# Optimal Portfolio Analysis for the Czech Republic, Hungary and Poland During 1994 – 1995 Period

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## Abstract

The objective of this paper is to assess the investment opportunities emerging in the newly developing stock markets of Eastern Europe. The Czech Republic, Hungary, and Poland, representative of the emerging stock markets of Eastern Europe, are examined from the perspective of a US investor who invests solely in the US markets. During the period November 24, 1994 to May 12, 1995, the most advantageous investment strategy is derived using optimization algorithms, comparing the optimal portfolio in the stock markets of a select group of Eastern European countries against the S&P 500 Index, representative of the US stock markets. Based on market volatility, sovereign risks, and foreign exchange the risks and rewards of investing in these countries are appraised. The results show that the risk-adjusted return, yielded from the optimal portfolio, exceeds or equals the return realizable from investing in stock markets with lesser degrees of risk.

**Portfolio Analysis of Eastern European Stock Markets**

**I. Introduction**

In 1993, the Warsaw Stock Exchange rose 800% in United States dollars and set a new world record. This paper provides an analysis of the risk return trade-off of investing in newly emerging Eastern European stock markets such as Poland, as well as Hungary and the Czech Republic.[1] An optimal portfolio is derived based on historic observations, and then evaluated utilizing reliable performance measures. In the end, to the dismay of those wishing to profit from the Eastern European markets, the results show this investment decision to be a far less grandiose prospect than originally conceived. Investors should invest at most up to 2% of their stock portfolio in these markets.

The most dramatic changes in Eastern Europe are associated with stock market developments in the Czech Republic, Hungary, and Poland. The opening of these markets has attracted both foreign investors and foreign financial institutions. The entrepreneurial private investor now plays an increasingly important role and is strongly being encouraged to enter the markets. Such developments have created the need for collection and dissemination of current, accurate data. The doors have opened to privatization acquisition and joint ventures accompanied with repatriation of earnings. At the same time, state enterprises are undergoing major changes. Some state enterprises have been dismantled while others are being reconfigured as holding companies with the injection of new government capital, or split into separate companies that may be viable. Still others have been completely liquidated. It has been said that in no place in the world has there been so dramatic a shift in both culture and economic institutions. As economic restructuring takes place, studying the performance of these Eastern European stock markets is a worthwhile undertaking. The Czech Republic, Hungary, and Poland have attracted attention from potential Western investors because of their willingness to accept economic change and their apparent determination to pursue systematic structural reforms. Subsequent to the creation of the new Czech Republic in January 1993,[2] The Prague Stock Exchange opened in April 1993 and presently has 10 listed issues and 976(!) unlisted issues.[3] Foreign direct investment, with the notable exception of Volkswagen in the Skoda enterprise, has generally been small, and usually made by Czechs living abroad.[4]

Of the three markets in this study, Hungary has been the prime location in Eastern Europe for Western investment, and as a result, it has been the largest recipient of foreign capital in Eastern Europe. The introduction of new and higher levels of taxation, however, has resulted in a loss of its attraction for foreign investors.[5] Furthermore, Hungary's dependency on private financial markets means that it has to continuously reassure its foreign lenders of its economic soundness (see Tai). The Hungarian currency, the forint, is internally convertible. This convertability means that any Hungarian can open a foreign currency account and firms are able to repatriate their profits, including the initial hard currency investment. There are no limits on the amount of profit a firm may convert back into hard currency. Such policies reduce the risk to foreign investors.

The Budapest Stock Exchange was the first Eastern European exchange to open in 1990. It operates for two and a half hours each day, five days a week. Trading volume is low. Only six or seven shares of the 28 listed are actively traded on the exchange.[6] Liquidity is low - the stock exchange was capitalized at $US 811 million at year-end 1993. Share prices are subject to wild fluctuations as the small size of the market tends to exaggerate any activity. Total equity traded in 1993 amounted to $US 181 million. Nonetheless, the Budapest Stock Exchange is experiencing growth. It was estimated that 70 percent of all equity transactions in 1993 were originated from foreign investors drawn by low valuations.

The Warsaw Stock Exchange was opened in 1991, and as mentioned previously, rose 800% in United States dollars in 1993, setting a new world record. Foreign investment accounts for 25-30 percent of the $US 2 billion capitalization of the market. The stock market has only 23 stocks listed. The market is considered to be too small for institutional investors who believe it to be overpriced and over-regulated, although Western pension funds and institutional investors are now beginning to enter. The growth in the Warsaw stock market capitalization has been spectacular, from $US 142 million at the end of 1991 to an estimated $US 2 billion at the end of 1993, see Paliwoda, (1995), Dobosiewicz, (1992).[7] Poland's foreign currency accounts are difficult to open which leaves companies exposed to the vagaries of the domestic currency, the zloty.[8] It is necessary to use cash whenever possible as the currency quickly devalues. On the relations between Stock Return

Differentials and Exchange Rates, see Ajayi, Mehdian and Shachmurove (1996).

