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Global yield curve dynamics and interactions: A dynamic Nelson–Siegel approach

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ABSTRACT

The popular Nelson–Siegel [Nelson, C.R., Siegel, A.F., 1987. Parsimonious modeling of yield curves. Journal of Business 60, 473–489] yield curve is routinely fit to cross sections of intra-country bond yields, and Diebold–Li [Diebold, F.X., Li, C., 2006. Forecasting the term structure of government bond yields. Journal of Econometrics 130, 337–364] have recently proposed a dynamized version. In this paper we extend Diebold–Li to a global context, modeling a potentially large *set* of country yield curves in a framework that allows for both global and country-specific factors. In an empirical analysis of term structures of government bond yields for the Germany, Japan, the UK and the US, we find that global yield factors do indeed exist and are economically important, generally explaining significant fractions of country yield curve dynamics, with interesting differences across countries.

1. Introduction

The yield curve is of great interest both to academics and market practitioners. Hence yield curve modeling has generated a huge literature spanning many decades, particularly as regards the term structure of government bond yields. Much of that literature is unified by the assumption that the yield curve is driven by a number of latent factors (e.g., Litterman and Scheinkman (1991), Balduzzi et al. (1996), Bliss (1997a,b) and Dai and Singleton (2000)). Moreover, in many cases the latent yield factors may be interpreted as level, slope and curvature (e.g., Andersen and Lund (1997) and Diebold and Li (2006)). The vast majority of the literature studies a single country's yield curve in isolation and relates domestic yields to domestic yield factors, and more recently, to domestic macroeconomic factors (e.g., Ang and Piazzesi (2003), Diebold et al. (2006) and Mönch (2006)).

Little is known, however, about whether common *global* yield factors are operative, and more generally, about the nature of dynamic cross-country bond yield interactions. One might naturally conjecture the existence of global bond yield factors, as factor structure is routinely present in financial markets, in which case understanding global yield factors is surely crucial for understanding the global bond market. Numerous questions arise. Do global yield factors indeed exist? If so, what are their dynamic properties? How do country yield factors load on the global factors, and what are the implications for cross-country yield curve interactions? How much of country yield factor variation is explained by global factors, and how much by country-specific factors, and does the split vary across countries in an interpretable way? Has the importance of global yield factors varied over time, perhaps, for example, increasing in recent years as global financial markets have become more integrated?

In this paper we begin to address such questions in the context of a powerful yet tractable yield curve modeling framework. Building on the classic work of Nelson and Siegel (1987) as dynamized by Diebold and Li (2006), we construct a hierarchical dynamic factor model for sets of country yield curves, in which country yields may depend on country factors, and country factors may depend on global factors. Using government bond yields for the US, Germany, Japan, and the UK, we estimate the model and extract the global yield curve factors. We then decompose the variation in country yields and yield factors into the parts dues global and idiosyncratic components. Finally, we also explore the evolution (or lack thereof) of global yield curve dynamics in recent decades.

Our generalized Nelson–Siegel approach is related to, but distinct from, existing work that tends to focus on spreads

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between domestic bond yields and a "world rate" (e.g., Al Awad and Goodwin (1998)), implicit one-factor analyses based on the international CAPM (e.g., Solnik (1974, 2004) and Thomas and Wickens (1993)), multi-factor analyses of long bond spreads (e.g., Dungey et al. (2000)), and affine equilibrium analyses (e.g., Brennan and Xia (2006)). Instead we work in a rich environment where each country yield curve is driven by country factors, which in turn are driven both by global and country-specific factors. Hence we achieve an approximate global bond market parallel to the global real-side work of Lumsdaine and Prasad (2003), Gregory and Head (1999) and Kose et al. (2003).

We proceed as follows. In Section 2 we describe our basic global bond yield modeling framework, and in Section 3 we discuss our bond yield data for four countries. In Section 4 we provide full-sample estimates and variance decompositions for the global yield curve model, and in Section 5 we provide sub-sample results. We conclude in Section 6.

2. Modeling framework

Diebold and Li (2006) and Diebold et al. (2006, 2005) show that, in a US closed-economy environment, a generalized Nelson–Siegel model accurately approximates yield curve dynamics and provides good forecasts. Here we extend that framework to a multi-country environment, allowing for both global and country-specific factors.

