Job Search Models

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Motivation

- We want to have dynamic models of the job market.
- Example of questions we are interested in.
- Lucas: "theoretical sterility of involuntary unemployment."
- Prescott: "unemployment is a state of mind."
- Equilibrium models of unemployment.

Search Models

- We will begin with a simple model of job search.
- Matching is costly. Think about getting a date.
- We can bring our intuition to the job market. Why?
- Useful to illustrate many ideas and for policy analysis.
- Contributions of:
  1. Stigler (1961).
  2. McCall (1970).
- Static problem versus sequential.

Stigler's Model

- Risk-neutral agent.
- Easier to think as an agent asking for bids.
- Samples offers i.i.d. from $F(w)$.
- Decide ex-ante how many offers n she is going to ask for.
- Each offer has a cost c.

Optimal Number of Offers I

- Remember that:

$$M_n = \mathbb{E}\min\left(w_1, w_2, ..., w_n\right) = \int_0^\infty (1 - F(w))^n \, dw$$

- Then, gain of additional offer is:

$$\begin{aligned}
G_n &= M_{n-1} - M_n \\
&= \int_0^\infty (1 - F(w))^{n-1} \, dw - \int_0^\infty (1 - F(w))^n \, dw \\
&= \int_0^\infty (1 - F(w))^{n-1} \, dw - \int_0^\infty (1 - F(w))^{n-1} (1 - F(w)) \, dw \\
&= \int_0^\infty (1 - F(w))^{n-1} F(w) \, dw
\end{aligned}$$

Optimal Number of Offers II

- Then G_n is a decreasing function with $\lim_{n \to \infty} G_n = 0$.

- Optimal rule: set n such that

$$G_n \geq c > G_{n+1}$$

- Basic problem of static decisions: What if I get the lowest possible price in my first offer?

McCall's Model

- An agent searches for a job, taking market conditions as given.

- Preferences:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t x_t$$

  where

$$x_t = \begin{cases} = w \text{ if employed} \\ = z \text{ if unemployed} \end{cases}$$

- Interpretation of w and z.

Job Offers

- An unemployed agent gets every period one offer i.i.d. from $F(w)$.

- Offer can be rejected (unemployed next period) or accepted.

- No recall of offers (no restrictive because of stationarity of the problem).

- Job last forever (neither quitting nor firing).

Bellman Equations

- Value function of employed agent:

$$W(w) = w + \beta W(w)$$

Clearly: $W(w) = \frac{w}{1-\beta}$.

- Value function of unemployed agent:

$$U = z + \beta \int_0^\infty \max\{U, W(w)\}\, dF(w)$$

Then:

$$U = z + \beta \int_0^\infty \max\left\{U, \frac{w}{1-\beta}\right\} dF(w)$$

Reservation Wage

- There exist a reservation wage w_R

$$W(w_R) = U = \frac{w_R}{1-\beta}$$

such that if $w \geq w_R$ the worker should accept the offer and reject otherwise.

- Then:

$$w_R = T(w_R) = (1-\beta) z + \beta \int_0^{\infty} \max\{w_R, w\} \, dF(w)$$

that is a contraction (i.e. $\lim_{N\to\infty} T^N(w_0) = w_R$ and w_R is unique).

Characterizing Strategy I

Note:

$$\frac{w_R}{1-\beta} = z + \beta \int_0^\infty \max\left\{\frac{w_R}{1-\beta}, \frac{w}{1-\beta}\right\} dF(w) \Rightarrow$$

$$\int_0^{w_R} \frac{w_R}{1-\beta} dF(w) + \int_{w_R}^\infty \frac{w_R}{1-\beta} dF(w) =$$

$$= z + \beta \int_0^{w_R} \frac{w_R}{1-\beta} dF(w) + \beta \int_{w_R}^\infty \frac{w}{1-\beta} dF(w) \Rightarrow$$

$$w_R \int_0^{w_R} dF(w) - z = \beta \int_{w_R}^\infty \frac{\beta w - w_R}{1-\beta} dF(w)$$

Characterizing Strategy II

- Adding $w_R \int_{w_R}^{\infty} dF(w)$ to both sides:

$$w_R - z = \frac{\beta}{1-\beta} \int_{w_R}^{\infty} (w - w_R)\, dF(w)$$

- Interpretation

$$\underbrace{w_R - z}_{\text{Cost of Search one more time}} = \underbrace{\frac{\beta}{1-\beta} \int_{w_R}^{\infty} (w - w_R)\, dF(w)}_{\text{Expected Gain of one more search}}$$

- Sequential nature of the problem.

- Integrating by parts $\int_{w_R}^{\infty} (w - w_R)\, dF(w) = \int_{w_R}^{\infty} (1 - F(w))\, dw$ and then:

$$w_R - z = \frac{\beta}{1-\beta} \int_{w_R}^{\infty} (1 - F(w))\, dw$$

Characterizing Strategy III

Note that

$$\begin{aligned}
w_R - z &= \frac{\beta}{1-\beta}\int_{w_R}^{\infty}(w-w_R)\,dF(w) + \frac{\beta}{1-\beta}\int_0^{w_R}(w-w_R)\,dF(w) \\
&\quad -\frac{\beta}{1-\beta}\int_0^{w_R}(w-w_R)\,dF(w) \\
&= \frac{\beta}{1-\beta}\int_0^{\infty}(w-w_R)\,dF(w) - \frac{\beta}{1-\beta}\int_0^{w_R}(w-w_R)\,dF(w) \\
&= \frac{\beta}{1-\beta}\int_0^{\infty} w\,dF(w) - \frac{\beta}{1-\beta}w_R - \frac{\beta}{1-\beta}\int_0^{w_R}(w-w_R)\,dF(w) \\
&= \frac{\beta}{1-\beta}\mathbb{E}w - \frac{\beta}{1-\beta}w_R - \frac{\beta}{1-\beta}\int_0^{w_R}(w-w_R)\,dF(w)
\end{aligned}$$

