Ramsey Fiscal Policy

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Optimal Fiscal Policy

- We can use dynamic equilibrium theory to think about the design and implementation of optimal policy.

- Reasons for a non-trivial problem: absence of a lump-sum tax.

- We will focus first in the case of full commitment: Ramsey problems.

- Two approaches:

  1. Primal approach: we search directly for allocations by maximizing a social planner's problem subject to an implementability constraint. Then, we decentralize the allocation.

  2. Dual approach: we search directly for optimal taxes.

A Nonstochastic Economy

- Preferences:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

- Budget constraint:

$$c_t + k_{t+1} + \frac{b_{t+1}}{R_t} = \left(1 - \tau_t^l\right) w_t l_t + \left[1 + \left(1 - \tau_t^k\right)(r_t - \delta)\right] + b_t$$

- Technology: representative firm

$$c_t + g_t + k_{t+1} = F(k_t, l_t) + (1 - \delta) k_t$$

- Government:

$$g_t = \tau_t^k (r_t - \delta) k_t + \tau_t^l w_t l_t + \frac{b_{t+1}}{R_t} - b_t$$

Competitive Equilibrium

A Competitive Equilibrium is an allocation $\{\widehat{c}_t, \widehat{l}_t, \widehat{k}_t, \widehat{g}_t\}_{t=0}^{\infty}$, a price system $\{\widehat{w}_t, \widehat{r}_t, \widehat{R}_t\}_{t=0}^{\infty}$, and a government policy $\{\widehat{g}_t, \widehat{\tau}_t^k, \widehat{\tau}_t^l, \widehat{b}_t\}_{t=0}^{\infty}$ such that:

1. Given prices and the government policy, households maximize.

2. Given prices, firms minimize costs.

3. Government satisfies its budget constraint.

Note that 3. plus the budget constraint of households deliver market clearing.

Ramsey equilibrium

- Fix a sequence of exogenously given government purchases $\{g_t\}_{t=0}^{\infty}$ (alternative: g_t can be a choice variable given some utility from government consumption).

- A Ramsey equilibrium is the best competitive equilibrium given $\{g_t\}_{t=0}^{\infty}$, k_0, b_0, and bounds on τ_t^k.

- Note that best is defined ex-ante.

Consolidating Budget Constraints

- Consolidate two consecutive budget constraints:

$$c_t + \frac{c_{t+1}}{R_t} + \frac{k_{t+2}}{R_t} + \frac{b_{t+2}}{R_t R_{t+1}} =$$

$$\left(1 - \tau_t^l\right) w_t l_t + \left(1 - \tau_{t+1}^l\right) \frac{w_{t+1} l_{t+1}}{R_t} + \left(1 + \left(1 - \tau_t^k\right) (r_t - \delta)\right) k_t$$

$$+ \left(\frac{1 + \left(1 - \tau_{t+1}^k\right) (r_{t+1} - \delta)}{R_t} - 1 \right) k_{t+1} + b_t$$

- By no arbitrage: $R_t = 1 + \left(1 - \tau_{t+1}^k\right)(r_{t+1} - \delta)$. Then:

$$c_t + \frac{c_{t+1}}{R_t} + \frac{k_{t+2}}{R_t} + \frac{b_{t+2}}{R_t R_{t+1}} =$$

$$\left(1 - \tau_t^l\right) w_t l_t + \left(1 - \tau_{t+1}^l\right) \frac{w_{t+1} l_{t+1}}{R_t} + \left(\left(1 - \tau_t^k\right) r_t + 1 - \delta\right) k_t + b_t$$

Asset Pricing

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- Define

$$Q(t|0) = \prod_{i=1}^{t} R_i^{-1}$$

where clearly $Q(0|0) = 1$.

