Capital Flows and the Venture Financing Cycle*

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Abstract

We develop a market equilibrium model that captures the dynamics of venture financing in the presence of imperfections in the venture capital market. Specifically, we study the effects of frictions associated with search, bargaining, and segmentation from other asset classes on (i) fund size, (ii) pre-money valuation, and (iii) average returns to investors. By solving the model, we provide closed-form predictions about which ventures will receive funding and about how ownership shares will be divided between the Venture Capitalist (*VC*) and the entrepreneur. Using numerical simulations to perform parametric analyses, we find the following: If the *VC* can more easily find investment opportunities, pre-money valuations will decline, while fund size and returns will increase. Secondly, if the pool of potential investees improves in terms of the return prospects, fund size and average returns will also increase despite an increase in pre-money valuations. Third, over the long run, more *VCs* will enter the industry, and because of competition, industry returns will shrink to a point where a no-arbitrage condition holds.

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1 Introduction

The financial economics literature on venture capital sheds light on important characteristics of the industry.[1] Theoretical studies provide us with a considerable understanding about the implications of the moral hazard and on the adverse selection problems faced by venture capitalists.[2] However, research is sparse on market-wide frictions other than on asymmetry of information.

In this paper we study the effects of frictions associated with search, bargaining, and segmentation from other asset classes. Specifically, it takes time and other resources for an investor to find a suitable venture investment opportunity. Similarly, entrepreneurs need to spend time and resources in their search for a Venture Capitalist (*VC*) who is willing to fund their venture and contribute to its success. When a *VC* and an entrepreneur search, they are uncertain about whom they will encounter and their intrinsic qualities. Once a meeting occurs, both parties decide whether to match or not; and, if they do, they bargain bilaterally over the terms of the transaction. Thus, the price of VC investments is endogenously determined, depending both on idiosyncratic characteristics of the venture in hand and on overall properties of the market. In general, time considerations are relevant as well. This is particularly true for *VCs* who raise funds without knowing both how much time it will take to find an acceptable entrepreneurial venture to invest in and about how long it will then take for the venture to come to fruition.[3] And, there are, of course, costs to holding liquid assets for uncertain lengths of time.

These frictions are reflected in the dynamic model of the venture capital industry we develop in this study. Specifically, in our model, venture financing occurs in

[1]See Gompers and Lerner (2001, 2004) and Kaplan and Strömberg (2001) for comprehensive summaries of the literature.

[2]See for example Admati and Pfleiderer (1994), Amit, Glosten and Muller (1990), Bergemann and Hege (1998), Hellmann (1998), Kirilenko (2001).

[3]As Cochrane (2005) stated "The risk facing a venture capital investor is as much when his or her return will occur as it is how much that return will be".

stages of uncertain duration in a decentralized market. In the first stage of the venture capital cycle, *VCs* raise funds from investors (limited partners) given an ex-ante uncertainty about the entrepreneurs they will end up funding. Once funds have been raised, *VCs* and entrepreneurs embark on the next stage: They search for each other and, upon finding a match, decide whether or not to form a partnership. Next – and still while realized returns remain uncertain – *VCs* and entrepreneurs bargain over the share of the equity each party will own. Finally, when the venture comes to fruition, *VCs* exit the investment with their share of the realized value, and return to the fundraising stage of the cycle. Entrepreneurs, on the other hand, are assumed to realize their share of the venture and exit the market.

While the amount of funds raised by specific venture capitalists is an endogenous decision and is affected by external factors, general questions that remain unanswered include: (i) how much capital limited partners should invest in the venture capital industry and (ii) which ventures should receive funding, given the current state of the economy and investment prospects inside and outside of the venture-financing industry. By formulating a search-theoretic multistage model of the venture capital cycle, we can answer these questions. Furthermore, our analysis clarifies how market factors lead to changes in fundraising and in valuations of ventures. Finally, in combination with our numerical simulations, we highlight a number of empirical predictions.

The theoretical analysis provides: (i) the amount of capital raised by venture capitalists and (ii) their selection criteria for investing in entrepreneurial ventures. We determine the outcome of any bilateral meeting, including whether or not a venture will be implemented and, in case it will, how both parties will split the venture's equity. Additionally, we show that VCs' equity share linearly increases according to the investment cost; and, regardless of whether VCs receive more shares, the post-money valuation increases for ventures that require greater investments. We argue that if there is an exogenous inflow of capital to the venture financing market, this will increase the pool of possible investments, and VCs consequently will be more selective in their investment decisions.

In examining the annual commitments of capital to the industry, we note substantial variation from year to year.[4] We apply the model to understand the determinants of fluctuations in the amount of capital that is committed to the industry. For that simulate the model in two ways: first, we consider the short term, where the number of VCs in the industry is assumed to be fixed; and second, we simulate the long term, where the number of VCs in the industry adjusts to ensure that a no-arbitrage condition holds.

In the short run we found that better venture prospects yield a greater present value of investing in the VC industry, even though the pre- and post-money valuations increase. As well, when VCs have a higher probability of encountering entrepreneurs, there will be more capital flowing into the industry, but this time the valuations will decrease. Finally, when the risk-free asset yield increases, less capital will be allocated to the VC market and fewer ventures will be funded in the short run. In the long run, however, VCs will exit the market, reducing the competition for ventures and raising the returns from investing in VC.

One of the questions that have puzzled market analysts concerns why investment opportunities in the private equity market are not priced solely on the expected present value of future returns. Our modeling approach suggests an answer: Market participants cannot simply pick their most desirable investment. Rather, they encounter opportunities over time and, by comparing their estimated gain with the expected cost of searching again, they decide whether to accept or reject a particular match. As our paper suggests, both the decision to match and the decision about how

[4] For example, according to Dow Jones/Venture Source (2006), in 1997 $17 billion were committed to U.S. venture capital firms. This amount tripled by 1999 to $57.6, and in 2000 it reached an all-time high of $83.5 billion. Following the "burst of the bubble," new capital commitments to the industry declined to $13.1 billion in 2002 and to only $9.4 billion in 2003. By 2005, fundraising by VCs had recovered and reached close to $30 billion. Similar fluctuations have been recorded in the amounts of capital invested in companies and in the number of deals that closed.

to divide up equity share ownership depend on market-wide factors. Thus, the pre- and post-money valuations hinge not only on the venture's expected present value but also on characteristics of the entire market.

The theoretical model presented in this paper, which captures empirically observed market imperfections, corroborates empirical evidence about market behavior. Gompers and Lerner (2000) suggest a positive relationship between the valuation of venture capital investments and inflows of capital into this market.[5] They show empirically that this connection is most likely not driven by improvements in investment prospects. Rather, they observe that inflows of capital to the VC market may cause price increases because of its intrinsic frictions and segmentation from other asset markets. Gompers, Kovner, Lerner and Scharfstein (2005) show that the fluctuation in venture capital investment is a response to changes in fundamentals. We show how shocks to market characteristics change both capital flows into the industry and returns in the VC industry. Consistent with the findings of Gompers et. al. (2005), we establish, for example, that when the venture prospects improve, more capital will flow into the market without degradation of the present value of the returns from investment.

1.1 Search-theoretic approach to the VC industry

As stated earlier, much of the growing financial economics literature on the VC industry contains a considerable number of theoretical and empirical papers that explore how *VCs* screen, select, finance, monitor, and advise their portfolio companies.[6] This literature largely investigates the topic through the lens of partial equilibrium, and it studies optimal contractual arrangements between *VCs* and entrepreneurs to

[5] This result challenges the assumption that the private equity market is perfect and frictionless. As stated in Gompers and Lerner (2000) "If markets are perfect, inflows of money into the venture capital funds should be unrelated to the valuations of private companies."

[6] Besides the references cited in footnote 3, we point to Aghion and Tirole (1994), Bascha and Walz (2001), Casamatta (2003), Cumming and MacIntosh (2002), Hellmann and Puri (2000, 2002), Kaplan and Strömberg (2003) and Schmidt (2003).

mitigate adverse selection and moral hazard problems. Few papers consider any market-wide frictions on the VC industry other than information asymmetry.

Search theory has been used to provide a rigorous yet tractable framework for addressing questions related to markets with frictions. Mortensen (1970) and Pissarides (1979) pioneered a series of studies concerned with individual workers' job search decisions and their macroeconomic implications.[7] Our model draws on this literature and incorporates the idea that when a meeting between a VC and an entrepreneur occurs (which itself is governed by a matching function), individuals bargain over the value created by the match. Recently, search theory has been applied in other areas in economics.[8] Lagos and Wright (2005) began a line of research in which the decision to accumulate money is endogenous. We revisit their original insight and extend it to venture capitalists that may raise liquid assets during a fundraising stage and invest in a subsequent decentralized market. Further, in our environment, investors' choice of the amount of capital to commit to the VC asset class hinges on such factors as the estimated yield from an alternative asset class, market frictions, outcome probabilities, and more.

Search theory has also been used in the field of finance. Duffie, Gârleanu, and Pedersen (2005), Vayanos and Wang (2005), Vayanos and Weill (2005), and Weill (2005a,b) relate commodity pricing and liquidity by modeling asset trading as a decentralized exchange process that resembles the equilibrium search model of Kiyotaki and Wright (1989). Lagos (2006) and Lagos and Rocheteau (2006) address similar issues but incorporate a more recent generation of the search-based model of exchange, which allows more general portfolios and preferences.

A few papers have considered search frictions in the venture capital industry. Included are the studies of Inderst and Muller (2004) and Michelacci and Suarez (2004).

[7] See Rogerson, Shimer and Wright (2005) for a survey on labor search models.

[8] Examples include Kiyotaki and Wright (1993), who used search models to justify the existence of money; Burdett and Coles (1997, 1999), who contributed to the marriage literature; and Jovanovic (1982), who used it in industrial organization.

These authors assume that all meetings between entrepreneurs and venture capitalists are the same, implying that there is no uncertainty about the characteristics of a future encounter. This assumption, however, avoids considerations about which projects should be implemented and the optimal fund size, two points our paper explores. In the private equity market, heterogeneity in returns and differential costs among different investments opportunities are prevalent. These differences could arise from better business ideas, differential skills, or experience. Jovanovic and Szentes (2006) consider a model with heterogeneous entrepreneurs, who decide whether to match with a venture capitalist, a bank, or to pursue the given venture alone. Unlike the setting of our model, they assume that *VCs* have unlimited wealth and extract the total surplus from entrepreneurs in every match.[9]

As in our work, Inderst and Muller (2004) study the impact of capital supply on prices and bargaining. However, in their model, the amount of capital *VCs* carry is exogenous, and, therefore, the capital committed to the industry changes only as the number of *VCs* varies. Our model adds to their work by incorporating an explicit fundraising stage, by providing selection criteria for ventures, and by showing that, in addition to the changes in the number of *VCs* (extensive margin effect), there may also be a change in the amount of capital each *VC* raises (intensive margin effect). Both effects are important to understand the observed adjustments in the market.[10]

Therefore, to analyze the questions about how much capital investors commit to the VC industry and how *VCs* choose ventures to fund, we must expand these previous frameworks and consider a more comprehensive model of different stages of the

[9] Silveira and Wright (2006a) suggest a model where innovators can sell ideas to entrepreneurs in decentralized markets, with random matching and bargaining. Yet, in their model, Silveira and Wright assume that entrepreneurs and innovators don't form partnerships to pursue the venture and they never exit the market.