Investors willing to assume the additional risk present in these markets, have been well compensated. Yet, many market analysts have pointed out that such markets are abnormal, in that they tend to be characterized as thin, narrow, and driven by poorly informed individuals rather than by fundamentals. It cannot be assumed, however, that investing in emerging stock markets is, on the whole, riskier than investing in more developed countries (see, for example, Friedman and Shachmurove, 1996, 1997, Shachmurove, 1996). What can be concluded, on the other hand, is that the international investor is better off investing in a diversified portfolio rather than restricting his investment efforts to a single emerging market that is currently yielding high returns. The reason why global diversification is effective is that stock markets are not highly correlated and thus investing in them reduces the overall risk of the portfolio (see Tang and Choi, 1998, and Aizenman, 1997).

The remainder of the paper is organized as follows: Section II discusses theoretical issues. Section III presents the empirical results. Finally, section IV provides a summary.

## II. Theoretical Concepts

This section presents a brief survey of the theoretical concepts used in this paper. The theoretical concepts are optimization algorithms and portfolio evaluation techniques. Optimization algorithms are mathematical procedures that solve multiple variable problems simultaneously. The results are optimal given the information provided in the formulation of the problem. Funds are allocated into different investments in such a way that return is maximized for a given variability or risk. In order to screen investments according to their return and risk characteristics, a few statistical measures are used. These statistics include geometric mean, variance, beta, and lower partial moment (LPM).[9] These have been found to offer adequate measures of the return and risk inherent in an investment, see Levy and Sarnat (1984). In this case, the budget constraint is that all allocations will sum up to 100 percent of the available total investment. In addition to the variance, both beta and LPM statistics can be formulated and used in quadratic programming analysis.[10] The ranking of assets by their risk/return statistics provides an initial screen of individual assets (see Shawky, Kuenzel and Mikhail, 1997).

Optimization algorithms only provide tradeoffs between risk and return. There will be optimized high return - high risk portfolios, optimized medium return - medium risk portfolios, and optimized low return - low risk portfolios. At this point, the portfolio holder has to decide which portfolio will maximize the utility of the investor. Evaluation techniques are used to assess the optimal solutions derived by comparing them to other investment alternatives such as the S&P 500, or a portfolio consisting of equally weighted initial allocations of the assets present in the optimal portfolio derived (see, for example, Shachmurove, 1998A, 1998B).

## 1.     Optimization Algorithms
In this paper two methods for choosing the optimal portfolio are presented. The first is the Markowitz Variance-Covariance Analysis. The second method is the Lower Partial Moment Analysis.

## 1.A.     Markowitz Variance-Covariance Analysis
Markowitz (1959) developed the basic variance-covariance analysis. Low or negative correlations between assets are used to reduce the overall variability or risk of the portfolio. The variance of the portfolio is calculated as follows:

$$(1) \qquad Vp = \sum_{i=1}^{k} \sum_{j=1}^{k} X_i \cdot X_j \cdot Cov_{ij},$$

where Vp is the portfolio variance, k the number of assets in the portfolio, X the share of asset i or j within the portfolio, and $Cov_{ij}$ the covariance between assets i and j, and is calculated by:

$$(2) \qquad Cov_{ij} = \sigma_i \cdot \sigma_j \cdot r_{ij},$$

where $\sigma_i$ is the standard deviation for asset i, and $r_{ij}$ the correlation coefficient between assets i and j.

The expected return of the portfolio is determined by:

$$k$$

(3)    $Ep = \sum X_i \cdot E(R_i),$

$\quad\quad\quad i=1$

where Ep is the expected return of the portfolio, and $E(R_i)$ the expected return for asset i.

Using the above formulas, quadratic programming is set up to maximize return and minimize variance as follows:

(4)    $Min\ z = Vp - \lambda \cdot Ep$

$\quad\quad\quad\quad\quad k$

$\quad\quad s.t, \quad \sum X_i = 1,$

$\quad\quad\quad\quad\quad i=1$

where $\lambda$ is the slope of the objective function. The term $\lambda$ can be varied from zero to infinity in order to solve for different points on the efficient frontier.[11]  The result of these portfolios is that they map the efficient frontier, where each portfolio represents the lowest risk for a given return or the highest return for a given risk, see Markowitz, (1959).

### 1.B.    Lower Partial Moment (LPM) Analysis

In Lower Partial Analysis (LPM), the variance is simply replaced with the lower partial moment, (or, with the semivariance, which is a special case of lower partial moment with n=2).[12]  The same expected return and risk equations hold true as does the quadratic formulation as follows:

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad k \quad\quad\quad k$

(5)    $LPM_{2,p} = \quad \sum \quad \sum X_i \cdot X_j \cdot SD_i \cdot SD_j \cdot r_{ij}$

$\quad\quad\quad\quad\quad\quad\quad\quad i=1 \quad\quad j=1$


(6)    $Min\ z = LPM_{2,p} - \lambda \cdot Ep,$

where $LPM_{2,p}$ is the semivariance of portfolio p, k the number of assets, SDi the semideviation (square root of the semivariance) for asset i, and $r_{ij}$ the correlation between assets i and j, see Bawa, (1975), Fishburn, (1977), and Nawrocki, (1991).

## 2. Portfolio Evaluation Measures

After a portfolio has been selected, its performance needs to be evaluated. Performance measures that account for both risk and return need to be computed. Portfolio evaluation measures consist of Terminal Wealth, Sharpe's Utility Measure, Sharpe, Treynor and Jensen Measures, Reward to Semivariance, and Stochastic Dominance.