- Also, we have

$$\frac{Q(t|0)}{Q(t+1|0)} = \frac{1}{\beta} \frac{u_c(t)}{u_c(t+1)}$$

Resource Constraint Again

- Using asset prices to iterate on the budget constraint:

$$\sum_{t=0}^{\infty} Q(t|0)\, c_t = \sum_{t=0}^{\infty} Q(t|0) \left(1 - \tau_t^l\right) w_t l_t + \left(1 + \left(1 - \tau_0^k\right)(r_0 - \delta)\right) k_0 + b_0$$

subject to

$$\lim_{T\to\infty} \left(\prod_{i=1}^{T} R_i^{-1}\right) k_{T+1} = \lim_{T\to\infty} Q(T-1|0)\, k_{T+1} = 0$$
$$\lim_{T\to\infty} Q(T|0)\, b_{T+1} = 0$$

- Role of transversality conditions.

Necessary Conditions

- Necessary conditions for households:

$$\beta^t u_c(t) \dot{-} \lambda Q(t|0) = 0$$
$$-\beta^t u_l(t) - \lambda Q(t|0)\left(1 - \tau_t^l\right) w_t = 0$$
$$-\lambda Q(t|0) + \lambda Q(t+1|0)\left(1 + \left(1 - \tau_{t+1}^k\right)(r_{t+1} - \delta)\right) k_t = 0$$

- Given $Q(0|0) = 1$, we can find

$$Q(t|0) = \beta^t \frac{u_c(t)}{u_c(0)}$$

and:

$$\frac{u_l(t)}{u_c(t)} = \left(1 - \tau_t^l\right) w_t$$

- From firms' problem:

$$\begin{aligned} r_t &= F_k(t) \\ w_t &= F_l(t) \end{aligned}$$

Budget Constraint

- Substituting necessary conditions in the budget constraint of household:

$$\sum_{t=0}^{\infty}\beta^t\frac{u_c(t)}{u_c(0)}c_t=\sum_{t=0}^{\infty}\beta^t\frac{u_c(t)}{u_c(0)}\frac{u_l(t)}{u_c(t)}l_t+\left(1+\left(1-\tau_0^k\right)(r_0-\delta)\right)k_0+b_0$$

- Rearranging terms:

$$\sum_{t=0}^{\infty}\beta^t\left(u_c(t)\,c_t-u_l(t)\,l_t\right)-\underbrace{u_c(0)\left\{\left(1+\left(1-\tau_0^k\right)(r_0-\delta)\right)k_0+b_0\right\}}_{A\left(c_0,l_0,\tau_0^k,b_0\right)}=0$$

- You can think about extra term as an implementability constraint with associated lagrangian Φ.

Social Planner

- Define $W(c_t, l_t, \Phi) = (u(c_t, l_t) \dot{+} \Phi(u_c(t) c_t - u_l(t) l_t))$

- We get the social planner's objective function:

$$\sum_{t=0}^{\infty} \beta^t W(c_t, l_t, \Phi) +$$

$$\theta_t (F(k_t, l_t) + (1-\delta) k_t - c_t - g_t - k_{t+1}) - \Phi A\left(c_0, l_0, \tau_0^k, b_0\right)$$

- Interpretation.

- Convex set?

Necessary Conditions

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- If solution is interior:

$$W_c(t) = \theta_t, \ t \geq 1$$
$$W_l(t) = -\theta_t F_l(t), \ t \geq 1$$
$$\theta_t = \beta\theta_{t+1}(F_k(t+1) + 1 - \delta), \ t \geq 0$$
$$W_c(0) = \theta_t + \Phi A_c$$
$$W_l(0) = -\theta_0 F_n(0) + \Phi A_l$$

- Playing with conditions:

$$\begin{aligned} W_c(t) &= \beta W_c(t+1)(F_k(t+1) + 1 - \delta), \ t \geq 1 \\ W_l(t) &= -W_c(t) F_l(t), \ t \geq 1 \\ W_l(0) &= [\Phi A_c - W_c(0)] F_l(t) + \Phi A_l \end{aligned}$$