[10] A couple of papers propose static search models of the VC market, a simplified approach that does not capture the important timing considerations and delays faced by entrepreneurs and *VCs*. Keuschnigg (2003) studies optimal taxation of VC-backed firms and Boadway, Secrieru, and Vigneault (2006) are concerned with the relationship between VC activity, occupational choice and unemployment.

venture financing cycle. Silveira and Wright (2006b) made a preliminary attempt to build a search model of the different stages of the venture investment cycle that showed the importance of liquidity in such an environment. We extend this work by introducing heterogeneity (in costs and in expected returns) among matches, and by considering a market where *VCs* raise investment funds given an alternative asset class. Since we wish to create a parsimonious model that highlights search, heterogeneity, and liquidity frictions, we assume complete and symmetric information. This excludes the problems of moral hazard and adverse selection, which are profoundly important in the venture capital market, yet they have been extensively examined in the received literature.

The remainder of the paper is organized as follows: In section 2, we depict the model environment and specify its key elements. In section 3, we characterize the equilibrium condition and derive the main general propositions. We also provide the results of parametric analysis of numerical simulations. In section 4, we examine adjustments to industry size that result from a shock in the economy. Section 5 summarizes the empirical implications of our theory, and in section 6 we conclude the paper with a discussion.

2 Model

2.1 Environment

Consider a discrete time and infinite horizon environment with two classes of risk neutral agents: *entrepreneurs (Es)* and *venture capitalists (VCs)*. There is a population with measures N_e and N_v of each, where e denotes entrepreneurs and v denotes *VCs* in what follows. Entrepreneurs have an idea in mind and the ability to execute the project but are in need of financial resources for implementation. Venture capitalists are financial intermediaries, who are engaged in raising funds from investors, searching for an investment opportunity, evaluating and selecting ventures, monitoring and adding value to their invested firm, and finally exiting the investment. In our formulation, each *VC* has the capital resources, human resources and time to invest in one entrepreneurial venture at a time. Similarly, each entrepreneur seeks to find one *VC* to invest in the venture.[11]

In our setting, a venture cycle is composed of four stages of different (and sometimes stochastic) length. Consistent with Gompers and Lerner (2001), in the first stage of the cycle, to which we refer as *fundraising*, the general partners (*VCs*) raise capital – denoted by m and measured in real terms – to finance an entrepreneur. The amount of funds raised from the limited partners (investors) depends on the characteristics of two competing investment alternatives faced by the limited partners of a venture capital firm: On the one hand, they can choose to invest in the venture capital market, which yields uncertain returns at a random point in time, or, on the other hand, they can invest in a risk-free asset that pays a return r every period.[12] Thus, the perceived attractiveness of each investment alternative is the key determinant of the amount of funds to be raised by the *VC*, which in turn will be used to invest in entrepreneurial ventures.[13]

The second stage of the cycle, which we refer to as *venture search*, centers on establishing a match between *VCs* and entrepreneurs. During this phase both *Es* and *VCs* are assumed to meet randomly, according to a process that depends on the number of both types of players in the economy. Let $\alpha_e \in (0, 1)$ be the probability that an entrepreneur meets an *VC* and $\alpha_v \in (0, 1)$ be the probability that a *VC* meets a entrepreneurs in any given period.[14] When a *VC* and an entrepreneur meet, they

[11] In our formulation, we do not consider syndication among VC firms. We allow for only one investment round during each cycle.

[12] In reality, there are more than two asset classes, yet for the sake of parsimony we only consider two classes.

[13] We do not consider investment in start-ups by angels who use their own funds, and assume that all investments are carried out by venture capital firms. Yet, we note that a special case of our model would apply to angel investors.

[14] In this formulation, we consider that agents have at most one encounter per period of time.

observe the realization of two random variables $\tilde{S}$ and $\tilde{C}$: the venture's value in case of success (S) and its implementation cost (C). The pair $(S, C) \in [0, \bar{S}] \times [0, \bar{C}]$ is drawn from $G_S(\tilde{S}|C)$ and $G_C(\tilde{C})$ respectively. All projects are assumed equal in case of failure, having scrap value of size F.

After each encounter, agents decide whether or not to match. If a match is consummated, the VC must invest C in the entrepreneurial venture, with this money used for inputs (e.g. physical plant, wages, equipment, marketing expenses etc). Hence, for a match to occur, a new venture must satisfy two criteria: First, it must be feasible, that is $m \geq C$; secondly, it must have an expected return that is high enough for both agents to agree to participate. If a project satisfies both criteria, agents bargain over the firm's equity, specifying the shares of equity to be owned by each agent. In our model, a generalized Nash bargaining program determines the shares owned by the VC and entrepreneur, denoted by μ and $(1 - \mu)$ respectively.[15] Since searching for a partner happens in a decentralized environment, we assume that agents are uncertain about future encounters and the corresponding characteristics of the projects. In addition, since agents discount the future at rate $\frac{1}{1+r}$, which creates an opportunity cost of waiting for a potentially better match, thereby forgoing one period of investment at rate r.

The third stage – called *implementation* – begins once a match between a VC and a entrepreneur is consummated. During every period of this phase, the venture can come to fruition – revealing itself as a success or failure and allowing the VC to exit – or it can continue its maturation process, leaving agents uncertain about future returns. For any given period, σ denotes the probability that the venture is ready for exit with value S (success) while δ denotes the probability the venture maturated but created value F (failure), where $\sigma + \delta < 1$. The third possibility is that the venture continues the maturation process, which occurs with probability $(1 - \sigma - \delta)$.

[15] Our modelling framework does not consider debt financing of entrepreneurial ventures.

Once the venture comes to fruition (whether as a success or failure), the last stage of the cycle, named *exit*, starts. At this point, *VCs* receive the proportional fraction of the value (μS or μF) and return the proceeds to the limited partners/investors. *VCs* will then adjust their asset holdings and begin a new cycle. Entrepreneurs receives their fraction $(1-\mu)$ and exit the industry.

The venture cycle, which we described above, is depicted by figure 1.

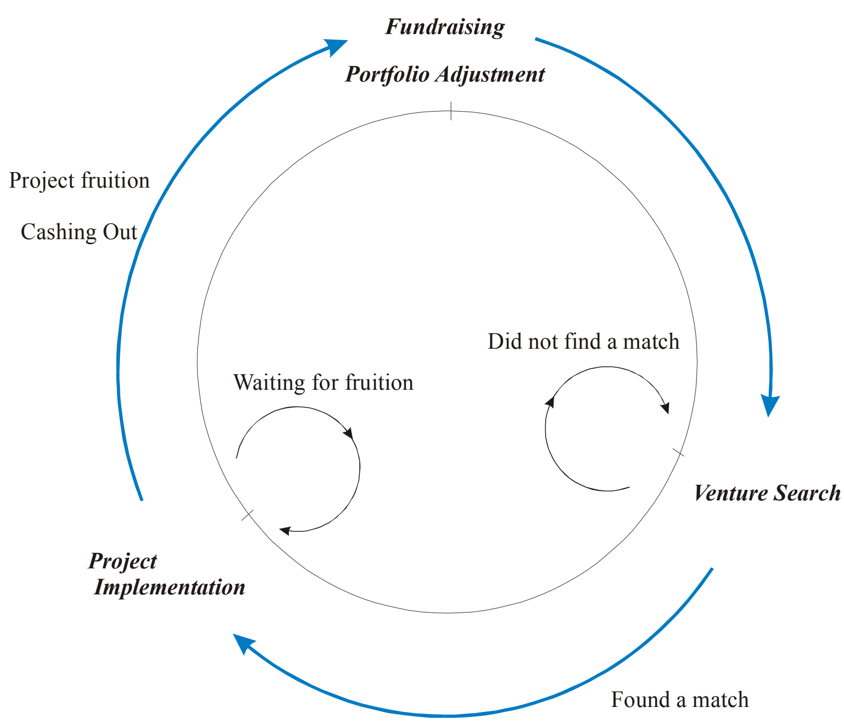

Figure 1: The venture cycle.

2.2 Value functions

We begin by describing a typical problem for an entrepreneur. Let J_e^0 represent the value function of an entrepreneur searching for a VC to finance the venture idea. If the entrepreneur meets a VC (which happens with probability α_e) and the cost of developing the venture is feasible from the VC's perspective (that is, the VC has enough funds to finance the project), the Es has the choice of accepting the match or waiting for another encounter - maybe with a more suitable VC who can provide, for instance, a higher S or lower C.[16] As will be shown below, for a given cost C, there is a threshold S_C^* such that both parties agree to implement the venture only if S of the proposed match is greater than S_C^*. If E neither meets a VC or the costs of developing the venture C is greater than the equity m available to the VC or the venture's value in case of success S is below the acceptance value, than E waits for a subsequent encounter with a VC, which has a present value of $\frac{J_e^0}{1+r}$.[17] This is,

$$J_e^0 = \frac{1}{1+r}\left\{ \alpha_e \int_0^m \int_{S_C^*}^{\bar{S}} J_e^1(S,C) dG_S(\tilde{S}|C) dG_C(\tilde{C}) \right. \tag{1}$$

$$\left. + \left[1 - \alpha_e \int_0^m \int_{S_C^*}^{\bar{S}} dG_S(\tilde{S}|C) dG_C(\tilde{C}) \right] J_e^0 \right\}.$$

Similarly, $J_e^1(S,C)$ is E's value function following a match with a VC, engaged in a venture that has value S in case of success, and requires investment C. In any period of this stage, the venture returns S with probability σ and F with probability δ. In either case, E owns a fraction $(1-\mu)$ of the firm's value. With probability $(1-\sigma-\delta)$ the project does not maturate in the current period. Then,

$$J_e^1(S,C) = \frac{1}{1+r}\left[\sigma(1-\mu)S + \delta(1-\mu)F \ + (1-\sigma-\delta)J_e^1(S,C)\right]. \tag{2}$$

For future reference, (2) implies

$$J_e^1(S,C) = \frac{(1-\mu)}{r+\sigma+\delta}(\sigma S + \delta F) = \frac{(1-\mu)\pi(S)}{r+\sigma+\delta}. \tag{3}$$

To simplify notation we define $\pi(S) \equiv \sigma S + \delta F$ as denoting the expected value of the

[16] It is assumed that the ability of a VC to finance a project is capped by the amount m raised in the first stage.

[17] Given that all VCs are assumed to satisfy an optimality condition, which is explained later, they will leave the *fundraising* stage with the same amount of equity.

firm.

We now proceed to describe the VC's value functions, which is similar to E's function presented above. Let $J_v^0(m)$ be the value function of a VC searching for an E having capital m in hand. In other words, $J_v^0(m)$ is the present value of raising equity m and making them available for investment in an entrepreneurial venture. Therefore,

$$J_v^0(m) = \frac{1}{1+r}\left\{\alpha_v \int_0^m \int_{S_C^*}^{\bar{S}} J_v^1(S,C)dG_S(\tilde{S}|C)dG_C(\tilde{C}) + \left[1-\alpha_v \int_0^m \int_{S_C^*}^{\bar{S}} dG_S(\tilde{S}|C)dG_C(\tilde{C})\right] J_v^0(m)\right\}. \tag{4}$$

It is assumed that the amount of equity m brought to the venture capital industry which exceeded C is kept by the VC to be returned to investors together with the project proceeds. An alternative formulation, which does not change our major findings, is to assume that VCs invest the excess equity in a risk-free asset with a yield of r or that they return excess equity to investors immediately after the match is consummated.[18]

When the VC invests C in a venture that has value S in case of success, the value function is

$$J_v^1(S,C) = \frac{1}{1+r}\left[\sigma W(\mu S + m - C) + \delta W(\mu F + m - C) + (1-\sigma+\delta)J_v^1(S,C)\right]. \tag{5}$$

This value function implies that in any period of the *implementation stage,* if the venture comes to fruition, the VC will cash-out and reenter the fundraising stage with

[18] Under such hypothesis

$$J_v^0(m) = \frac{1}{1+r}\left\{\alpha_v \int_0^m \int_0^{\bar{S}} (J_v^1(S,C)+m-C)dG_S(\cdot)dG_C(\cdot) + \left[1-\alpha_v \int_0^m \int_0^{\bar{S}} dG_S(\cdot)dG_C(\cdot)\right] J_v^0(m)\right\}.$$

the proceeds from the last project – μS with probability σ and μF with probability δ – plus any excess equity that had not been used $(m - C)$.[19]

Let $W(\hat{m})$ be VC's value function of reentering the *fundraising* stage with capital $\hat{m}$ that equals the proceeds from the last cycle plus any excess cash that was not invested. These funds will be returned to the investors, who will, in turn, decide on the desired asset allocation, determining the new amount of funds m to commit to the VC industry and initiating a new cycle. The asset allocation decision is based on the expected return from VC investments when compared to the rate of return from the risk-free investment.[20] Thus,

$$W(\hat{m}) = \hat{m} + \max_{m} \left\{ \max \left[0, J_v^0(m) - m \right] \right\}. \tag{6}$$

The term $J_v^0(m) - m$ represents the difference between the expected present value from investing m in the VC industry and the present value of investing the same amount in the alternative risk-free asset. This difference could be either positive or negative in the short run, when the number of *VCs* in the market is assumed to be fixed.