### 2.A.    Terminal Wealth

The Terminal Wealth Measure answers the following question: How much money did the investor make? Terminal wealth is the k-th power of the geometric mean, or simply the product of the individual returns. It is the only important performance measure for long term evaluation. This is a result of the fact that risk-return measures are not accurate because of the decreasing importance of liquidity risk as the investment horizon increases.

$$(7) \qquad \text{Terminal Wealth} = \prod_{t=1}^{k} R_t$$

where $\Pi$ is the multiplication operator, k the number of periods and $R_t$ the rate of return at time period t.

### 2.B.    Sharpe's (1966) Utility Measure

The Sharpe Utility Measure uses an estimate of the investor's risk tolerance rather than the riskless rate of return as an indicator of the investor's utility function. The risk tolerance ranges from zero to one. The higher the risk tolerance the higher the proportion of the portfolio invested in riskier assets, see Sharpe and Alexander, (1990). The measure is defined as follows:

$$(8) \qquad \text{Utility} = \text{Return - (Variance/Risk Tolerance)}.$$

Risk tolerance is defined as the amount of risk an investor is willing to assume. The risk tolerance is determined by the nature of each particular investor. Investors who are risk-

averse tolerate lower amounts of risk compared to their risk-neutral and risk-loving counterparts. Risk-averse investors penalize the expected rate of return of a risky investment by a certain percentage to reflect the risk involved. Risk-neutral investors look solely at the expected returns of investments, thus risk levels are not a factor for them. Finally, risk-loving investors adjust expected returns upwards when there is risk present, see Bodie, Kane and Marcus, (1993).

## 2.C.    Sharpe (1966), Treynor (1966) and Jensen (1968) Measures

The Sharpe (1966), Treynor (1966), and Jensen (1968) Measures are defined as follows:

$$(9) \qquad \text{Sharpe} = (R_p - R_f) / \sigma_p$$

$$(10) \qquad \text{Treynor} = (R_p - R_f) / \beta_p$$

$$(11) \qquad \text{Jensen } (a_p) = (R_p - R_f) - \beta_p(R_m - R_f) - e_t,$$

where $R_p$ is the return on the portfolio, $R_f$ the riskless rate of return, $\sigma_p$ the standard deviation of the portfolio, and $\beta_p$ is the portfolio's beta.

Both the Sharpe and the Treynor Measures use reward to risk ratios. The Sharpe Measure, uses standard deviation in its denominator, while the Treynor Measure uses the beta value. The Jensen Alpha, which is based on the Capital Asset Pricing Model (CAPM), looks at investment performance by calculating the intercept ($a_p$) of the regression line: $R_p - R_f = a_p + \beta_p(R_m - R_f) + e_t$. This variable ($a_p$) is called the Jensen Alpha of the portfolio. When the portfolio fares better than the market, $a_p$ is greater than 0. When it under-performs the market, $a_p$ is less than 0. If $a_p$ is positive and significantly different than zero, the portfolio is considered successful. On the other hand, if $a_p$ is less than zero, the portfolio is a failure. Therefore, the higher the value of $a_p$, the greater the abnormal rate of return achieved by the portfolio in excess of the market, see Jensen, (1968), and Levy and Sarnat, (1984). These three measures are difficult to estimate since they are statistically biased, see Ang and Chua, (1979). The effect of the bias is that each of the measures may rank the performance of a group of portfolios differently from the other measures.

## 2.D.    Reward to Semivariance

Reward to Semivariance is defined as follows:

(12)    Reward to Semivariance = $(R_p - R_f)/SD_p$,

where $SD_p$ is the semideviation of the portfolio.  This ratio is preferred over alternative ones as studies have revealed that the Sharpe (1966), Treynor (1966), and Jensen (1968) Measures are statistically biased. Various causes of the biases have been proposed.  These causes are: the existence of unequal borrowing and lending rates, the failure to consider higher moments of return distributions, and the elusive "true" holding period, see Ang and Chua, (1979). The shortcoming of this ratio is that it assumes a fixed utility function by setting n=2.   This shortcoming can be overcome by utilizing the more general reward-to-LPM ratio, as the degree, n, can then be manipulated to match the investor's utility function, see Klemkosky, (1973).

## 2.E.    Stochastic Dominance

Stochastic dominance is an effective evaluation technique for judging the performance of portfolios, due to the fact that it does not make any assumptions concerning the underlying probability distribution of security returns, and is based on a very general utility function. The disadvantage of stochastic dominance models is that they do not take the correlations between assets into account.  First Degree Stochastic Dominance (FSD) places no restrictions on utility functions except that they be non-decreasing.  Thus, FSD acts as a preliminary screening that eliminates those options that no rational investor would choose.   Second Degree Stochastic Dominance (SSD) applies only to risk-averse investors by assuming a concave utility function.  All efficient sets included in SSD are also present in FSD, but not necessarily vice versa.  Finally, Third Degree Stochastic Dominance (TSD) further assumes decreasing absolute risk aversion, and hence is only applicable to yet a smaller group of investors.   Decreasing absolute risk aversion means that the risk premium an investor is willing to pay to get rid of a given risk decreases as his wealth increases.  This implies that he

becomes more risk-neutral at higher levels of wealth, see Porter, Wart and Ferguson, (1973), Francis and Archer, (1979), Francis, (1980), Saunders, (1980), Elton and Gruber, (1984), and Levy and Sarnat, (1984).

