

MATHEMATICS FOR ECONOMISTS  
ECON-897 Lecture Notes: Part I

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<sup>1</sup>These are only slightly revised class notes prepared by previous instructors of this course, most recently by Gabriel Bassaluzzo. All remaining errors are, of-course, my responsibility.



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