Characterizing Strategy IV

- Now:

$$w_R - z = \frac{\beta}{1-\beta}\mathbb{E}w - \frac{\beta}{1-\beta}w_R - \frac{\beta}{1-\beta}\int_0^{w_R}(w - w_R)\,dF(w) \Rightarrow$$

$$(1-\beta)(w_R - z) = \beta\mathbb{E}w - \beta w_R - \beta\int_0^{w_R}(w - w_R)\,dF(w) \Rightarrow$$

$$w_R - z = \beta(\mathbb{E}w - z) - \beta\int_0^{w_R}(w - w_R)\,dF(w)$$

- Integrating by parts $\int_0^{w_R}(w - w_R)\,dF(w) = -\int_0^{w_R}F(w)\,dw$ and then:

$$w_R - z = \beta(\mathbb{E}w - z) + \beta\int_0^{w_R}F(w)\,dw$$

Comparative Statics

Factors that affect search strategy:

1. Value of unemployment z.

   Unemployment Insurance: Length and generosity of unemployment insurance vary greatly across countries. US replacement rate is 34%. Germany, France and Italy the replacement rate is about 67%, with duration well beyond the first year of unemployment.

2. Distribution of offers.

   Minimum Wages: If the minimum wage is so high that it makes certain jobs unprofitable, less jobs are offered and job finding rates decline.

Problems of the Model

Rothschild (1973): Where does the distribution $F(w)$ come from?

Diamond (1971): Why is the distribution not degenerate?

Answers

1. Exogenously given: different productivity opportunities.

2. Endogenous: Lucas and Prescott model of islands economy.

Lucas and Prescott (1974)

- Continuum of workers.
- Workers are risk adverse (or there are complete markets).
- A large number of separated labor markets (Islands).
- There is a firm in each island subject to productivity shocks.
- Wage is determined competitively in each island.

Firms

- Each island has an aggregate production function:

$$\theta f(n)$$

where θ is a productivity shock, n is labor, and f has decreasing returns to scale.

- θ evolve according to kernel $\pi(\theta, \theta')$.

- There is a stationary distribution of θ.

Worker

- At the beginning of the period, worker observes:

    1. Productivity θ.

    2. Amount of worker on the island x.

    3. Distribution of islands in the economy $\Psi(\theta, x)$.

- They decide whether or not to move:

    1. If it stays, workers will get wage $w(\theta, x)$.

    2. If it moves, it does not work this period and picks which island to move to.

Equilibrium within the Island

- Firms maximize:

$$w(\theta, x) = \theta f'(n(\theta, x))$$

- Markets clear:

$$n(\theta, x) \leq x$$

Value Function for the Worker

- The Bellman equation for the worker is given by:

$$v(\theta, x) = \max \left\{ \beta v_u, w(\theta, x) + \beta \int v\left(\theta', x'\right) d\theta \right\}$$

where v_u is the value of search.

- Three cases:

1. $v(\theta, x) = \beta v_u$: some workers are leaving the market.
2. $v(\theta, x) > \beta v_u$: no worker is leaving the market. Some may or may not arrive.
3. $v(\theta, x) < \beta v_u$: cannot happen.

Case 2

- No worker is leaving but some workers are arriving:

$$v_u = \int v\left(\theta', x'\right) d\theta$$

  Thus:

$$v(\theta, x) = \theta f'(n(\theta, x)) + \beta v_u.$$

- No worker is leaving and no workers are arriving:

$$v(\theta, x) = \theta f'(n(\theta, x)) + \beta \int v\left(\theta', x'\right) d\theta \leq \theta f'(n(\theta, x)) + \beta v_u$$

A New Expression

- Putting all these cases together:

$$v(\theta, x) = \text{max}\left\{\beta v_u, \theta f'(n(\theta, x)) + \text{min}\left\{\beta v_u, \beta \int v\left(\theta', x'\right) d\theta\right\}\right\}$$

- Functional equation on $v(\theta, x)$.

- Unique solution.

Evolution of the Labor Force

- Some agents leave the market. Then $x' = n(\theta, x)$ solves:

$$\theta f'(n(\theta, x)) + \beta \int v\left(\theta', x'\right) d\theta = \beta v_u$$

- No worker is leaving but some will arrive next period. Then x' solves:

$$\int v\left(\theta', x'\right) d\theta = v_u$$

- No worker is leaving and no workers will arrive next period. Then:

$$x' = x$$

Stationary Distribution

- The evolution of (θ, x) is then governed by a function $\Gamma\left(\theta', x' \middle| \theta, x\right)$ that embodies the equations above.

- Then the stationary distribution solves:

$$\Psi(\theta, x) = \int \Gamma\left(\theta', x' \middle| \theta, x\right) \Psi(\theta, x)\, d\theta$$

- From the stationary distribution we can find v_u.

Álvarez and Veracierto (1999)

- Now, instead of going to their favorite island, unemployed workers search for a new job randomly.

- Every period they find one island from distribution $\Psi(\theta, x)$.

- They decide whether to accept it or reject it.

- Endogenous distribution of wage offers.