### III.    Empirical Results

The database consists of observations from January 1, 1988 till May 12, 1995. For the purposes of this paper, an optimal portfolio for the period ranging from November 24, 1994 till May 12,1995 is used as the basis of the following discussion. This period was chosen for discussion for several reasons. One reason is that studies completed on it have resulted in the largest number of optimal portfolios for any period, and are consequently, more likely to reveal the optimal asset allocation. Another is that the T-bill rate for that period was the lowest rate recorded over the span of the study, which encourages investors to turn to more active investment strategies in hopes of securing higher returns. The average U.S Treasury bill interest rate of 6.03 percent is assumed to represent the risk free interest rate for that period. The stock markets of Hungary, Poland, and the Czech Republic are studied and their performances contrasted with the S&P 500.[13] The objective is to evaluate the investment opportunities presented by these emerging markets from the perspective of a US investor who invests solely in the US markets.

Table 1 indicates that there are four different portfolios on the optimization frontier, each of which optimizes one or more particular variables. These variables are annual return, periodic return, standard deviation, probability of loss, utility, shortfall probability, and the Reward/Semivariance (R/SV) ratio. The algorithm that is used is the Markowitz Critical Line Algorithm which computes corner portfolios on the efficient frontier, see Markowitz, (1959). Of the four optimal portfolios on the frontier, the one with the highest Reward/Semivariance ratio is the optimal one for the purpose of this study. This is portfolio number 2. It has a R/SV ratio of 0.41 which significantly exceeds the corresponding R/SV ratios of the three other portfolios. An annual return of 37.36 percent can be made on an investment in this portfolio.

Table 2 shows the component securities of the optimal portfolio which was presented in Table 1: S&P 500 - 97.75% percent, Poland - 2.25 percent. The portfolio has a standard

deviation of 0.44 percent. It has a shortfall probability, defined as the probability of realizing a return below the risk free rate, of 0.41. It is somewhat of a surprising result that the optimal portfolio is so little diversified. This interesting result is in contradiction with some empirical papers which point out the benefits of diversifying in emergent markets, see Harvey (1995). When one takes into account other risks, however, such as foreign exchange and sovereign risks which are discussed later, it is found that of the four investment vehicles studied in this paper, the S&P 500 is the most attractive.

Table 3 provides a short summary of the individual assets. The annualized return in Hungary is -6.67 percent with a standard deviation of 1.07 percent. In Poland, the return is 83.91 percent and deviation 2.52 percent. In the Czech Republic, the return is -50.54 percent annually with a deviation of 0.95 percent. It is not surprising that the Polish market is the most volatile of the markets, given the high return that it offers. There is also some additional risk inherent in investing in foreign stocks markets that is not reflected in the stated standard deviation. Both an inadequate legal infrastructure and inadequate General Accepted Accounting Principles, compounded by different clearing and settlement procedures, turn one's investment decision into a more risky venture. Moreover, there are additional risks which foreign investors need to consider before venturing into these markets. These risks are foreign exchange risk and sovereign risk. Foreign exchange risk is defined as the risk that a return denominated in a foreign currency will have a decreased value in the domestic currency due to a movement between the two currencies. Sovereign risk refers to the risk of a foreign government interceding in its market and acting in a manner that has an adverse impact on one's investments, see Grabbe, (1991). These risks are present in the Polish market, as well as in the other Eastern European markets. Shortfall probabilities are 0.52 in Hungary, 0.47 in Poland, and 0.63 in the Czech Republic.

Table 4 shows that the portfolio beta is 0.99, very close to the market beta of 1.0. The Sharpe Measure is 0.24, the Treynor Measure is 0.10 and the Jensen Alpha value is 0.002. To understand these results better, they are compared to the corresponding market values. The Sharpe Measure for the S&P 500 is 0.23, the Treynor Measure is 0.10 and the Jensen value is, by definition, 0. The portfolio, therefore, provides more reward per unit of risk, whether variance or beta, than does the S&P 500.

The results are also compared to those computed for a portfolio consisting of equally weighted initial allocations to all securities in the optimal portfolio. The optimal portfolio outperforms the equally weighted portfolio on all counts. The performance measures considered are periodic return, Sharpe Measure, Treynor Measure, Jensen Alpha, beta, T-test, R-squared test, terminal wealth, portfolio utility and the Reward/SemiVariance (R/SV) ratio. Furthermore, the portfolio provides a higher return than that predicted by the Capital Asset Pricing Model (CAPM), given its beta and the average market return. Since the Jensen Measure is greater than zero, this means that the portfolio performs better than the market. These results are summarized in Table 4.

The conclusions mentioned above are summarized in Figures 1 through 9. Figure 1 illustrates the variance covariance efficient frontier, see Markowitz, (1959). Figures 2-4 reveal how well the investment fared in the Czech Republic, Hungary and Poland respectively. Figure 5 shows the portfolio utility with respect to time for the optimal portfolio (EV2), the equally weighted portfolio, and the S&P 500. Figure 6 does the same with the Sharpe ratio, while Figures 7, 8 and 9 deal with the Treynor Measure, Jensen Alpha and R/SV ratio, respectively.