If $J_v^0(m) - m < 0$, the prospect of investing in the VC industry is worse than the risk-free asset, then *VCs* will not raise any capital. But if $J_v^0(m) - m \geq 0$ the prospect of investing in the VC industry is better than the risk-free asset and investors can benefit from transferring funds to *VCs*. In this case, m is defined by the solution to the maximization problem in (6), which has the following first order condition

$$J_v^{0\prime}(m) = 1. \tag{7}$$

[19]Considering the formulation described in footnote 18, equation 5 is modified to

$$J_v^1(S, C) = \frac{1}{1+r} \left[\sigma W(\mu S) + \delta W(\mu F) + (1 - \sigma + \delta) J_v^1(S, C) \right].$$

[20]This is equivalent to saying that, while in the *fundraising* market, *VCs* can borrow at market rate r. We thank Guido Menzio for suggestion this approach.

Equation (5) can be simplified by inserting $W(\cdot)$ in and defining $\mathcal{M} \equiv \max\left[0, J_v^0(m) - m\right]$. It becomes

$$J_v^1(S,C) = \frac{1}{r+\sigma+\delta}\left\{\mu\pi(S) + (\sigma+\delta)\left[m - C + \mathcal{M}\right]\right\}. \tag{8}$$

In the long-run, there will be an adjustment in the number of *VCs* in the industry until a no-arbitrage condition between the two investment alternatives is satisfied. Such condition is

$$J_v^0(m) - m = 0. \tag{9}$$

Definition 1 (Long-run) *The long-run equilibrium is characterized by the endogenous variables* (m, μ, S_C^*) *and the population of venture capitalists* N_v *such that the no-arbitrage condition (9) holds.*

2.3 Bargaining

Once a *VC* and an entrepreneur meet and observe the feasibility of the investment (namely $m \geq C$), the bargaining over the share of the venture that will be owned by the *VC* (μ) begins. This bargaining process is modelled as a generalized Nash bargaining problem. The solution can be understood either as a generalization of the axiomatic model of Nash or as the strategic model of Rubinstein, since the latter has a reduced form that approaches the former as the time between bargaining rounds shrinks (Binmore et al., 1986).

Let θ and $\Omega_v(S,C)$ denote the *VC*'s bargaining power and the surplus of the match, respectively. Similarly, $1-\theta$ and $\Omega_e(S,C)$ denote the entrepreneur's, *E*'s, bargaining power and the surplus of the match, respectively. The problem then is

$$\max_{0\leq\mu\leq1}\left[\Omega_v(S,C;\mu)\right]^{\theta}\left[\Omega_e(S,C;\mu)\right]^{1-\theta}. \tag{10}$$

The surpluses are determined by comparing the benefit of implementing the prospective venture to the benefit of rejecting it and searching again next period. Thus, $\Omega_e(S,C;\mu) = J_e^1(S,C) - J_e^0$ and $\Omega_v(S,C;\mu) = J_v^1(S,C) - J_v^0(m)$. Substituting (3) and (8) into (10) yields

$$\max_{0\leq\mu\leq1} \left[\mu\pi(S) - d_v\right]^{\theta} \left[(1-\mu)\pi(S) - d_e\right]^{1-\theta},$$

where $d_v = (r+\sigma+\delta)J_v^0(m) - (\sigma+\delta)(m - C + \mathcal{M})$ and $d_e = (r+\sigma+\delta)J_e^0$.

The problem above is how to split a pie of (expected) value $\pi(S)$ given each agent's threat point for not engaging in the venture. The solution is to first give each agent its threat-point benefit (d_v and d_e) and then to allocate the remaining value ($\pi(S) - d_v - d_e$) according to the relative bargaining powers (i.e. fraction θ to *VC* and $(1-\theta)$ to entrepreneur).

The solution of the bargaining problem defines $\mu(S,C)$, which is given by [21]

$$\begin{aligned} \mu\pi(S) &= d_v + \theta[\pi(S) - d_v - d_e] = \theta\left[\pi(S) - d_e\right] + (1-\theta)d_v \\ &= \theta\left[\pi(S) - (r+\sigma+\delta)J_e^0\right] + (1-\theta)\left[(r+\sigma+\delta)J_v^0(m)\right. \qquad (11) \\ &\quad \left.-(\sigma+\delta)(m - C + \mathcal{M})\right]. \end{aligned}$$

The greater the bargaining power of an agent, the larger its share of the ownership of the venture. For instance, if $\theta = 1$ *VCs* own the entire firm's value net of Es' benefit for not accepting the match. If, however, $\theta = 0$ *VCs* own just the opportunity cost of participating in the venture (d_v), the Es will keep the remaining value.

The following result shows that agents are willing to match only when it is beneficial to both parties and generate positive surpluses for each one.

Lemma 1 *For $\theta \in (0,1)$, $\Omega_v(S,C) > 0$ if, and only if, $\Omega_e(S,C) > 0$.*

[21] We simplify notation and most of the time write $\mu(S,C)$ as simply μ.

All proofs are provided in Appendix A. From the proof of Lemma 1, we get that $\Omega_v(S, C) = \frac{\theta}{1-\theta}\Omega_e(S, C)$ for $\theta \in (0, 1)$. That is, each agent's surplus from the project is proportional to its bargaining power in bilateral matches. Further, when a *VC* has no bargaining power ($\theta = 0$), then $\Omega_v(S, C) = 0$, and when a *VC* has all the bargaining power ($\theta = 1$), then $\Omega_e(S, C) = 0$.

Since the surpluses for both agents are increasing in S, the nature of the optimal search strategies is governed by a cutoff rule determining a threshold S^*_C: An affordable venture is implemented only if $S \geq S^*_C$. Thus, we define S^*_C as the threshold value of a successful outcome required to accept a match with an upfront investment C.

Lemma 1 implies that the reservation value function for entrepreneurs and *VCs* are the same. Thus, at S^*_C, *VCs* and *Es* are equally drawn to match or to keep looking for a better business opportunity in the next period. Therefore, $\Omega_v(S^*_C, C) = \Omega_e(S^*_C, C) = 0$. This implies,

$$\begin{aligned} \Omega_e(S^*_C, C) &= 0 \Longrightarrow (1 - \mu(S^*_C, C))\pi(S^*_C) = d_e & (12) \\ \Omega_v(S^*_C, C) &= 0 \Longrightarrow \mu(S^*_C, C)\,\pi(S^*_C) = d_v, & (13) \end{aligned}$$

where $\pi(S^*_C) = \sigma S^*_C + \delta F$. Combining the two equations above, we have

$$\begin{aligned} \pi(S^*_C) &= d_e + d_v \\ &= (r + \sigma + \delta)\left[J^0_v(m) + J^0_e\right] - (\sigma + \delta)(m - C + \mathcal{M}). \qquad (14) \end{aligned}$$

The joint surplus of the partnership is positive if and only if $S \geq S^*_C$. This implies that matches are consummated by mutual consent when both agents benefit from it. Also, any match with $S \geq S^*_C$, will induce venture implementation with $0 \leq \mu \leq 1$.

The proposition below states that the higher the cost C of a project, the greater is the equity μ owned by a *VC*. In addition, μ increases linearly in C and, and the rate of change decreases for higher values of S.

Proposition 1 *For given S and $\theta \in (0,1)$, equity $\mu(S,C)$ owned by a VC is linearly increasing in C and its slope is decreasing in S.*

The following result shows that the expected value of the equity in hand, namely $\mu\pi(S)$, is increase in S; even tough μ – the fraction of the venture owned by the VC – could either increase or decrease for a higher realization of S.

Proposition 2 *For $\theta \in (0,1)$, $\mu\pi(S)$ is increasing in S. Further, μ is increasing in S if, and only if, $\theta > \mu(S_C^*, C)$.*

2.4 Valuations

The value of the enterprize prior to the investment is commonly referred to as the pre-money valuation; and immediately after the transaction is consummated the firm's value is referred to as post-money valuation. According to Hellmann (2002), the post-money valuation "calculates the value of the venture on the basis of the stake purchased by the investor". Using our notation, if a VC finds a venture that requires investment C and, after bargaining with the entrepreneur, the VC obtains a fraction μ of the equity; then the post-money valuation (represented by Γ_{post}) is given by $\Gamma_{post} = \frac{C}{\mu(S,C)}$. The pre-money valuation is given by $\Gamma_{pre} = \frac{C}{\mu(S,C)}[1 - \mu(S,C)]$, which can be understood as the cost per newly-issued share times the outstanding shares prior to the transaction.[22] The pre money valuation, referred to by Gompers and Lerner (2000) as the price of an investment, is decreasing in the fraction of equity owned by a VC (μ). In our model μ is determined endogenously and, therefore it

[22]The shares purchased in a venture capital investment are in general newly-issued shares, leading to a change in the total number of outstanding shares. In our model, the post-money number of shares is assumed to equal 1. Therefore, the pre-money outstanding shares are $1 - \mu$, and the newly-issued shares are μ.

is possible to study the pre-money valuation (investment price) for different ventures and for different market conditions.

From Proposition 1, it is known that investments with higher cost C yields a greater share μ to the *VC*. Because of such adjustment, it is not clear if the post-money valuation does also increase in C. The proposition below shows that even tough *VCs* receive more shares, i.e. μ increases, the post-money valuation Γ_{post} increases in the investment cost.

Proposition 3 *The post-money valuation* Γ_{post} *is increasing in the investment cost* C.

3 Equilibrium

We now focus on the steady state of a rational expectations equilibrium. In this setting, *VCs* and entrepreneurs aim to maximize their respective objective functions, subject to matching and exit conditions. At the steady state equilibrium, the number of Es entering the market for venture capital equal the number exiting it after a liquidity event.

Over the long-run, the number of *VCs* in the market changes in order to maintain the no-arbitrage condition, i.e. $J_v^0(m) - m = 0$. When this condition is satisfied, investors are indifferent to the choice of whether to invest m in a risk-free asset, earning r per period, or whether to devote the same amount of equity capital to the VC industry, given its intrinsic uncertainty about the magnitude and timing of a return.

In the short-run however, a different situation exists. Two cases are possible: A) $\mathcal{M} = J_v^0(m) - m$ – the expected present value of investing in the VC market is larger than the return to be obtained in the risk-free arena, and B) $\mathcal{M} = 0$ – investors prefer not to commit new funds to the VC industry. Further, when $\mathcal{M} = 0$, venture capitalists will not be able to raise funds from investors and the threshold investment decision becomes irrelevant. If however, $\mathcal{M} = J_v^0(m) - m$, then funds will be committed to the VC industry. This is the case we examine in detail below.

