

# Search-Theoretic Models of Money

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## Motivation

- What is money?
- Why do we use money as a society?
- Use search theory to model existence of money.
- Contrast with other two approaches:
  1. Money in DSGE models (CIA, MIU).
  2. Money in OLG models.

## Essential Models of Money

- Hahn (1965): money is essential if it allows agents to achieve allocations they cannot achieve with other mechanisms that also respect the enforcement and information constraints in the environment.
- Why do we care about essential models of money?
- Three frictions that will make money essential:
  1. Double-coincidence of wants problem.
  2. Long-run commitment cannot be enforced.
  3. Agents are anonymous: histories are not public information.
- Money is a consequence of these frictions in trade: medium of exchange.

## Three Generations of Models

1. 1 unit of money, 1 unit of good: Kiyotaki and Wright (1993).
2. 1 unit of money, endogenous units of good: Trejos and Wright (1995).
3. Endogenous units of money, endogenous units of good: Lagos and Wright (2005).

## Environment

- $[0, 1]$  continuum of anonymous agents.
- Live forever and discount future at rate  $r$ .
- $[0, 1]$  continuum of goods. Good  $i$  is produced by agent  $i$ .
- Goods are non-storable: no commodity money.
- Unit cost of production  $c \geq 0$ .

## Double-Coincidence of Wants Problem

- I do not produce what I like (non-restrictive: home production, specialization).
- $iWj$ : agent  $i$  likes to consume good produced by agent  $j$  :.
  1. utility  $u > c$  from consuming  $j$ .
  2. utility 0 otherwise.
- Probabilities of matching:

$$p(iWi) = 0$$

$$p(jWi) = x$$

$$p(jWi|iWj) = y$$

## First Generation: Fixed Money and Fixed Good

- Exogenously given quantity  $M \in [0, 1]$  of an indivisible unit of storable good.
- Holding money yields zero utility  $\gamma$ : fiat money.
- Initial endowment:  $M$  agents are randomly endowed with one unit of money.
- Agents holding money cannot produce (for example because you need to consume before you can produce again).
- We eliminate (non-trivial) distributions.

## Trades

- Pairwise random matching of agents with Poisson arrival time  $\alpha$ .
- Bilateral trading is important, randomness is not (Corbae, Temzelides, Wright, 2003).
- Upon meeting, agents decide whether to trade. Then, they part company and re-enter the process.
- History of previous trades is unknown.
- Exchange 1 unit of good for 1 unit of good (barter) or 1 unit of money.

## Individual Trading Strategies

- Agents never accept a good in trade if he does not like to consume it since it is not storable.
- They will barter if they like the both agents in the pair like each other goods.
- Would they accept money for goods and viceversa?
- We will look at stationary and symmetric Nash equilibria.

## Probabilities

- You meet someone with arrival rate  $\alpha$ .
- This person can produce with probability  $1 - M$ .
- With probability  $x$  you like what he produces.
- With probability  $\pi = \pi_0\pi_1$  (endogenous objects to be determined) both of you want to trade.
- If  $\pi > 0$ , we say that money circulates.

## Value Functions

- Value functions with money,  $V_1$ :

$$rV_1 = \alpha x (1 - M) \pi (u + V_0 - V_1)$$

- Value functions without money,  $V_0$ .

$$rV_0 = \alpha x y (1 - M) \pi (u - c) + \alpha x M \pi (V_1 - V_0 - c)$$

- Renormalize  $\alpha x = 1$  by picking time units:

$$\begin{aligned} rV_1 &= (1 - M) \pi (u + V_0 - V_1) \\ rV_0 &= y (1 - M) \pi (u - c) + M \pi (V_1 - V_0 - c) \end{aligned}$$

## Individual Trading Strategies

- Net gain from trading goods for money:

$$\Delta_0 = V_1 - V_0 - c = \frac{(1 - M)(\pi - y)(u - c) - rc}{r + \pi}$$

- Net gain from trading money from goods:

$$\Delta_1 = u + V_0 - V_1 = \frac{(M\pi + (1 - M)y)(u - c) + ru}{r + \pi}$$

## Equilibrium Conditions for $\pi_0$ and $\pi_1$

- Clearly:

$$\pi_j \begin{cases} = 1 \\ \in [0, 1] \\ = 0 \end{cases} \quad \text{as } \Delta_j \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases}$$

- Plug those into the individual trading strategies, and check them.

## Characterizing $\pi$

- Clearly  $\Delta_1 > 0$ . Hence  $\pi_1 = 1$ , i.e., the agent with money always wants to trade.

- For  $\pi_0$ , you have

$$\Delta_0 = \frac{(1 - M)(u - c)\pi_0}{r + \pi_0} - \frac{(1 - M)y(u - c) + rc}{r + \pi_0}$$

- Then,  $\Delta_0$  has the same sign as

$$\pi_0 - \frac{rc + (1 - M)y(u - c)}{(1 - M)(u - c)} = \pi_0 - \hat{\pi}$$

## Multiple Equilibria

- Nonmonetary equilibrium: we have an equilibrium where  $\pi_0 = 0$ .
- Monetary equilibrium: if

$$c < \frac{(1 - M)(1 - y)}{r + (1 - M)(1 - y)}u$$

then  $\hat{\pi} < 1$  and  $\pi_0 = 1$  is an equilibrium as well.