In addition to the variance-covariance analysis, the Lower Partial Moment algorithm is applied to the optimal portfolio. The Lower Partial Moment algorithm computes the LPM/CLPM (Lower Partial Moment/ Covariance Lower Partial Moment) matrix, given the investor's level of risk aversion. Table 5 shows that the application of this algorithm to the data creates an optimal portfolio that provides an annual return of 37.15 percent, and has a R/SV ratio of 0.41. These results are practically identical to the return and R/SV ratio generated by the critical line algorithm.

Table 6 shows that the portfolio is composed of allocations in the following proportions: 98.26 percent S&P 500, 1.74 percent Poland. In terms of portfolio allocation, the results generated from the Lower Partial Moment algorithm are similar to those generated from the critical line algorithm, described above.

Finally, to complete the analysis, the risk/return performance of the securities in the portfolios is evaluated by utilizing First, Second, and Third Degree Dominance techniques. Table 7 lists the assets for each degree of dominance, and displays their corresponding

statistical variables. The best risk/return performance is provided by those securities listed under Third Degree Dominance. Under First Degree Dominance all the assets except the Czech Republic are included. The Czech Republic is not included because it has a lower likelihood of achieving the same level of return as the other markets, given a specific level of risk. Under Second and Third Degree Dominance, only Poland and the S&P 500 are listed. The reason why the other securities are not included under Second and Third Degree Dominance is that the cumulative probability of either Poland or the S&P achieving a given return, each taken separately, minus the cumulative probabilities of the other securities achieving the same return, also taken separately, are always non-negative. The results of applying the Stochastic Dominance models confirm the composition of the portfolio arrived at by using both the optimal Markowitz Variance-Covariance and the Lower Partial Moment models.

## IV.    Conclusion

This paper studies the daily stock market returns of three Eastern European countries, and the prospect of investing in them for the purposes of diversification from the perspective of a US investor. The period November 24, 1994 to May 12, 1995 is used as the basis of the analysis. An optimal portfolio is generated, and then evaluated with appropriate performance measures. The optimal portfolio, generated through the application of the Markowitz Critical Line algorithm, is one that allocates 97.75 percent in S&P 500, and 2.25 percent in Poland. It has an annualized return of 37.36 percent, a R/SV ratio of 0.41, a standard deviation of 0.44 percent and a shortfall probability of 0.41. The portfolio's beta is 0.99, which is virtually identical to the corresponding market beta of 1.0, which means that the portfolio is as volatile as the market (as represented by the S&P 500). The Sharpe Measure is 0.24, the Treynor Measure is 0.10 and the Jensen Alpha is 0.002.

A prudent investor evaluates the merits of an investment opportunity, even when the portfolio chosen seems to achieve a desired result. In efficient markets, securities are never priced inefficiently. If an investment yields an annual return of 37.36 percent, for a risk level that appears to be below that of lower returning assets, then one of the assumptions is wrong. The return is either not consistently as high as one initially believed, or one's perceived risk of

the higher yielding asset is lower than it actually is.

The focus of this analysis is the Eastern European markets. While the Reward-to-Risk ratios might be appealing, based on stock return volatilities, additional risk factors need to be both examined and accounted for. There is inherent risk in foreign investments. The risk manifests itself in two forms: foreign exchange risk and sovereign risk. The optimal portfolio derived above incorporates both of these risks, since it is based on allocation into foreign securities. Consequently, investors must demand a risk premium in order to be compensated for the additional risk they are bearing.

Nonetheless, it is important for the international investor to hold a well diversified portfolio, rather than to concentrate his investments in a single market. Due to the fact that stock markets are not highly correlated, their movements are not perfectly synchronized. Consequently, investing in a portfolio consisting of allocations in several markets, gives an investor the ability to diversify away the risk of an adverse movement in a given market having a substantial effect on the return of his portfolio.

Based on the Markowitz Variance-Covariance model and the more general Lower Partial Moment model, as well as Second and Third Degree Stochastic Dominance models, it seems that foreign investors should refrain from investment in these markets, except for about 2 percent of their total investment allocated to the Polish stock market.

The paper may be viewed as an independent test of market efficiency. As already has been noted, the stock market index in Poland increased by 800 percent in 1993, which set a world record for that year. Still two years later, the results of the optimal portfolios in this paper for the period November 1994 through May 1995, show that investors should invest no more than two percent in the Polish market. The conclusion to be made here is that if one takes foreign exchange and sovereign risks into account, these emerging markets do not significantly improve on the alternative of investing in the S&P 500.

## Appendix

### Statistical Measures

The statistical measures used are: geometric mean, variance, beta, and lower partial moment (LPM).

## 1. Geometric Mean

For the k numbers a, b, c, d, e, and f, the geometric mean is:

(A.1)    $[a \bullet b \bullet c \bullet d \bullet e \bullet f]^{(1/k)}$

For the purpose of determining rates of return, the method of computing a geometric mean is more accurate than a simple arithmetic mean, since it takes into account the compounding nature of interest over time.

## 2. Variance

(A.2)    $\sigma_i^2 = (1/k) \cdot \sum_{t=1}^{k} [R_{it} - E(R_i)]^2$,

where $R_{it}$ is the return to asset i in period t, and $E(R_i)$ the expected geometric mean return for asset i. Variance measures the magnitude of deviations from the mean. The greater the deviations, the greater the level of risk. Variance is important in the evaluation of potential investments. For a risk-averse individual choosing between two investments with equal expected returns, the investment with the lower variance is more attractive. Consequently, investments with higher risk - i.e. higher variance - must offer higher expected returns to compensate investors for the additional risk, see Markowitz (1959).