3.1 Equity raised

The decision on how much capital to attempt to raise from investors involves tradeoffs: On one hand, the VC would like to have a large pool of capital, which may enable the $VC$to find acceptable and affordable projects in less time. On the other hand, the opportunity cost of forgoing the risk-free rate is higher the greater the amount of capital raised. In addition, as we show below, raising more capital may adversely affect the threshold value S_C^* and the VC's share of the venture μ.

We proceed to characterize the optimal amount of equity to invest in the venture funding market. We first modify $J_v^0(m)$ by substituting $\mu\pi(S)$ from (11), and then plugging $\pi(S_C^*)$ from (23). This procedure yields [23]

$$rJ_v^0(m) = \frac{\alpha_v \theta \sigma}{r + \sigma + \delta} \int_0^m \int_{S_C^*}^{\bar{S}} (S - S_C^*) dG_S(\tilde{S}|C) dG_C(\tilde{C}). \tag{15}$$

This equation is used to find the derivative of $J_v^0(m)$ with respect to m, which, in turn, is combined with the first-order condition from equation (6). Indeed, Appendix

[23] From a similar procedure we show that

$$rJ_e^0 = \frac{\alpha_e(1-\theta)\sigma}{r + \sigma + \delta} \int_0^m \int_{S_C^*}^{\bar{S}} (S - S_C^*) dG_S(\tilde{S}|C) dG_C(\tilde{C}).$$

By combining this equation with (15), it is possible to conclude that $J_e^0 = \frac{(1-\theta)\alpha_e}{\theta\alpha_v} J_v^0(m)$.

B shows that after applying Leibniz rule twice, the equation (15) becomes

$$r = \frac{\alpha_v \theta \sigma}{r+\sigma+\delta} \left\{ \int\limits_{S^*_{C=m}}^{\bar{S}} (S - S^*_{C=m}) dG_S(\tilde{S}|m) G'_C(m) - \int\limits_0^m \int\limits_{S^*_C}^{\bar{S}} \frac{\partial S^*_C}{\partial m} dG_S(\tilde{S}|C) dG_C(\tilde{C}) \right\} \tag{16}$$

where we substituted $J^{0\prime}_v(m) = 1$, from (6).

Equation (16) defines the short-run optimality condition for $m(S^*_C)$, the amount of capital investors transfer to *VCs* as a function of the acceptance rule S^*_C. Note that such equation does not depend on the match-specific variables – S or C – since *VCs* must decide how much funds to raise before considering any specific venture investment.

3.2 Reservation function

The reservation function maps the cost of developing the venture C into the space of returns S, pinning down a value S^*_C, such that any affordable venture with $S \geq S^*_C$ is accepted by the entrepreneur and the *VC* in the match. Therefore, if $S \geq S^*_C$, then

$$\max\{J^1_e(S,C) - J^0_e, 0\} = J^1_e(S,C) - J^0_e \text{ and}$$
$$\max\{J^1_v(S,C) - J^0_v(m), 0\} = J^1_v(S,C) - J^0_v(m).$$

This allows us to simplify equations (1) and (4). Appendix C shows the procedure to modify and combine these equations with (11) and (14) to generate an explicit expression for S^*_C, namely

$$\sigma S^*_C + \delta F = \Delta\sigma \int\limits_0^m \int\limits_{S^*_C}^{\bar{S}} (S - S^*_C) dG_S(\tilde{S}|C) dG_C(\tilde{C}) + (\sigma+\delta)C \tag{17}$$

where $\Delta \equiv \left[(1-\theta)\frac{\alpha_e}{r} + \theta\frac{\alpha_v}{r+\sigma+\delta}\right]$.[24]

Equation (17) defines $S_C^*(m)$ as a function of the endogenous variable m given the exogenous parameter C. The left-hand side of (17) represents the expected value of investing in a venture where the successful outcome has value S_C^*. The right-hand side reflects the cost of accepting such project; the first term is the cost of committing and forgoing potentially better projects in the future, and the second term is the capital invested in the venture business.

We note that S_C^* is increasing in C, and the rate of increase rises with the venture's costs. This means that, to accept a venture, *VCs* and *Es* demand an increase in return greater than the proportional increase in the project's cost. Proposition 4 summarizes this result.

Proposition 4 *The reservation function S_C^* is increasing and convex in C.*

The equilibrium of the model is given by an m and S_C^* that simultaneously solve equations (16) and (17). Figure 2 depicts such functions in the space of possible realizations of (S, C). As shown in Proposition 3, S_C^* is an increasing convex function of C for any distributions G_S and G_C. Also, from equation (16), m is known to be independent of the specific realizations of S and C.

After a meeting, an E and a *VC* observe the realization of S and C, which is represented by a point in Figure 2. If this point is to the left of the m-curve ($C \leq m$), then the venture is affordable to the *VC*. If the point is to the right of the m-curve, then the match is not feasible and agents have to continue searching for other matches. Among the affordable ventures, only the ones with $S \geq S_C^*$ are consummated, having

[24] If the alternative approach mentioned in footnote 18 is adopted the the equilibrium equation would be

$$\sigma S_C^* + \delta S = \Delta \int_0^m \int_{S_C^*}^{\bar{S}} [1 - G_S(S|C)]\, d\tilde{S} dG_C(\tilde{C}) - rm + (r+\sigma+\delta)C.$$

Thus, the assumption about what to do with the left over funds causes only a minor variation on the equilibrium equation.

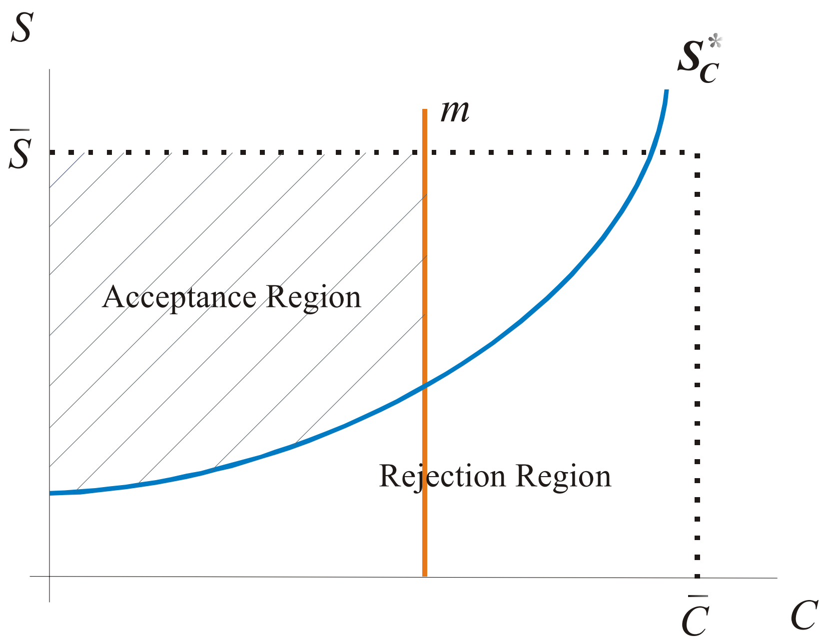

Figure 2: Acceptance and rejection regions.

been judged acceptable given their cost. Therefore, only ventures (S, C) that lie to the left of the m-curve and above the S^*_C-curve are implemented. All other projects are rejected either because of their high costs or low expected return (or both).

Despite the fact that the amount of capital *VCs* accumulate during the fundraising stage is determined endogenously, it is interesting to understand the impact of an exogenous change in m on the acceptance threshold S^*_C, keeping constant the market parameters.[25] We note that Es and *VCs* are assumed to meet in a decentralized market setting. When a change in the amount venture capital in the market occurs, the pool of feasible investments for each *VC* changes along with their threshold. The proposition below shows that, ceteris paribus, if *VCs* have more capital during the venture search stage, they will become more selective. Hence, the selection criteria $S^*_C(m)$ goes up as m increases.

[25] The next section will analysis this exercise and derive the adjustments of equilibrium variables μ, m and S^*_C given variations in exogenous variables.

Proposition 5 *The reservation function S_C^* is non-decreasing for exogenous increases of m. If all $C \in [0, \bar{C}]$ have a strictly positive probability, then S_C^* is strictly increasing in m.*

The result can be understood using Figure 3 below. An exogenous increase in m shifts the m-curve to the right. Because *VCs* have more funds, their pool of possible investments increases and they will become more selective in accepting venture investments, shifting the S_C^*-curve to the left. This implies that *VCs* now reject the low expected return ventures in area A (Figure 3), because they can accept the ones with the high expected return in area B (Figure 3).

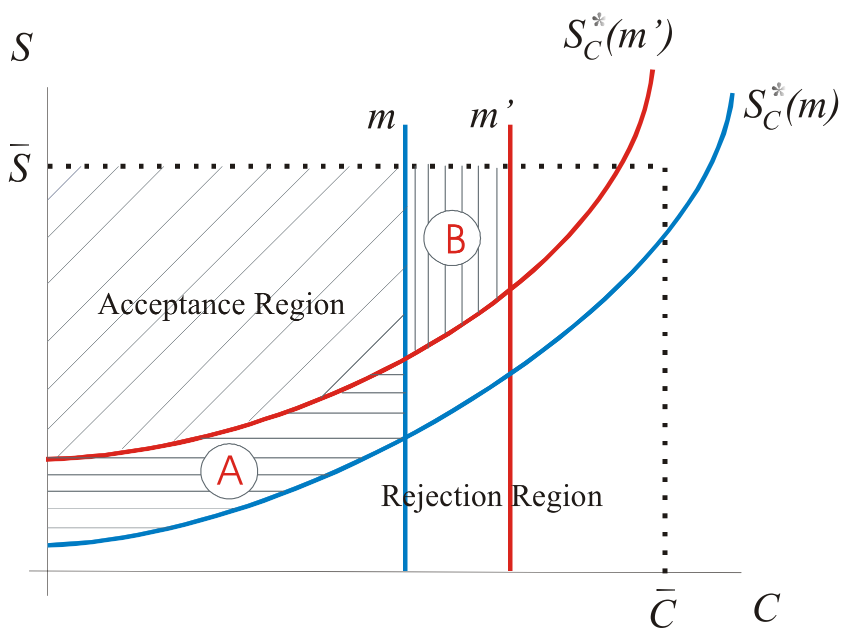

Figure 3: Effect of exogenous change in capital.

3.3 Numerical simulations

To further explore the properties of the endogenous variables and derive their adjustments given changes in the market for venture financing, assume that S and C

are distributed independently and uniformly between $[0, 1]$. Then (17) simplifies to

$$\sigma S_C^* = \frac{\Delta m \sigma}{2}(1 - S_C^*)^2 + (\sigma + \delta)C - \delta F. \tag{18}$$

Also, equation (16) becomes

$$r = \frac{\alpha_v \theta \sigma}{r + \sigma + \delta} \left\{ \frac{1}{2} \left(1 - S_{C=m}^*\right)^2 - \int_0^m \frac{\partial S_C^*}{\partial m} \left(1 - S_C^*\right) dC \right\}, \tag{19}$$

where $S_{C=m}^*$ can be found from (18). We can plot such equations and establish important relationships. The simulation results are shown in Appendix D.