Capital Taxation I: Basic Result

- Assume $\exists T \geq 0$ s.t. $g_t = g$ for. $t \geq T$ and $\exists$ a Ramsey Equilibrium that converges to a steady state in finite time. Then:

$$W_c(ss) = \beta W_c(ss)(F_k(ss) + 1 - \delta)$$

or

$$1 = \beta(F_k(ss) + 1 - \delta)$$

- Now, note that in the steady state of any decentralized equilibrium:

$$\frac{Q(t|0)}{Q(t+1|0)} = \frac{1}{\beta}\frac{u_c(ss)}{u_c(ss)} = \frac{1}{\beta} = \left(1 - \tau_{t+1}^k\right) r_{ss} + 1 - \delta$$

- Now, note that $r_{t+1} = F_k(ss)$. Hence,

$$1 = \beta\left(1 + \left(1 - \tau_{t+1}^k\right)(r_{ss} - \delta)\right)$$

Capital Taxation II: Zero Capital in Steady State

- If we compare .

$$1 = \beta \left(F_k \left(ss \right) + 1 - \delta \right)$$

with

$$1 = \beta \left(1 + \left(1 - \tau_{t+1}^k \right) \left(r_{ss} - \delta \right) \right)$$

we see that, Ramsey implies:

$$\tau_{t+1}^k = 0.$$

- Chamley (1986)-Judd (1985) result.

- Intuition and robustness.

- Relation with uniform taxation theorem and with the no taxation of intermediate goods.

Role of First Period Taxation

- Note that the first order condition of the objective function with respect to τ_0^k is

$$\Phi u_c(0) F_k(0) k_0$$

  which is positive as long as Φ is positive.

- Φ represents the welfare cost of distorted margins induced by taxation.

- Optimal policy in first period$\Rightarrow$war chest. Taxation of capital in first period is non-distorsionary.

- Relation with time inconsistency problem.

- Woodford's timeless perspective.

Capital Taxation III: A Stronger Result

- Now, assume that $u(c,l) = \frac{c^{1-\gamma}}{1-\gamma.} + v(l)$.

- Then $W_c(t) = c_t^{-\gamma} + \Phi\left(-\gamma c_t^{-\gamma}\right) = (1-\gamma\Phi)\, c_t^{-\gamma}$ and:

$$\begin{aligned} W_c(t) &= \beta W_c(t+1)\left(F_k(t+1)+1-\delta\right) \Rightarrow \\ (1-\gamma\Phi)\, c_t^{-\gamma} &= \beta(1-\gamma\Phi)\, c_{t+1}^{-\gamma}\left(F_k(t+1)+1-\delta\right) \end{aligned}$$

which implies:

$$\left(\frac{c_t}{c_{t+1}}\right)^{-\gamma} = \beta\left(F_k(t+1)+1-\delta\right)$$

- In the decentralize equilibrium:

$$\left(\frac{c_t}{c_{t+1}}\right)^{-\gamma} = \beta\left(1+\left(1-\tau_{t+1}^k\right)\left(r_{ss}-\delta\right)\right)$$

- Hence, for $t \geq 2 \Rightarrow \tau_{t+1}^k = 0$.

Capital Taxation IV: Extensions

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- Judd (1985).

- Jones, Manuelli, and Rossi (1997).

- Garriga (2001) and Erosa and Gervais (2002)

- Chari, Golosov and Tsyvinski (2004).

Stochastic Economy

- We follow same notation than in the basic RBC model.
- Preferences:

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu\left(s^t\right) u\left(c_t\left(s^t\right), l_t\left(s^t\right)\right)$$

such that:

$$\begin{gathered} c_t\left(s^t\right)+i_t\left(s^t\right)+b_t\left(s^t\right)= \\ \left(1-\tau_t^l\left(s^t\right)\right) w_t\left(s^t\right) l_t\left(s^t\right) \\ +\underbrace{\left[1+\left(1-\tau_t^k\left(s^t\right)\right)\left(r_t\left(s^t\right)-\delta\right)\right]}_{R_t^k\left(s^t\right)} k_t\left(s^{t-1}\right)+R_t^b\left(s^t\right) b_t\left(s^{t-1}\right) \end{gathered}$$

$$k_{-1} \text{ given}$$

Technology

- Production function

$$F\left(k_t\left(s^{t-1}\right), l_t\left(s^t\right), s^t\right)$$

- Competitive pricing ensures that:

$$\begin{aligned} r_t\left(s^t\right) &= F_k\left(k_t\left(s^{t-1}\right), l_t\left(s^t\right), s^t\right) \\ w_t\left(s^t\right) &= F_l\left(k_t\left(s^{t-1}\right), l_t\left(s^t\right), s^t\right) \end{aligned}$$