## 3. Beta

The beta (ß) of an asset measures the variability of an asset relative to the market index. It is a popular risk measure, and has been widely used for the past 25 years. It was developed to make the Modern Portfolio Theory (MPT) model operational, which is computationally complex when the variance is used. ß is determined using the following regression:

(A.3)    $R_{it} = a_i + ß_i \cdot R_{mt} + e_t$

(A.4)    $\sigma_e^2 = (1/k) \sum_{t=1}^{k} e_t^2$,

where,

(A.5) $\quad\quad\quad e_t = R_{it} - [a_i + \beta_i \cdot R_{mt}],$

$R_{it}$ is the return on asset i for period t, $a_i$ the intercept of the line, $\beta_i$ represents the slope of the line, and is defined as the tendency of the asset's returns to respond to swings in the broad market, $R_{mt}$ is the return to the market index for that same period t, and $e_t$ measures the deviation of $R_{it}$ from the line for period t. There are k observations, t = 1, 2, ..., k.

The beta of the market index $\beta_M$, is arbitrarily set at 1.0, and serves as a reference value with which to compare individual asset betas. If the beta of an asset is equal to 1.0, then both the asset and the market are equally risky, and will tend to move together. If $\beta$ is greater than 1.0, then the asset is more volatile than the market, and hence, more risky. If $\beta_i$ is less than 1.0, then the asset is less volatile than the market, and hence, less risky. Furthermore, beta also serves to determine the incremental risk an individual asset brings to a well diversified portfolio.

(A.6) $\quad\quad\quad \sigma_i^2 = \beta_i^2 \cdot \sigma_m^2 + \sigma_e^2$

The first component of the variance of an asset $(\beta_i^2 \cdot \sigma_m^2)$ is termed the systematic or non-diversifiable risk component, and is the risk inherent in the general market. The second component $(\sigma_e^2)$ is termed the unsystematic or diversifiable risk component, and can be diversified away as it is due not to the market in general, but rather, only to that particular asset, see Sharpe (1964).

## 4. Lower Partial Moment (LPM)

Both variance analysis and the use of betas to estimate risk levels presuppose a normally distributed set of securities and investors with quadratically defined utility functions. In order to address risk levels when these assumptions cannot confidently be made, the Lower Partial Moment (LPM) was developed. It was Harry Markowitz, see Markowitz, (1959) who first offered the use of semivariance analysis as a substitute for beta and variance analysis to handle skewed return distributions and investors who displayed utility functions

that were non-quadratic. Semivariance is a special case of LPM analysis, see Bawa (1975), and Fishburn (1977). Semivariance is defined as an n-degree LPM with n=2. The variable n refers to the degree that, deviations below a target return, are raised to.

$$\text{(A.7)} \qquad LPM_n(h) = (1/k)\cdot \sum_{t=1}^{k} Max(0, (h - R_t)^n),$$

where n is the degree of the LPM, h the target return the investor does not wish to go below, k the number of periods used to calculate the LPM, and $R_t$ the return for the asset for period t. A problem that often occurs when determining asset riskiness is the problem of non-normal distributions. For two distributions, one positively skewed and the other negatively skewed, it is possible that they both have the same mean and variance; that is, the variance measure might not differentiate between the two distributions. However, the LPM measure can handle non-normal distributions, and is able to differentiate between the two. In LPM analysis, n=1 is the boundary between risk-averse and risk-loving investors. If n>1, the investor is risk-averse and attempts to minimize risk for a given return, while for values of n<1, the investor is risk-loving and seeks additional risk. Furthermore, the use of LPM is less restrictive on assumptions of the investor's behavior than beta and variance analysis. It has been shown that the LPM can match the utility functions of investors who have been described in utility function literature. Decision makers in investment contexts frequently associate risk with failure to attain a target return. Examination of published utility functions which use the maximization of expected utility criterion lends support to the notion of a target return at which the utility undergoes a noticeable change. Depending on the context, the change point may be negative, zero or positive, see Fishburn, (1977).

## Endnotes

1. The dynamic linkages among the world's major markets have been studied since the late 1960s (Grubel, 1968, Granger and Morgenstern, 1970, Levy and Sarnat, 1970, Grubel and Fadner, 1971, Agmon, 1972, Bertoneche, 1979, Hilliard, 1979) and more recently (Schollhammer and Sands, 1985, Eun and Shim, 1989, Meric and Meric, 1989, Von Furstenberg and Jeon, 1989, 1991, Hamao, Masulis and Ng, 1990, Koch and Koch, 1991, Birati and Shachmurove, 1992, Chan, Gup and Pan, 1992, Malliaris and Urrutia, 1992,

Roll, 1992, and Friedman and Shachmurove, 1995).  While some have studied the East European economies, see Paliwoda, 1995, Dobosiewicz, 1992, Jedrzejczak, 1992, Kecskes, 1992, Schwartz and Tyson, 1992, and Svitek, 1992), this study is the first to investigate the dynamic linkages among national stock indexes of the newly emerging markets of Eastern Europe.