Finding 1 *Ceteris paribus, an increase in the probability of the successful outcome (σ) or in the scrap value of ventures (F) will induce an increase in the amount of capital flowing into the VC industry (m) and in the present value of funds allocated to the VC industry (J_v^0). Furthermore, the acceptance threshold value (S_C^*) and shares owned by the VCs (μ) will decrease.*[26]

Our simulations demonstrate that better venture prospects, namely increases in σ or F, induce a higher present value of investing in the VC industry, even though *VCs* receive a smaller fraction of the equity in the ventures they invest in. Also, the above changes induce a shift of the S_C^* and m-curves to the right, implying an increase in the acceptance region in Figure 2.

Given the better investments opportunities in the industry, investors transfer more funds to *VCs* which in turn can accept projects more easily. Noticing the overall increase in the expected return of projects and the additional flow of equity into the market, entrepreneurs will receive a greater fraction of the equity in their ventures, which in turn implies an increase in the price of VC investments in new ventures.

[26]The same result is true for a decrease in the probability that the venture fails (δ).

The finding above could offer an explanation to what happened in the industry during the mid-1990s, when the supply of funds to *VCs* increased considerably. Contrary to expectations, instead of facing a deterioration, investors observed an increase in returns (Lerner, 2002). Such improvement is attributed to better entrepreneurial opportunities, namely the rapid diffusion of internet access and the development of the web.

Finding 2 *Ceteris paribus, as the probability of VCs meeting Es (α_v) increases or as the probability of Es meeting VCs (α_e) decreases, the present value of VC investment (J_v^0) and the flow of capital (m) to the industry increase. Furthermore, there will be an increase in the fraction of the venture's equity (μ) VCs receive in each investment.*

There are two effects to consider: First, an increase in α_v allows *VCs* to meet entrepreneurs more frequently, meaning that *VCs* will need less time on average to find acceptable and affordable projects. This may happen due to an increase in the number of entrepreneurs in the market (N_e) or a decrease in the number of *VCs* (N_v). Also, as α_e decreases, so do *Es*' outside option. Therefore, in a bilateral meeting, entrepreneurs will offer a greater fraction of the venture in order to persuade *VCs* to engage in the match. These forces increase the value of investing in the industry, stimulating more investment in the venture capital industry.[27]

The second effect is that *VCs* do not need as much money to find good and affordable projects in a given timeframe as they would have otherwise. Since carrying capital is costly, such an effect induces *VCs* to reduce the amount of capital they raise. Given our assumptions, the simulations show that the former force dominates the latter and, overall, more money flows into the industry.

The preceding analysis of valuation can be used to extend the results of Findings 1 and 2 of the numerical simulations. Specifically, we note from Finding 1 that if

[27] This effect is similar to what would happen if the bargaining power of the *VCs* (θ) increased. Therefore, as θ increases so does equilibrium m.

investment prospects in the market improve on average (namely higher σ or F), then pre and post-money valuations increase. Further, from Finding 2 we observe that in a market setting where *VCs* are able to find entrepreneurs more easily (or it is harder for entrepreneurs to find *VCs*), pre and post-money valuation decrease.

Despite the fact that Findings 1 and 2 generate parallel outcomes, the driving force is somewhat different. Thus, the findings above could provide alternatives explanations for empirical observations. For example, the observed increase in committed capital to the venture capital industry and returns in the late-1990s could be a consequence of an increase in the number of entrepreneurs looking for financing, without any significant improvement in the quality of the ventures they had to offer (as suggested by Finding 1).[28]

Finding 3 *Ceteris paribus, as the return of the risk-free asset (r) increases, both the present value of investing (J_v^0) and the amount of capital (m) in the VC industry decrease.*

When alternative investments become more attractive, the opportunity cost of allocating funds to the VC industry increases. In addition, every period waiting to find an investment in the first place or for maturation of an existing investment is more costly, since it means forgoing a greater outside return. Our simulations show that investors will react by allocating fewer funds to the industry and keeping roughly the same threshold criterium as before. Thus, the acceptance region in Graph 2 will decrease and fewer projects will receive investment capital.

[28]The findings differ on the prediction about changes in the equity contract μ. While in Finding 1, Es increase their bargaining power by bringing better ideas to VCs (i.e. μ decreases); in Finding 2, VCs can find investments more easily and μ increases.

4 Endogenous number of Venture Capitalists

In this section, we study how the number of *VCs* in the industry and the amount of capital they raise vary in response to shocks in the economy that either change the outside investment return r or increase the flow of entrepreneurs into the venture funding market. Starting from a steady state equilibria, we will show the impact of each shock in the short run, when the number of *VCs* is fixed, and on the long run, when *VCs* can enter or exit the venture market.

We note that in the preceding sections α_v and α_e, the probabilities of encounters between *VCs* and Es, were exogenous parameters of the model since the number of *VCs* was given and constant across cycles. Further, we recall that in steady state equilibrium, every entrepreneur who leaves the market is replaced by a new one, who will begin a search for a *VC*. This may be a useful characterization of the short run, but it is reasonable to allow for market entrance and exit in the long run.

Let u_v and u_e denote the number of unmatched *VCs* and Es at a given moment. Suppose further that unattached agents find each other according to an encounter technology outlined in Pissarides (2000) that determines the total number of meetings, $\mathcal{H}$ per period, as function of u_v and u_e. As common in the literature, the function $\mathcal{H}$ is assumed to be homogeneous of degree one, increasing, concave and continuously differentiable. Then, the probability of encounters in any given period is

$$\alpha_v = \frac{\mathcal{H}(u_v, u_e)}{u_v} = \mathcal{H}(1, \frac{u_e}{u_v}) = \mathcal{H}(1, \frac{1}{\tau}) \text{ and}$$

$$\alpha_e = \frac{\mathcal{H}(u_v, u_e)}{u_e} = \mathcal{H}(\tau, 1),$$

where $\tau = \frac{u_v}{u_e}$ measures the market tightness from the point of view of *VCs*.

When there is a fixed number of entrepreneurs, as the number of *VCs* in the market increase, u_v and τ increase as well while α_v decreases since each *VC* will find it more difficult to locate an unmatched entrepreneur. In addition, entrepreneurs meet *VCs* more frequently, and, therefore, α_e increases. Inversely, for a fixed number of *VCs*, if there is an exogenous increase in the number of entrepreneurs seeking financing, then α_e decreases and α_v increases.

Let $m(\tau)$ and $S_C^*(\tau)$ denote the amount of capital in the industry and the reservation function given the market tightness for *VCs* in the industry. In the short run, τ is fixed, while in the long run, τ adjusts such that the no-arbitrage condition, given by (9), holds. The long-run τ is denoted by τ^*. Hence, $m(\tau^*)$ and $S_C^*(\tau^*)$ satisfy both the short-run optimality conditions – given by (16) and (17) – and the no-arbitrage condition, which can be written as

$$rm\left(\tau^*\right) = \frac{\alpha_v(\tau^*)\theta\sigma}{r+\sigma+\delta} \int_0^{m(\tau^*)} \int_{S_C^*(\tau^*)}^{\bar{S}} \left[S - S_C^*(\tau^*)\right] dG_S(\tilde{S}|C) dG_C(\tilde{C}). \tag{20}$$

Figure 3 below illustrates the short-run and long-run equilibrium for $m(\tau)$. The vertical axis measures the extra benefit of investing in the VC industry relative to the outside risk-free asset. The horizontal axis measures the amount of equity m raised by each *VC* in the industry given the market's tightness τ.

After a shock, $J_v^0(m) - m \neq 0$ can occur in the short run. If investing in the *VC* market yields an expected present value return greater than the risk-free asset ($J_v^0(m) - m > 0$), then more *VCs* will enter the industry, increasing τ and ultimately decreasing the benefit of investing in the industry. *VC* entry stops when the no-arbitrage condition holds. Similarly, if $J_v^0(m) - m < 0$, then *VCs* will exit, increasing the return of investing in the venture market. In both cases, *VCs* will exit or enter this market so that $J_v^0(m) - m = 0$ holds in the long run.

Given a change in the risk-free return r or in the number of entrepreneurs looking for financing u_e, one can calculate the short-run to long-run adjustment and separate two distinct effects: A) changes in the amount of capital raised by each *VC* ($\Delta m =$

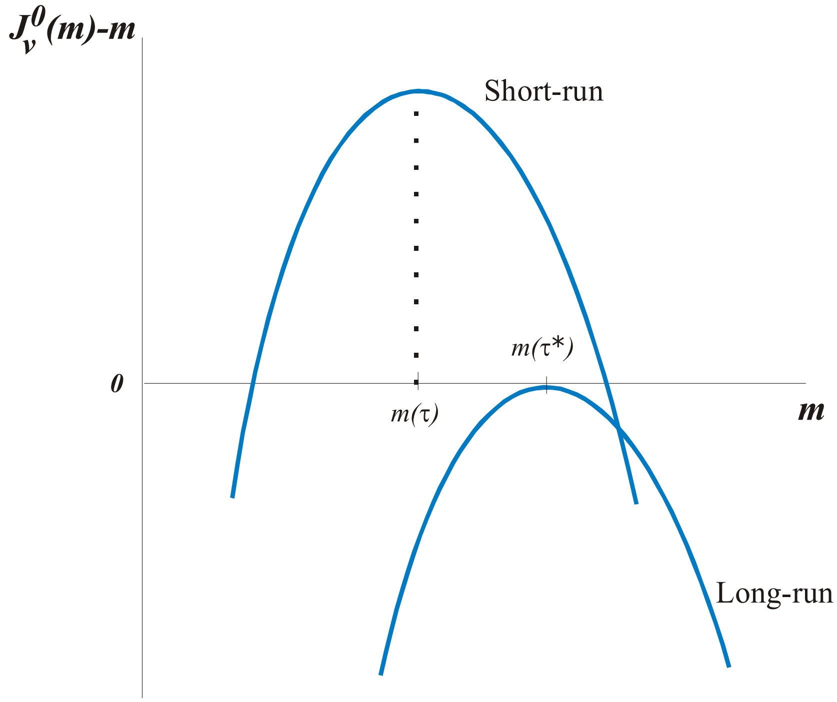

Figure 4: Short and long-run outcomes.

$m(\tau) - m(\tau^*)$), and B) changes in the market tightness for *VCs* ($\Delta\tau = \tau - \tau^*$). The first represents an adjustment in the intensive margin of the market, where *VCs* adjust m, changing the probability of implementing a project. The second effect corresponds to a variation in the extensive margin, in which *VCs* may enter or leave the market for entrepreneurial financing.

Assuming that S and C are distributed independently according to a uniform distribution between $[0, 1]$, it is possible to plot the equilibrium equations and draw conclusions about the VC industry's behavior in the long run.[29]

[29] Under such hypothesis, the long-run arbitrage condition (20) is given by

$$rm(\tau^*) = \frac{\alpha_v(\tau^*)\theta\sigma}{r + \sigma + \delta}\frac{1}{2}\int_0^{m(\tau^*)} [1 - S_C^*(\tau^*)]^2 dC. \tag{21}$$

Given our previous assumptions, one can take the matching function $\mathcal{H}$ as a Cobb-Douglas function with elasticity η (i.e. $\mathcal{H} = u_v^\eta \cdot u_e^{1-\eta}$). Then $\alpha_v = \tau^{\eta-1}$ and $\alpha_e = \tau^\eta$. It is possible to form a system of equations combining (18), (19) and (21), and find long-run values for m, S_C^* and τ^* and measure their changes given shocks in the market.