- Law of motion for capital:

$$k_{t+1}\left(s^t\right) = i_t\left(s^t\right) + (1-\delta)\, k_t\left(s^{t-1}\right)$$

Government

- Budget constraint:

$$
\begin{aligned}
g_t(s^t) &= b_t(s^t) - R_t^b(s^t)b_t(s^t) + \\
&\quad \tau_t^l\left(s^t\right) w_t\left(s^t\right) l_t\left(s^t\right) + \tau_t^k\left(s^t\right)\left(r_t\left(s^t\right) - \delta\right) k(s^{t-1})
\end{aligned}
$$

with b_{-1} given.

- Policy:

$$
\pi = \left\{\pi_t(s^t)\right\}_{t=0}^{\infty} = \left\{\tau_t^l\left(s^t\right), \tau_t^k\left(s^t\right), R_t^b(s^t)\right\}_{t=0}^{\infty}
$$

Note: state contingent rule.

Ramsey Equilibrium

- Allocation rule: $x(\pi)$ maps policies into allocations (consumption, labor, capital).

- Price rules: $w(\pi)$ and $r(\pi)$ maps policies into prices.

- A Ramsey equilibrium is an allocation rule $x(\cdot)$, price rules $w(\cdot)$ and $r(\cdot)$ and a policy π such that:

  1. π maximizes household utility.
  2. households maximize for any π'.
  3. prices equate marginal productivities.
  4. Government budget constraint is satisfied.

Proposition

The allocation in a Ramsey Equilibrium solve the Ramsey problem:

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu\left(s^t\right) u\left(c_t\left(s^t\right), l_t\left(s^t\right)\right)$$

s.t.

$$R.C.: \; c_t\left(s^t\right)+g_t\left(s^t\right)+k_t\left(s^t\right) = F\left(k_t\left(s^{t-1}\right), l_t\left(s^t\right), s^t\right)+(1-\delta)\, k_t\left(s^{t-1}\right)$$

and

$$\begin{aligned} I.C.: \quad & \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu\left(s^t\right)\left(u_c\left(s^t\right) c_t\left(s^t\right) - u_l\left(s^t\right) l_t\left(s^t\right)\right) \\ & = u_c\left(s_0\right)\left(R_0^k\left(s_0\right) k_{-1} + R_0^b\left(s_0\right) b_{-1}\right) \end{aligned}$$

Social Planner Problem

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu\left(s^t\right) W\left(c_t\left(s^t\right), l_t\left(s^t\right), \Phi\right)$$
$$+\theta_t\left(s^t\right)\left(\begin{array}{c} F\left(k_t\left(s^{t-1}\right), l_t\left(s^t\right), s^t\right)+(1-\delta) k_t\left(s^{t-1}\right) \\ -c_t\left(s^t\right)-g_t\left(s^t\right)-k_t\left(s^t\right) \end{array}\right)$$
$$-\Phi u_c\left(s_0\right) \Psi\left(k_{-1}, b_{-1}, s_0\right)$$

where

$$W\left(c_t\left(s^t\right), l_t\left(s^t\right), \Phi\right)=$$
$$u\left(c_t\left(s^t\right), l_t\left(s^t\right)\right)+\Phi\left(u_c\left(s^t\right) c_t\left(s^t\right)-u_l\left(s^t\right) l_t\left(s^t\right)\right)$$

and

$$\Psi\left(k_{-1}, b_{-1}, s_0\right)=R_0^k\left(s_0\right) k_{-1}+R_0^b\left(s_0\right) b_{-1}$$