2.  The Czech Republic has a population of about 10.3 million.  The right wing coalition is committed to privatization and radical economic reform.  The Gross Domestic Product has risen by about one percent, inflation is about 10 percent (a Value Added Tax (VAT) was introduced in January 1993 increased prices by 23 percent) and the unemployment rate is 8 percent.

3.  A recent development is the establishment of a Business School in Prague with assistance from the University of Pittsburgh, USA, which has introduced a Masters in Business Administration (MBA) program.  This is an important development which will enhance the integration of the Czech financial market with the international arena.

4.  See, Business Eastern Europe, February 28, 1994.

5.  Hungary has a population of 10.3 million, similar to the Czech Republic and has relatively high unemployment and inflation rates, 12.6 percent and 22.5 percent, respectively.  Most of Hungary's foreign debt, unlike Poland's, is privately held.  This makes debt rescheduling, debt relief and debt forgiveness, which has been extended to countries such as Poland, not an option.

6.  Of these, only Ibusz is a privatized enterprise; Konzum, Skala-Coop, Strada-Skala, Novotrade, and Trade-Coop are all retail or trade cooperative enterprises.  Successful privatization include Fotex, a Hungarian-American joint venture involved in photographic services; the Muszi electronics cooperative; Dunaholding, a partly state-owned finance company; Danubius hotel and spa chain; Matav telecommunications, Martfu, a state-owned brewery; and Technoimpex, a state-owned enterprise.

7.  One can compare this figure to Thailand's market capitalization of $US 99 billion or the $US 124 billion capitalized in Mexico.

8.  Poland is the largest country in Eastern Europe with a projected population of 40 million by the year 2000.  Poland is the first post-communist country which has shown positive results from market transformation, consistently enjoying real economic growth over the last few years.  However, it has a 16 percent unemployment rate, and inflation is high running at over 45 percent on an annual base.

9.  These statistics are described in the Appendix.

10. There are two decisions that need to be made during portfolio allocations: choosing between asset classes such as stocks, bonds, foreign currency, etc. (strategic optimization) and choosing between securities in any given asset class (tactical optimization).  The

majority of investors prefer to optimize across asset classes, that is, they perform strategic optimization. Few, however, optimize within a given asset class, ignoring tactical optimization. There is evidence to support the concept of tactical optimization. For example, an equity market index with optimized allocations will outperform indexes with equal or value weighted allocations see Haugen, (1990a,b).

11. The algorithm used is the Critical Line Algorithm. It starts with the highest return portfolio which, by definition, includes the highest return asset. Each asset is then evaluated using a critical value (pivot conditions) to determine which is the next asset to enter the portfolio. As assets enter into the portfolio, it becomes more diversified and will have lower risk as well as return. Each portfolio derived is called a corner portfolio. A corner portfolio is when an asset either enters or exits the portfolio. The result of these corner portfolios is that they map the efficient frontier, where each portfolio represents the lowest risk for a given return or the highest return for a given risk.

12. See the Appendix.

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**Table 1 :  Markowitz Covariance Analysis**

| Name | Sec | Ann Ret. | Per Ret. | Std Dev. | ProbLoss | Utility | Pr(R<Rf) | R/SV |
|------|-----|----------|----------|----------|----------|---------|----------|------|
| EV1 | 1 | 83.91 | 0.24 | 2.52 | 0.46 | -0.01 | 0.47 | 0.15 |
| EV2 | 2 | 37.36 | 0.13 | 0.44 | 0.39 | 0.12 | 0.41 | 0.41 |
| EV3 | 3 | 31.98 | 0.11 | 0.42 | 0.40 | 0.10 | 0.42 | 0.35 |
| EV4 | 3 | 23.98 | 0.09 | 0.41 | 0.42 | 0.08 | 0.44 | 0.25 |

**Table 2 :  Characteristics of the Optimal Portfolio Chosen (EV2) Based Upon Reward/Semivariance Criteria (R/SV)**

| | |
|---|---|
| Annualized Return | 37.36 |
| Periodic Return | 0.13 |
| Standard Deviation | 0.44 |
| Semi Deviation | 0.25 |
| Skewness | -0.02 |
| Beta | 0.99 |
| Pr(R< 15.00% Annual) | 0.44 |
| Pr(R<  6.03% Annual) | 0.41 |
| Pr(R< 0.00% Annual) | 0.39 |
| Reward/Variance | 0.24 |
| Reward/Semivariance | 0.41 |
| Portfolio Utility | 0.12 |

| Portfolio #  2 EV2 | Allocations |
|--------------------|-------------|
| S&P 500 | 97.75 |
| Poland | 2.25 |

## Table 3 : Summary of Individual Assets

|  | S&P500 | Hungary | Poland | Czech Rep. |
|---|---|---|---|---|
| Annualized Return | 36.43 | -6.67 | 83.91 | -50.54 |
| Return - Daily | 0.12 | -0.03 | 0.24 | -0.28 |
| Standard Deviation | 0.44 | 1.07 | 2.52 | 0.95 |
| Semi Deviation | 0.25 | 0.85 | 1.51 | 0.83 |
| Beta | 1.00 | 0.28 | 0.77 | 0.63 |
| Skewness | 0.05 | -0.93 | 0.49 | 0.11 |
| Kurtosis | 4.13 | 7.30 | 3.57 | 4.40 |
| Terminal Wealth | 1.16 | 0.97 | 1.34 | 0.72 |
| Risk Penalty | 0.01 | 0.05 | 0.25 | 0.04 |
| Utility | 0.12 | -0.07 | -0.01 | -0.32 |
| Pr(Return < 0.00%) | 0.39 | 0.51 | 0.46 | 0.62 |
| Pr(Return < 6.03%) | 0.41 | 0.52 | 0.47 | 0.63 |
| Reward/Variance | 0.23 | -0.05 | 0.09 | -0.32 |
| Reward/Semivariance | 0.40 | -0.06 | 0.15 | -0.37 |
| Reward/Beta | 0.11 | -0.18 | 0.29 | -0.48 |