Finding 4 *Consider an increase in the risk-free asset return* (r)*: In the short run, there will be a decrease in the expected extra return from investing in the VC industry* ($J_v^0 - m$)*. In the long run,* VCs *will exit the market, decreasing the market tightness* τ*. Consequently, the present value of investing in VC* (J_v^0) *increases until it reaches the long-run equilibrium value* (m)*.*

As noted in Finding 3, as r rises, it becomes more costly to invest in VC because of the higher yield of risk-free assets. Therefore, in the short run, investors will choose the alternative investment. In the long run, *VCs* will exit the market, and the market tightness τ ultimately will decrease. The remaining *VCs* will find entrepreneurs more easily and will also be able to bargain for a greater share of each venture invested. Thus, the value of investing in VC (J_v^0) increases.

Our simulations show that in the long run, the amount of capital raised by each *VC* (m) will also increase. However, J_v^0 will increase further, pushing $J_v^0 - m$ towards zero as τ decreases and the no-arbitrage condition (9) is satisfied.

Finding 5 *If there is an increase in the number of entrepreneurs in the economy* (N_e) *or there is an improvement in venture prospects (namely an increase in* σ *or* F), *then the expected extra return from investing in the VC industry* ($J_v^0 - m$) *increases in the short run. In the long run,* VCs *will enter the market, increasing the market tightness* τ*. Consequently, the present value of investing in VC* (J_v^0) *decreases until it reaches the long-run equilibrium value* (m)*.*

If the number of *VCs* is fixed, a sudden increase in the number of *Es* in the economy will cause an immediate decrease in τ. Because of the reasons mentioned above, the prospect of investing in the *VC* industry improves for higher values of τ. Hence, in the short run, investors will allocate more funds to the VC industry. In the long run, more *VCs* will enter the market, causing an increase in the market tightness τ. This increase has the effect of hindering the probability of *VCs* finding a match, and VC's ownership share in a venture μ will fall. As a result, the value of investing in the venture industry (J_v^0) decreases.

As with the preceding case, the amount of capital raised by each VC (m) will also decrease. However, $J_v^0 - m$ goes to zero as τ decreases and the no-arbitrage condition (9) holds after the adjustment in τ.

5 Empirical Predictions

This section reviews some of the empirical predictions of our model, discusses the available evidence, and suggests new avenues of research.

i) *Valuations and venture capitalists' equity shares*

Our model predicts that if the market environment changes so that the expected proceeds from the ventures in the market increase (specifically, an increase in F or σ), then the *VCs*' equity share of the venture decreases, and pre- and post-money valuations increase. Gompers and Lerner (2000) found supporting evidence: They say, "The marginal impact of a doubling in public market values is a 15-35% increase in the valuation of private equity transactions" (p. 283).

The framework we present also suggests that an exogenous increase in the number of entrepreneurs or a decrease in the number of *VCs* in the market (which induces a decrease in the market tightness for venture capitalists) will cause an increase in the *VCs*' equity share and a decrease in the valuations. Inderst and Muller (2004) found a similar result in a theoretical model, and Gompers and Lerner (2000) detected this relation empirically.

Further, when market conditions remain intact, our model predicts that for enterprises that require higher upfront investments (denoted by C), *VCs* are able to obtain a greater equity share (denoted by μ). While this observation is consistent with the practice, our model also revealed that the post-money valuation, namely

($\frac{C}{\mu}$), of such ventures is higher than for low-investment cost ventures. As both the numerator and the denominator increase, the value of the ratio is not obvious. Our theoretical prediction about the post-money valuation may be tested empirically.

ii) *Capital inflows and expected present value of VC investment*

Our study suggests that if the number of *VCs* decreases or the number of entrepreneurs increases in the market (implying a decrease in the market tightness for *VCs*), then the expected present value of investing in VC, the amount of capital raised, and the shares owned by *VCs* in each investment will all increase. Kortum and Lerner (2000), point to a surge of patents in the late 1980s and 1990s. If such increase in patents is associated with an increase in the number of entrepreneurs who seek venture capital, our model suggests that fund inflow should have increased during that period. Data provided by Dow Jones/Venture Source (2006) validates our prediction.

Further, the model predicts that improvements in ventures' prospects (higher F or σ) will induce increases both in the expected present value of investing in VC and in the amount of capital *VCs* raise from investors. This prediction is consistent with Lerner (2002), who describes a correlation between the advent of the Internet, an increase in the expected return from VC investment, and inflow of money into the industry.

Our framework also implies that better investment prospects cause capital flow into the industry to increase but the shares *VCs* get in their investee firms decrease, a point highlighted by Gompers and Lerner (2000). We note that our model considers capital inflows and the *VCs*' shares as endogenous variables that vary in response to changes in the fundamentals of the market and are therefore correlated.[30]

[30] Gompers and Lerner (2000) tested if the pricing of ventures investments were affected by exogenous increases in fund flows or if investment pricing and fund inflow are actually correlated with improvements in enterprises' prospects. They concluded that fund inflows varies exogenously and impacts prices, yet "the interpretation of these results is not without ambiguities" (pg. 284). Gompers, Kovner, Lerner and Scharfstein (2005) showed that the fluctuation in venture capital investment is a response to changes of fundamentals.

Finally, the model suggests that if the return on alternative investments increases then both the expected present value of investing in the VC industry and capital inflow will decrease.

iii) ***Acceptance criteria***

Our model predicts a positive and convex relation between the investment cost of a venture and the required expected value for acceptance.[31] This means that, as investment costs increase, the promised payoff value of the venture has to rise at an increasing rate. Further, the model predicts that if there is an improvement in the prospects of new ventures throughout the economy, *VCs* will increase the amount of funds they raise. This, in turn, will result in a decrease in the acceptance threshold of investments (denoted by S_C^*). Dow Jones/Venture Source (2006) industry data provides empirical support for both of these predictions, which were also noted by Lerner (2002).

iv) ***Number of venture capitalists and market tightness***

As discussed above, our model predicts that if there is an increase in the return from an alternative asset then the VC investment becomes less attractive in the short-run. Over time, therefore, *VCs* will exit the market. This reduces competition and increases the expected present value of investing in VC until it equals the returns from expected alternative investment opportunities.

Further, we suggest that if there is a sudden increase in the number of entrepreneurs in the market seeking funds or if the expected value of ventures in the market increases then the following two scenarios will occur: In the short-run, VC investments will become more attractive, inducing the entry of new *VCs* into the market.

[31] Throughout the paper, we emphasized the heterogeneity of firms observing the investment cost C and the value in the realization of the successful outcome S. Instead of S, we could say more generally that *VCs* rely on the expected value of the enterprise $\pi(S) = \sigma S + \delta F$ to make their assessment about whether or not to invest.

In the long run, however, competition among *VCs* will increase, thereby reducing the extra benefit from VC investments to a point where the present value of such investments equals the return from alternative investments.

Data from Ernst and Young (2006) suggest that there has been massive consolidation in the venture capital industry and a decline in the number of *VCs* in the market. Our model suggests that such decline could be the long-run effect of either a previous increase in return from competing assets, a decrease in the number of entrepreneurs in the market or a decrease in the expected proceeds from enterprises in the market. It may be interesting to empirically disentangle these effects and find each one's impact in the current, post-Internet-bubble era.

We close this section by pointing to a new avenue for empirical research in the venture capital field. Our model makes use of an implicit aggregate matching function that determines the probability that entrepreneurs and *VCs* will encounter each other given the number of market participants. It is a way to capture the search frictions with little added complexity. We chose functional forms that have been used in prior studies of the labor market (see Petrongolo and Pissarides (2001) for a survey), and we believe that search frictions affect the venture capital market in a similar fashion to the labor market. These markets, however, may differ in several important aspects, and these differences could affect the encounter rates between venture capitalists and entrepreneurs. For example, unlike labor market matches, in the VC market a match is associated with illiquid capital commitments. We believe that a fruitful line of research would be to estimate the matching function for the VC industry, to understand its behavior over time and to find variables that are likely to affect the matching function.

6 Discussion

We develop a multi-stage dynamic model of the venture capital market that reflects the imperfections that were empirically suggested by Gompers and Lerner (2000) and then goes further to consider the impact of other frictions. Specifically, we propose a multi-stage cycle characterized by three characteristics: uncertain matching of *VCs* and entrepreneurs, bargaining over equity shares, and random venture outcomes. We study the impact of market changes on the amount of capital that is committed to the industry and on the selection criteria of investors and entrepreneurs. We determine the outcome of any bilateral meeting, including whether or not a venture will be implemented and, in case it will, how both parties split the venture's equity.

By comparing the expected return from an investment in the VC industry with the return from a risk-free asset, our model delivers a closed-form solution to the problems venture capitalists face of how much capital they should raise from their investors and then what criteria they should use to select entrepreneurial ventures to invest in. We predict which ventures will be funded after being judged affordable and valuable. The analytical solutions depend on several parameters of the model, including, for instance, the probabilities of an encounter between an entrepreneur and a *VC*, venture success rates, and the ex-ante distribution of the capital needs of ventures and their potential to create value.

The analysis presented in this study is a first step toward broadening our understanding of a range of frictions that affect venture capital markets. It is possible to extend our model in several ways.[32] First, we could allow additional heterogeneity among agents. For example, a group of *VCs* could have more market experience and, therefore, greater probability of success with the ventures they invest in (higher σ or

[32] For instance, appendix E extends our results by calculating the flow of agents in each stage of the cycle, the expected duration of each stage, and the average cost and values of the funded ventures.

lower δ). Second, we could introduce a cost that entrepreneurs have to pay in order to enter the market for venture financing. This would address an important distortion and allow us to understand the determinants of market entrance for entrepreneurs. This may be relevant because some entrepreneurs may decide, as a result, not to search for *VCs*. Finally, it would be interesting to extend this model and allow *VCs* to engage in more than one project at a time and to consider the effect of syndication among *VCs*. Nonetheless, we believe the theory developed in this study provides new and important insights into the dynamics of the venture capital market.

Appendix A. Proofs.

Proof of Lemma 1. The surplus of a VC is $\Omega_v(S,C) = \mu\pi(S) - (r+\sigma+\delta)J_v^0(m) + (\sigma+\delta)(m - C + \mathcal{M})$. Substituting the bargaining in yields

$$\Omega_v(S,C) = \theta\left\{\pi(S) - (r+\sigma+\delta)\left[J_e^0 + J_v^0(m)\right] + (\sigma+\delta)(m - C + \mathcal{M})\right\}.$$

Proceeding in the same fashion, E's value function can be written as

$$\Omega_e(S,C) = (1-\theta)\left\{\pi(S) - (r+\sigma+\delta)\left[J_e^0 + J_v^0(m)\right] + (\sigma+\delta)(m - C + \mathcal{M})\right\}.$$

Hence, the result must be true. □

Proof of Proposition 1. Note that J_e^0, $J_v^0(m)$ and $\mathcal{M}$ do not depend on the specific realization of C. Then, from (11),

$$\frac{\partial\mu}{\partial C} = \frac{(\sigma+\delta)(1-\theta)}{\pi(S)} \geq 0 \text{ and } \frac{\partial\mu^2}{\partial C\partial S} = \frac{-\sigma(\sigma+\delta)(1-\theta)}{\pi(S)^2} \leq 0. \quad \square$$

Proof of Proposition 2. Again, note that $J_e^0, J_v^0(m)$ and $\mathcal{M}$ do not depend on S. Then, from (11),

$$\frac{\partial\mu\pi(S)}{\partial S} = \theta\sigma > 0$$

Also, from (14)

$$\begin{aligned}
\frac{\partial\mu}{\partial S} &= \frac{-\sigma}{\pi(S)^2}\left\{(1-\theta)\left[(r+\sigma+\delta)J_v^0(m) - (\sigma+\delta)(m - C + \mathcal{M})\right] - \theta(r+\sigma+\delta)J_e^0\right\} \\
&= \frac{-\sigma}{\pi(S)^2}\left\{\pi\left(S_C^*\right)\left[\mu(S_C^*) - \theta\right]\right\}
\end{aligned}$$

It is worth to note that $\mu(S_C^*) - \theta$ could be either positive or negative. For example, if $\theta = 1,$ then $\mu = \theta - \frac{\theta d_e}{\pi(S)} < \theta$. And if $\theta = 0$, then $\mu = \frac{d_v}{\pi(S)} > 0$. □

Proof of Proposition 3. As argued the post-money valuation is given by $\Gamma_{post} = \frac{C}{\mu(S,C)}$. Thus,

$$\frac{\partial \Gamma_{post}}{\partial C} = \frac{\mu - C\frac{\partial \mu}{\partial C}}{\mu^2}.$$

Such derivative is positive if, and only if, $\mu - C\frac{\partial \mu}{\partial C} > 0$.