Ramsey Equilibrium

$$W_c\left(s^t\right) = \sum_{s_{t+1}} \beta^t \mu\left(s_{t+1}|s^t\right) W_c\left(s^{t+1}\right)\left(F_k\left(s^{t+1}\right) + 1 - \delta\right),\ t \geq 1$$

$$-\frac{W_l\left(s^t\right)}{W_c\left(s^t\right)} = F_l\left(s^{t+1}\right),\ t \geq 1$$

$$W_c\left(s_0\right) - \Phi u_{cc}\left(s_0\right)\Psi\left(k_{-1}, b_{-1}, s_0\right) =$$

$$\sum_{s^1} \beta^t \mu\left(s^1|s_0\right) W_c\left(s^1\right)\left(F_k\left(s^1\right) + 1 - \delta\right)$$

$$\frac{W_l\left(s_0\right) - \Phi\left\{u_{cl}\left(s_0\right)\Psi\left(k_{-1}, b_{-1}, s_0\right) + u_c\left(s_0\right)\left(1 - \tau_0^k\left(s_0\right)\right)F_{kl}\left(s_0\right)\right\}}{W_c\left(s_0\right) - \Phi u_{cc}\left(s_0\right)\Psi\left(k_{-1}, b_{-1}, s_0\right)}$$

$$= -F_l\left(s_0\right)$$

Decentralizing Ramsey

- We need to move from the allocation derived before to a policy $\pi = \left\{\tau_t^l\left(s^t\right), \tau_t^k\left(s^t\right), R_t^b(s^t)\right\}_{t=0}^{\infty}$.

- First note that, from the solution of the necessary conditions, we can evaluate:

$$\tau_t^l\left(s^t\right) = 1 - \frac{1}{F_l\left(s^t\right)} \frac{u_l\left(s^t\right)}{u_c\left(s^t\right)}$$

- What about the $R_t^b\left(s^t\right)$ and $\tau_t^k\left(s^t\right)$?

- We use

$$
\begin{aligned}
u_c\left(s^t\right) &= \sum_{s_{t+1}|s^t} \beta^t \mu\left(s_{t+1}|s^t\right) u_c\left(s^{t+1}\right) R^b_{t+1}\left(s^{t+1}\right) \\
u_c\left(s^t\right) &= \sum_{s_{t+1}|s^t} \beta^t \mu\left(s_{t+1}|s^t\right) u_c\left(s^{t+1}\right) R^k_{t+1}\left(s^{t+1}\right) \\
R^k_{t+1}\left(s^{t+1}\right) &= 1 + \left(1 - \tau^k_{t+1}\left(s^{t+1}\right)\right)\left(F_k\left(s^{t+1}\right) - \delta\right)
\end{aligned}
$$

plus the budget constraint of household for each state.

- If there are N states period per period, we have $N+2$ equations (there is one of the previous equations that disappears because of Walras law) in $2N$ unknowns $R^b_t\left(s^t\right)$ and $\tau^k_t\left(s^t\right) \Rightarrow N-1$ degrees of indeterminacy.

Origin of Indeterminacy

- Take budget constraint of household, multiply by $\beta^t \mu\left(s_{t+1}|s^t\right) u_c\left(s^{t+1}\right)$, sum up over s_{t+1}, and use necessary conditions on bonds, capital, and the fact that

$$b_t\left(s^t\right) = \sum_{t=\tau+1}^{\infty} \sum_{s^t} \beta^{t-\tau} \mu\left(s_t|s^\tau\right) \frac{u_c\left(s^t\right) c_t\left(s^t\right) - u_l\left(s^t\right) l_t\left(s^t\right)}{u_c\left(s^\tau\right)} - k_{\tau+1}\left(s^\tau\right)$$

  to get an expression that does not depend on $R_t^b\left(s^t\right)$ and $\tau_t^k\left(s^t\right)$.

- Hence, we can rearrange policy in different equivalent ways.