- Mixed-monetary equilibrium:  $\pi_0 = \hat{\pi}$ . However, not robust (Schevchenko and Wright, 2004).

## Welfare

- Define welfare as the average utility:

$$W = MV_1 + (1 - M) V_0$$

- Then:

$$rW = (1 - M) [(1 - M) y + M\pi] (u - c)$$

- Note that welfare is increasing in  $\pi$ .

## Welfare $\pi = 1$

- Note:

$$\begin{aligned} rW &= (1 - M) [(1 - M)y + M] (u - c) \\ &= -My + M^2y \end{aligned}$$

- Maximize  $W$  with respect to  $M$  :

$$M^* = \frac{1 - 2y}{2 - 2y} \text{ if } y < \frac{1}{2}$$

$$M^* = 0 \text{ if } y \geq \frac{1}{2}$$

- Intuition: facilitate trade versus crowding out barter.

Welfare  $\pi = 0$

- Note:

$$rW = (1 - M) [(1 - M) y] (u - c)$$

- Monotonically decreasing in  $M \Rightarrow M^* = 0$ .
- Result is a little bit silly: it depends on the absence of free disposal of money. Otherwise, welfare is independent of  $M$ .

## Welfare $\pi$

- Note:

$$rW = (1 - M) [(1 - M)y + M\pi] (u - c)$$

- Monotonically increasing in  $M$  in the  $[0, 1]$  interval.

## Comparison with Alternative Arrangements

- Imagine that we have the credit arrangement: “produce for anyone you meet that wants your good.”

- Value function

$$rV_c = u - c$$

- Clearly

$$rV_c > rW$$

- However, this arrangement is not self-enforceable: histories are not observed.

## Second Generation: Endogenous Prices

- We make the very strong assumption that we exchanged one good for one unit of money.
- What if we let prices to be endogenous? Shi (1995) and Trejos and Wright (1995).
- We set  $y = 0$  and we let goods to be divisible.
- When agents meet, they bargain about how much  $q$  will be exchanged, or equivalently, about price  $1/q$ .

## Utility and Cost Functions

- Utility is  $u(q)$  and cost of production is  $c(q)$ .
- Assumptions:

$$\begin{aligned}u(0) &= c(0) = 0 \\u'(0) &> c'(0) \\u'(0) &> 0, u''(0) \leq 0 \\c'(0) &> 0, c''(0) \geq 0\end{aligned}$$

- Also,  $\hat{q}$  and  $q^*$  are such that

$$\begin{aligned}u'(\hat{q}) &= c'(\hat{q}) \\u'(q^*) &= c'(q^*)\end{aligned}$$

## Value Functions and Bargaining

- Take  $q = Q$  as given. Then:

$$\begin{aligned}rV_1 &= (1 - M) [u(Q) + V_0 - V_1] \\rV_0 &= M [V_1 - V_0 - c(Q)]\end{aligned}$$

- Bargaining is the generalized Nash bargaining solution:

$$\begin{aligned}q &= \arg \max [u(q) + V_0(Q) - T_1]^\theta \times [V_1(Q) - c(Q) - T_0]^\theta \\&\quad u(q) + V_0 \geq V_1 \\&\quad V_1 - c(q) \geq V_0\end{aligned}$$

where  $T_j$  is the threat point of the agent with  $j$  units of money.

- We will set  $T_j = 0$  and  $\theta = 1/2$ .

## Equilibria

- Necessary condition taking  $V_0(Q)$  and  $V_1(Q)$  as given:

$$[V_1(Q) - c(q)] u'(q) = [u(q) + V_0(Q)] c'(q)$$

- The bargaining solution defines a function

$$q = e(Q)$$

and we look at its fixed points.

- Two fixed points:
  1.  $q = 0$ : nonmonetary equilibrium.
  2.  $q = q^e > 0$ : monetary equilibrium.

## Efficiency

- Note that the efficient outcome is  $q^*$ , i.e.  $u'(q^*) = c'(q^*)$ .

- In the monetary equilibrium:

$$u'(q^e) = \frac{u(q^e) + V_0(q^e)}{V_1(q^e) - c(q^e)} c'(q^e) > u'(q^*)$$

since  $u(q^e) + V_0(q^e) > V_1(q^e) - c(q^e)$ .

- Hence  $q^* > q^e$ , or equivalently, the price is too high.

## Third Generation: Endogenous Prices and Goods

- Relax the assumption that agents hold 0 or 1 units of money.
- Problem: endogenous distribution of money that we (and the agents!) need to keep track of.
- Computational: Molicco (2006).
- Theoretical:
  1. Families: Shi (1997).
  2. Two markets: Lagos and Wright (2005).