As showed in the proof of Proposition 1, $\frac{\partial \mu}{\partial C} = \frac{(\sigma+\delta)(1-\theta)}{\pi(S)}$. When substituting μ given by equation (11), we get

$$\begin{aligned}
\mu - C\frac{\partial \mu}{\partial C} &= \theta\left[\pi(S) - (r+\sigma+\delta)J_e^0\right] + \\
&\quad (1-\theta)\left[(r+\sigma+\delta)J_v^0(m) - (\sigma+\delta)\left(m - C + \mathcal{M}\right)\right] - C\left(\sigma+\delta\right)\left(1-\theta\right) \\
&= \theta\left[\pi(S) - (r+\sigma+\delta)J_e^0\right] + (1-\theta)\left[rJ_v^0(m) + (\sigma+\delta)\left(J_v^0(m) - m - \mathcal{M}\right)\right]
\end{aligned}$$

For any venture that is implemented, it must be true that the entrepreneur surplus ($\Omega_e(S,C)$) is non-negative. Therefore,

$$\pi(S) - (r+\sigma+\delta)J_e^0 \geq \Omega_e(S,C) \geq 0.$$

and the first term in squared brackets is positive. Further, as discussed earlier, $\mathcal{M} = J_v^0(m) - m$ and the term in the second squared brackets becomes simply $rJ_v^0(m)$, which is positive. Therefore, $\frac{\partial \Gamma_{post}}{\partial C} > 0.\square$

Proof of Proposition 4. Applying the implicit function theorem to (17) yields

$$\frac{\partial S_C^*}{\partial C} = \frac{(\sigma+\delta)}{\sigma + \Delta\sigma \int_0^m \int_{S_C^*}^{\bar{S}} dG_S(\tilde{S})dG_C(C)} > 0.$$

This term is positive.

The only term that depends on C is S_C^* found at the lower limit of the integral in the denominator. Given an increase in C, the lower limit rises and the denominator value decreases. This implies $\frac{\partial^2 S_C^*}{\partial C^2} > 0$. □

Proof of Proposition 5. The proof follows the same line as the one for Proposition 4. The implicit function theorem to (17) implies

$$\frac{\partial S_C^*}{\partial m} = \frac{\Delta\sigma \int\limits_{S_{C=m}^*}^{\bar{S}} \left(S - S_{C=m}^*\right) dG_S(\tilde{S}) G_C'(m)}{\sigma + \Delta\sigma \int\limits_0^m \int\limits_{S_C^*}^{\bar{S}} dG_S(\tilde{S}) dG_C(C)}.$$

Thus, if $G_C'(m) > 0$, the term is strictly positive. □

Appendix B. Derivation of $J_v^{0\prime}(\mathbf{m})$.

The derivative of $J_v^0(m)$ with respect to m is found by applying Leibniz rule to equation (15) twice. Let $\Omega = \int\limits_{S_C^*}^{S} (S - S_C^*) dG_S(S|C)$. Then, by Leibniz,

$$\frac{\partial \Omega}{\partial m} = -\int\limits_{S_C^*}^{\bar{S}} \frac{\partial S_C^*}{\partial m} dG_S(\tilde{S}|C).$$

Then the same procedure is applied to the outside integral. This gives

$$rJ_v^{0\prime}(m) = \frac{\alpha_v \theta \sigma}{r + \sigma + \delta} \left\{ \int\limits_{S_{C=m}^*}^{\bar{S}} (S - S_{c=m}^*) dG_S(\tilde{S}|C) G_C'(m) + \int\limits_0^m \frac{\partial \Omega}{\partial m} dG_C(\tilde{C}) \right\}$$

Combining the two equations above and observing that $J_v^{0\prime}(m) = 1$, from equation (7), we reach equation (16).

Appendix C. Derivation of S_C^*.

After substituting $J_e^1(S)$ and $J_v^1(S)$ in equations (1) and (4)(respectively), we find that

$$rJ_e^0 = \frac{\alpha_e}{r+\sigma+\delta} \int_0^m \int_{S_C^*}^{\bar{S}} \left[(1-\mu)\pi(S) - (r+\sigma+\delta)J_e^0\right] dG_S(\tilde{S}|C)dG_C(\tilde{C}) \text{ and} \quad (22)$$

$$\begin{aligned} rJ_v^0(m) &= \frac{\alpha_v}{r+\sigma+\delta} \int_0^m \int_{S_C^*}^{\bar{S}} \left[\mu\pi(S) + (\sigma+\delta)(m - C + \mathcal{M})\right. \\ &\quad \left. - (r+\sigma+\delta) J_v^0(m)\right] dG_S(\tilde{S}|C)dG_C(\tilde{C}). \end{aligned} \quad (23)$$

What follows shows the procedure to modify and combine *Es*' value function. Such method is similar to the one for *VCs*, which we omit for sake of parsimony.

The term inside the squared brackets of (22) is simplified by first inserting equation (11) and then substituting the definition of S_C^* given by (14). This yields,

$$\begin{aligned} &(1-\mu)\pi(S) - (r+\sigma+\delta)J_e^0 = \\ &= (1-\theta)\left\{\pi(S) - (r+\sigma+\delta)\left[J_e^0 - J_v^0(m)\right] - (\sigma+\delta)(m - C + \mathcal{M})\right\} \\ &= (1-\theta)\sigma(S - S_C^*) \end{aligned}$$

Then, (22) becomes

$$rJ_e^0 = \frac{\alpha_e(1-\theta)\sigma}{r+\sigma+\delta} \int_0^m \int_{S_C^*}^{\bar{S}} (S - S_C^*) dG_S(\tilde{S}|C)dG_C(\tilde{C})$$

The same method for *VCs* yields

$$\mu\pi(S) + (\sigma+\delta)(m - C + \mathcal{M}) - (r+\sigma+\delta) J_v^0(m) =$$
$$= \theta\left\{\pi(S) - (r+\sigma+\delta)\left[J_e^0 - J_v^0(m)\right] - (\sigma+\delta)(m - C + \mathcal{M})\right\}$$
$$= \theta\sigma(S - S_C^*)$$

Then, (23) becomes

$$rJ_v^0(m) = \frac{\alpha_v\theta\sigma}{r+\sigma+\delta}\int_0^m\int_{S_C^*}^{\bar{S}}(S - S_C^*)dG_S(\tilde{S}|C)dG_C(\tilde{C}).$$

Since $\mathcal{M} = J_v^0(m) - m$ equation (14) becomes

$$\pi\left(S_C^*\right) = (r+\sigma+\delta)J_e^0 + rJ_v^0(m) + (\sigma+\delta)C$$

Then plug in the equations above for rJ_e^0 and $rJ_v^0(m)$ and reach (17).

Appendix D: Plots from the simulations

In the simulations below, $\tilde{S}$ and $\tilde{C}$ were assumed to be distributed independently and uniformly between [0,1]. To plot the endogenous variables S_C^*, m and μ against the exogenous parameters we must allow one parameter to vary and set the other values constant. For consistency, we maintain the same values for the constant variables over different plots.

When fixed, the chosen parameters values are $\theta = 0.6$, $\alpha = 0.01$, $\sigma = 0.4$, $\delta = 0.04$, $F = 0.3$, $\beta = 0.5$, $r = 0.01$ and $C = 0.5$. When varying, the range of values is shown in each graph. The results found do not depend on specific parameter values.

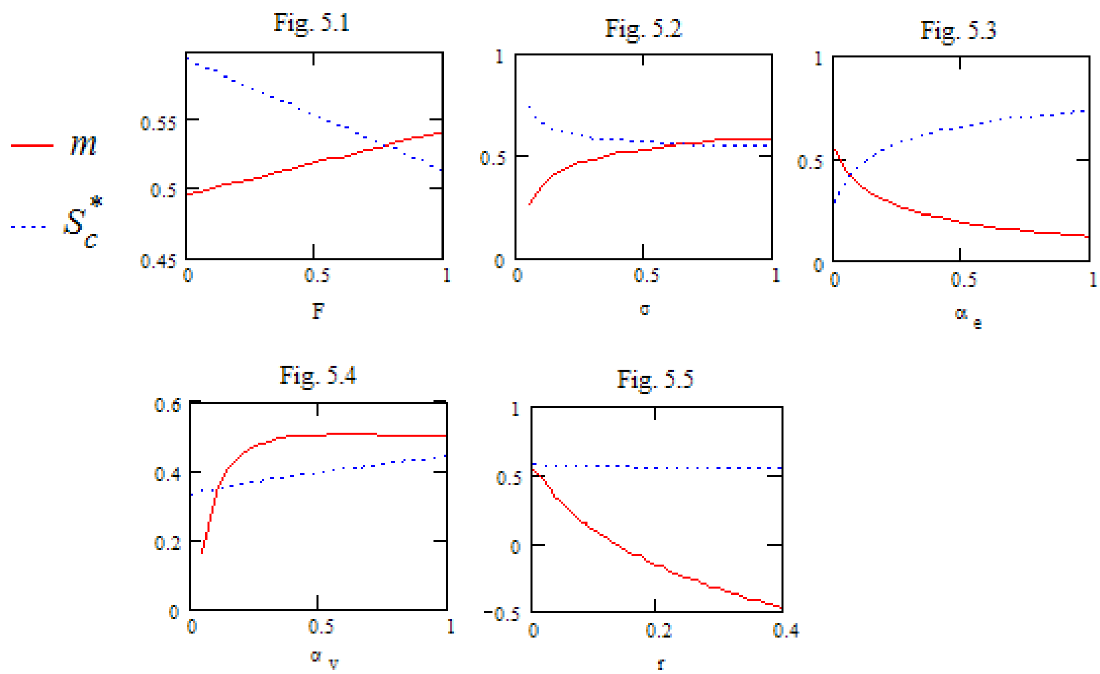

Figure 5: Plot of acceptance threshold S_C^* and raised capital m.