Indeterminacy of Capital Taxes

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If $R_t^b\left(s^t\right)$ and $\tau_t^k\left(s^t\right)$ satisfy the necessary conditions of the households, then so do $\widehat{R}_t^b\left(s^t\right)$ and $\widehat{\tau}_t^k\left(s^t\right)$ such that

$$\sum_{s_{t+1}|s^t} \beta^t \mu\left(s_{t+1}|s^t\right) u_c\left(s^{t+1}\right) R_{t+1}^b\left(s^{t+1}\right) = \sum_{s_{t+1}|s^t} \beta^t \mu\left(s_{t+1}|s^t\right) u_c\left(s^{t+1}\right) \widehat{R}_{t+1}^b\left(s^{t+1}\right) \tag{1}$$

$$\sum_{s_{t+1}|s^t} \beta^t \mu\left(s_{t+1}|s^t\right) u_c\left(s^{t+1}\right) \tau_{t+1}^k\left(s^{t+1}\right)\left(F_k\left(s^{t+1}\right)-\delta\right) = \sum_{s_{t+1}|s^t} \beta^t \mu\left(s_{t+1}|s^t\right) u_c\left(s^{t+1}\right) \widehat{\tau}_{t+1}^k\left(s^{t+1}\right)\left(F_k\left(s^{t+1}\right)-\delta\right) \tag{2}$$

$$\tau_{t+1}^k\left(s^{t+1}\right)\left(F_k\left(s^{t+1}\right)-\delta\right) k_{t+1}\left(s^t\right) - R_{t+1}^b\left(s^{t+1}\right) b_t\left(s^t\right) = \widehat{\tau}_{t+1}^k\left(s^{t+1}\right)\left(F_k\left(s^{t+1}\right)-\delta\right) k_{t+1}\left(s^t\right) - \widehat{R}_{t+1}^b\left(s^{t+1}\right) b_t\left(s^t\right) \tag{3}$$

- Proof: for the first two conditions, equate marginal utilities in necessary conditions of the households. The last one is just an arbitrage condition.

- Two alternatives:

  1. Uncontingent debt.

  2. Uncontingent capital tax.

- However, we cannot have simultaneously 1. and 2. and implement a Ramsey equilibrium.

Ex-Ante Capital Tax

- Note that even if state-by-state capital taxes are not pinned down, the payments across states are determined.

- Define

$$Q\left(s_{t+1}|s^t\right) = \beta^t \mu\left(s_{t+1}|s^t\right) \frac{u_c\left(s^{t+1}\right)}{u_c\left(s^t\right)}$$

- Then, we can find the ex-ante capital income tax rate:

$$\tau_{t+1}^{ek}\left(s^t\right) = \frac{\sum_{s_{t+1}|s^t} Q\left(s_{t+1}|s^t\right) \tau_{t+1}^{k}\left(s^{t+1}\right)\left(F_k\left(s^{t+1}\right) - \delta\right)}{\sum_{s_{t+1}|s^t} Q\left(s_{t+1}|s^t\right)\left(F_k\left(s^{t+1}\right) - \delta\right)}$$

- Result by Zhu (1992):

$$P^{\infty}\left(\tau_{t+1}^{ek}\left(s^{t}\right)=0\right)=1 \Leftrightarrow P^{\infty}\left(\frac{W_{c}\left(s^{t}\right)}{u_{c}\left(s^{t}\right)}=const.\right)=1$$

- Note that for $u(c,l)=\frac{c^{1-\gamma}}{1-\gamma}+v(l)$, we have

$$\frac{W_{c}\left(s^{t}\right)}{u_{c}\left(s^{t}\right)}=\frac{\left(1+\Phi\left(1-\gamma\right)\right)c_{t}\left(s^{t}\right)^{-\gamma}}{c_{t}\left(s^{t}\right)^{-\gamma}}=const.$$

- For other functions, $\tau_{t+1}^{ek}\left(s^{t}\right)\simeq 0$ (Chari, Christiano, Kehoe, 1994).

Numerical Properties

- Three main characteristics:

  1. $\tau_{t+1}^{el}\left(s^t\right)$ fluctuates very little.

  2. $\tau_{t+1}^{ek}\left(s^t\right)$ fluctuates a lot

  3. Public debt works as a shock absorber.

- Origin of welfare gains.

- What if we have balanced budget?