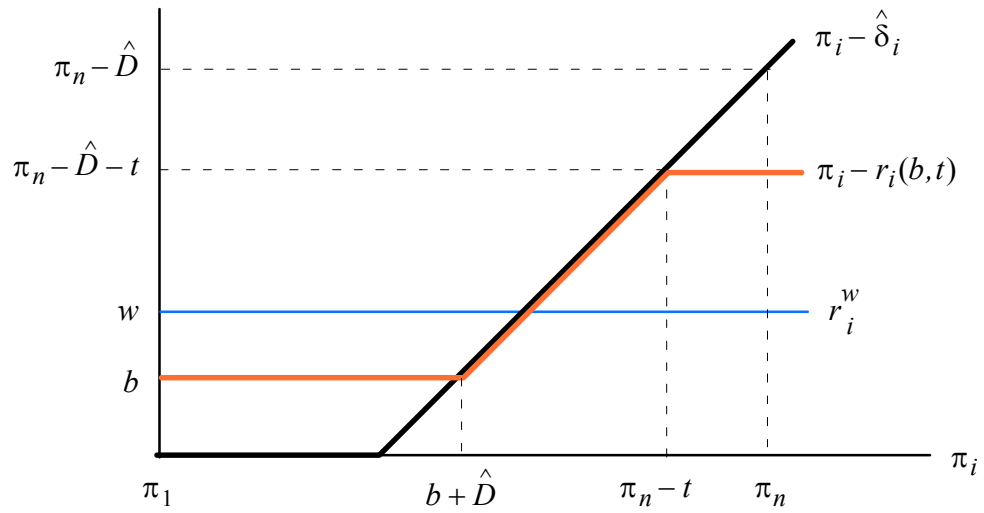


Figure 6: The entrepreneur's income as a function of output  $\pi_i$ , from the contracts  $\hat{\delta}$ ,  $r^w$ , and  $r(b, t)$  used to prove Theorem 5.