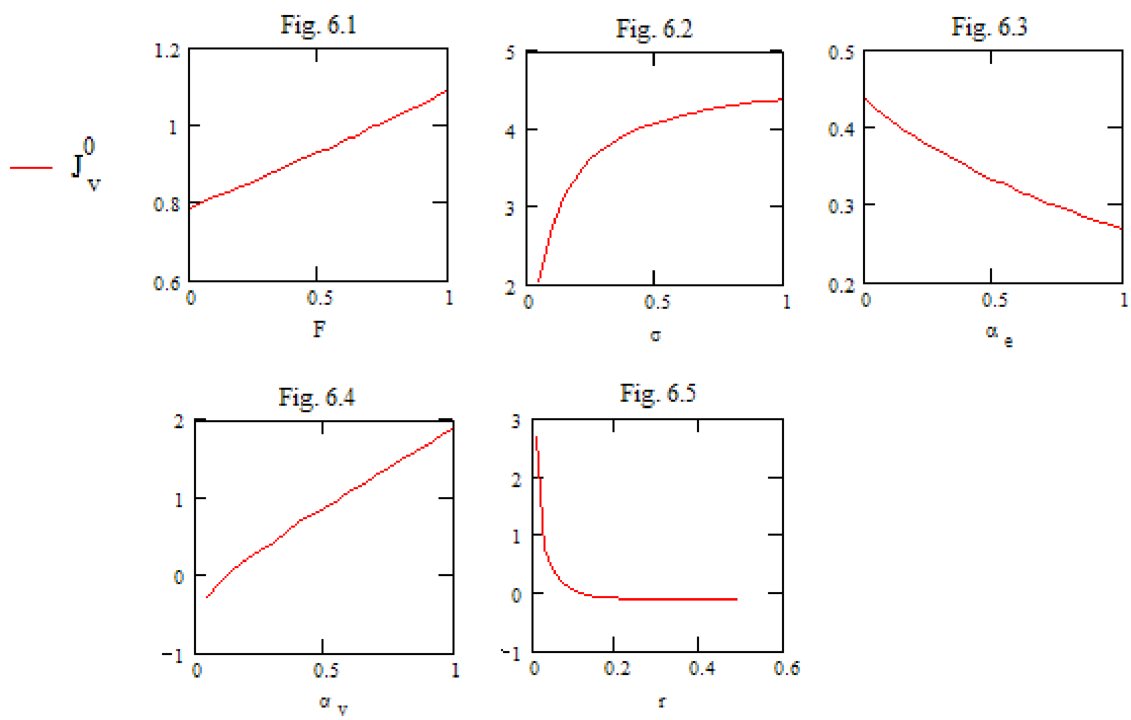

Figure 6: Plot of expected value of VC investment J_v^0.

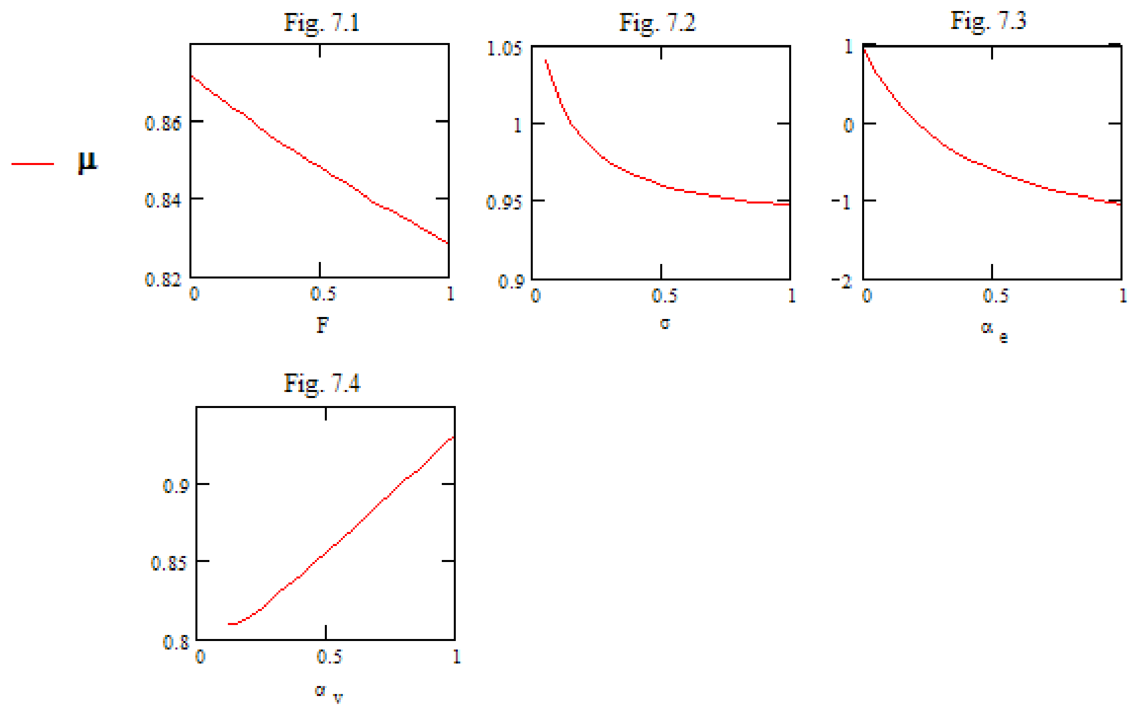

Figure 7: Plot of the fraction of equity owned by *VCs.*

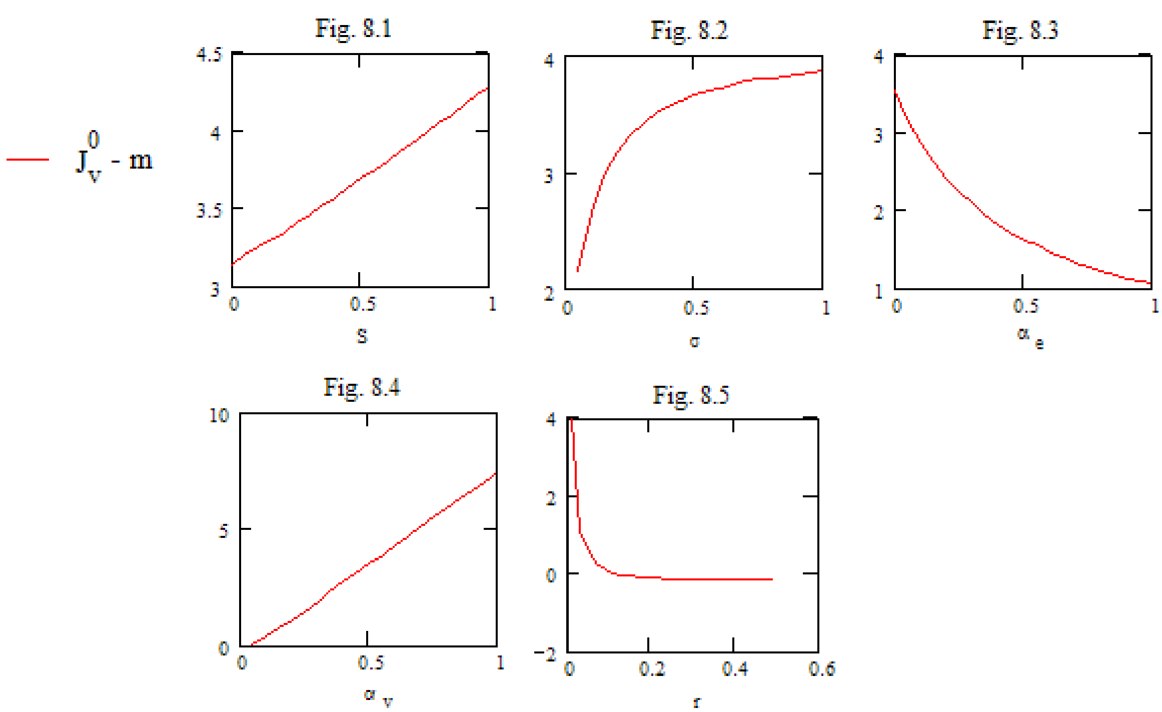

Figure 8: Plot of the extra return of VC investment over risk-free asset $J_v^0 - m$.

The statements in Finding 1 are derived from Figures 5.1, 5.2, 6.1, 6.2, 7.1 and

7.2. In a similar manner, Finding 2 is supported by plots 5.3, 5.4, 6.3, 6.4, 7.3 and 7.4. Finding 3 is corroborated by Figures 5.5 and 6.5.

The probabilities of encounters α_v and α_e are functions of τ only, where α_v is strictly decreasing and α_e is strictly increasing in τ.[33] Thus, we can use plots 8.3 and 8.4 to make inferences about the impact of τ on the extra return of VC investment over the risk-free asset $J_v^0(m) - m$. The theory presented in Section 4 in combination with Figures 8.3, 8.4 and 8.5 corroborate Finding 4. Similarly, the arguments in Section 4 and Figures 8.1 through 8.4 support Finding 5.

Appendix E: Expected durations and returns

We can calculate equilibrium properties of the venture financing market. More specifically, we will determine the flow of agents in each stage of the cycle, the expected duration of each stage (and of the whole investment) and finally the average cost and value of the funded projects in the venture finance market.

The probability that an unmatched agent will find a project that is affordable and acceptable (conditional on its cost and return) is called *search-stage hazard rate.* It is denoted by H^s. In the environment presented, H^s is the probability that an encounter occurs (α_j), and it is affordable ($C \leq m$) and acceptable ($S \geq S_C^*$). Thus, for $j \in \{v, e\}$,

$$H_j^s = \alpha_j \left\{ G_C(m) \left[1 - G_S(S_C^* | C) \right] \right\}.$$

In a similar manner, the probability a venture comes to maturation and agents move out of the implementation stage is called *implementation-stage hazard rate* and is represented by $H^i = \sigma + \delta$, equal for both agents. This is the probability that the venture maturated with success (σ) or with failure (δ).

Hence, the flow out of partnership is $(N_j - u_j)H^i$, while the flow into partnerships (agents leaving the search stage and moving into the implementation stage) is stated

[33] Note that α_e and α_v have opposite impacts in every one of the plotted endogenous variables.

as $u_j H_j^s$ for $j \in \{v, e\}$.

We can calculate the implied path of u_j and the fraction of the population which is unmatched. The change in the number of unmatched agents is given by $\dot{u}_j = (N_j - u_j)H^i - u_j H_j^s$. For any initial condition, $u_j \to u_j^*$, where $u_j^* = \frac{N_j H^i}{H^i + H_j^s}$.

Given these objects, we can calculate the expected duration of each stage. The probability of an agent of type j being unmatched for exactly d periods is $(1 - H_j^s)^{d-1} H_j^s$.[34] Thus, the expected duration of the *venture-search* stage is

$$\begin{aligned} ED_j^s &= \sum_{d=1}^{\infty} d(1 - H^s)^{d-1} H^s = \frac{1}{H_j^s} \\ &= \frac{1}{\alpha_j \left\{ G_C(m) \left[1 - G_S(S^* | C) \right] \right\}} \text{ for } j \in \{v, e\}. \end{aligned} \tag{24}$$

By a similar argument, the expected duration of the *implementation* stage is

$$ED^i = \frac{1}{H^i} = \frac{1}{\sigma + \delta}. \tag{25}$$

Therefore, the expected investment spell or total duration (denoted by ED_j) of funds in the VC market is $ED_j = ED_j^s + ED^i$. The term ED_v is the expected time funds will take to return to investors once they are allocated to the VC investment. Similarly, ED_e is the expected time entrepreneurs will take to receive the proceeds from the project once they entered the market for financing.

Finally, let EP be the average return of invested ventures at the time they payback; and let EC be the average cost of invested ventures at the time it receives funding in the venture capital market. Since the distributions are truncated (G_S from below S_C^* and G_C from above m), EP and EC are given by

$$EP = \int_0^m \int_{S_C^*}^{\bar{S}} \left[\frac{\mu \sigma S}{[1 - G_S(S_C^* | C)]} + \mu \delta F \right] dG_S(\tilde{S} | C) dG_C(\tilde{C}) \tag{26}$$

[34] This is the probability of rejecting exactly $d - 1$ partners and then accepting the d^{th} one.

$$EC = \int_{0}^{m} \int_{S_C^*}^{\bar{S}} \frac{C}{G_C(m)} dG_S(\tilde{S}|C) dG_C(\tilde{C}). \tag{27}$$

One can study the impact of changes of exogenous parameters on the expected spell of each stage, the average return and the average cost of the implemented projects by plugging the equilibrium m and S_C^* - defined by equations (17) and (19) - in (24), (25), (26) and (27) respectively.

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