## Table 4 :  A Comparison Between the Optimal Portfolio, the S&P 500, and an Equally Weighted Portfolio

| Portfolio | Per. Ret. | Std. Dev. | Semivar. |
|---|---|---|---|
| EV2 | 0.13 | 0.44 | 0.25 |
| Equal | 0.04 | 0.85 | 0.59 |
| S&P 500 | 0.12 | 0.44 | 0.25 |

| Portfolio | Term Wlth | Utility | R/SV |
|---|---|---|---|
| EV2 | 1.16 | 0.12 | 0.41 |
| Equal | 1.04 | 0.01 | 0.03 |
| S&P 500 | 1.16 | 0.12 | 0.40 |

| Portfolio | Ann. Ret | Beta | Skewness |
|---|---|---|---|
| EV2 | 37.42 | 0.99 | -0.03 |
| Equal | 9.62 | 0.68 | 0.02 |
| S&P 500 | 36.43 | 1.00 | 0.05 |

\* Signifies Significant Skewness at Two Standard Deviations

| | EV2 | Equal | S&P 500 |
|---|---|---|---|
| Periodic Return | 0.130 | 0.040 | 0.120 |
| Standard Dev. | 0.440 | 0.840 | 0.440 |
| Sharpe Measure | 0.240 | 0.020 | 0.230 |
| Treynor Measure | 0.100 | 0.020 | 0.100 |
| Jensen Alpha | 0.002 | 0.000 | 0.000 |
| Beta | 0.990 | 0.680 | 1.000 |
| T-Test | 88.840 | 4.130 | 99.000 |
| R-Square | 0.980 | 0.130 | 1.000 |
| Terminal Wealth | 1.160 | 1.040 | 1.160 |
| Utility Measure | 0.120 | 0.000 | 0.120 |
| Reward/Semivariance | 0.410 | 0.020 | 0.400 |

## Table 5:  Lower Partial Moment Quadratic Programming Analysis (Degree = 2)

| Name | Sec | AnnRet | PerRet | SemiDev | ProbLoss | Utility | Pr(R<Rf) | R/SV |
|---|---|---|---|---|---|---|---|---|
| LPMQ1 | 1 | 83.91 | 0.24 | 1.51 | 0.46 | -0.01 | 0.47 | 0.15 |
| LPMQ2 | 2 | 37.15 | 0.13 | 0.25 | 0.39 | 0.12 | 0.41 | 0.41 |

**Table 6: Characteristics of the Optimal Portfolio Chosen (LPMQ2) Based Upon**

**Reward/Semivariance Criteria (R/SV)**

| | |
|---|---:|
| Annualized Return | 37.15 |
| Daily Return | 0.13 |
| Standard Deviation | 0.44 |
| Semi Deviation | 0.25 |
| Skewness | 0.00 |
| Beta | 0.99 |
| Pr(R< 15.00% Annual) | 0.44 |
| Pr(R< 6.03% Annual) | 0.41 |
| Pr(R< 0.00% Annual) | 0.39 |
| Reward/Variance | 0.24 |
| Reward/Semivariance | 0.41 |
| Portfolio Utility | 0.12 |

| Portfolio (LPMQ2) | Allocations |
|---|---:|
| S&P 500 | 98.26 |
| Poland | 1.74 |

## Table 7:  First, Second and Third Degree Stochastic Dominance Analysis

**First Degree Stochastic Dominance**

| Asset | Mean | Variance | Skew | Kurtosis | Min | Max |
|-------|------|----------|------|----------|-----|-----|
| Poland | 1.0028 | 0.0006 | 0.4588 | 3.5757 | 0.9909 | 1.0775 |
| S&P | 1.0013 | 0.0000 | 0.0417 | 4.1622 | 0.9872 | 1.0133 |
| Hungary | 0.9998 | 0.0001 | -0.9470 | 7.3895 | 0.9552 | 1.0314 |

**Second Degree Stochastic Dominance**

| Asset | Mean | Variance | Skew | Kurtosis | Min | Max |
|-------|------|----------|------|----------|-----|-----|
| Poland | 1.0028 | 0.0006 | 0.4588 | 3.5757 | 0.9909 | 1.0775 |
| S&P | 1.0013 | 0.0000 | 0.0417 | 4.1622 | 0.9872 | 1.0133 |

**Third Degree Stochastic Dominance**

| Asset | Mean | Variance | Skew | Kurtosis | Min | Max |
|-------|------|----------|------|----------|-----|-----|
| Poland | 1.0028 | 0.0006 | 0.4588 | 3.5757 | 0.9909 | 1.0775 |
| S&P | 1.0013 | 0.0000 | 0.0417 | 4.1622 | 0.9872 | 1.0133 |