2.1. Single-country

The Diebold–Li factorization of the Nelson–Siegel yield curve for a single country (at a particular and arbitrary point in time) is

$$y_i(\tau) = l_i + s_i\left(\frac{1-e^{-\lambda_i\tau}}{\lambda_1\tau}\right) + c_i\left(\frac{1-e^{-\lambda_i\tau}}{\lambda_1\tau} - e^{-\lambda_i\tau}\right) + \nu_i(\tau), \tag{1}$$

where $y_i(\tau)$ denotes the continuously-compounded zero-coupon nominal yield on a τ-month bond, l_i, s_i, c_i and λ_i are parameters, and $\nu_i(\tau)$ is a disturbance with standard deviation $\sigma_i(\tau)$. Following Diebold and Li, we dynamize the model by allowing the parameters to vary over time,

$$y_{it}(\tau) = l_{it} + s_{it}\left(\frac{1-e^{-\lambda_{it}\tau}}{\lambda_{it}\tau}\right) + c_{it}\left(\frac{1-e^{-\lambda_{it}\tau}}{\lambda_{it}\tau} - e^{-\lambda_{it}\tau}\right) + \nu_{it}(\tau), \tag{2}$$

and we interpret l_{it}, s_{it}, and c_{it} as *latent factors*. In particular, as shown by Diebold and Li, they are level, slope and curvature factors, respectively, because their factor loadings are a constant, a decreasing function of τ and a concave function of τ. (Hence the notation l, s and c.) As the yield factors vary over time, this generalized Nelson–Siegel model can generate a variety of time-varying yield curve shapes.

Henceforth we will work with a simplified version of the yield curve (3). First, we will assume constancy of the parameters λ_{it} over countries and time. Following Diebold and Li (2006), there is little loss of generality from doing so, because, λ primarily determines the maturity at which the curvature loading is maximized. Second, because the curvature factor is normally estimated with low precision due to missing data at very short and/or very long maturities in most of the countries used in our study, and because curvature lacks clear links to macroeconomic fundamentals as shown in Diebold et al. (2006), we focus on the model with level and slope factors only. Hence we write

$$y_{it}(\tau) = l_{it} + s_{it}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \nu_{it}(\tau). \tag{3}$$

Note that (3) is effectively the measurement equation of a state space system with state vector $(l_{it}, s_{it})'$, as emphasized by Diebold et al. (2006). Hence the generalized Nelson–Siegel model does not need to be cast in state space form — it is *already* in state space form. Subsequently we will discuss the details of specific parameterizations of that state space form, but for now we simply note its immediate existence.

2.2. Multi-country

We now move to an N-country framework. We allow global yields to be depend on global yield factors,

$$Y_t(\tau) = L_t + S_t\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + V_t(\tau), \tag{4}$$

where the $Y_t(\tau)$ are global yields and L_t and S_t are global yield factors. We endow the global yield factors with simple autoregressive dynamics,

$$\begin{pmatrix} L_t \\ S_t \end{pmatrix} = \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix}\begin{pmatrix} L_{t-1} \\ S_{t-1} \end{pmatrix} + \begin{pmatrix} U_t^l \\ U_t^s \end{pmatrix}, \tag{5}$$

where the U_t^n are disturbances such that $EU_t^nU_{t'}^{n'} = (\sigma^n)^2$ if $t = t'$ and $n = n'$, and 0 otherwise, $n = l, s$.

Each country's yield curve remains characterized by (3), but we now allow the country common factors, l_{it} and s_{it}, to load on the global factors L_t and S_t, as well as country idiosyncratic factors:

$$l_{it} = \alpha_i^l + \beta_i^l L_t + \varepsilon_{it}^l \tag{6a}$$

$$s_{it} = \alpha_i^s + \beta_i^s S_t + \varepsilon_{it}^s, \tag{6b}$$

where $\{\alpha_i^l, \alpha_i^s\}$ are constant terms, $\{\beta_i^l, \beta_i^s\}$ are loadings on global factors, and $\{\varepsilon_{it}^l, \varepsilon_{it}^s\}$ are country idiosyncratic factors, $i = 1, \ldots, N$. Because we include constant terms in (6), we assume with no loss of generality that the country idiosyncratic factors have zero mean. In addition, we make two sets of identifying assumptions. First, because the magnitudes of global factors and factor loadings are not separately identified, we assume that innovations to global factors have unit standard deviation, that is, $\sigma^n = 1$, $n = l, s$.[1] Second, to identify the signs of factors and factor loadings, we assume that the US loadings on the global factors are positive; that is, we assume that $\beta_{us}^n > 0$, $n = l, s$.

As with the global factors, we allow the county idiosyncratic factors to have first-order autoregressive dynamics,

$$\begin{pmatrix} \varepsilon_{it}^l \\ \varepsilon_{it}^s \end{pmatrix} = \begin{pmatrix} \varphi_{i,11} & \varphi_{i,12} \\ \varphi_{i,21} & \varphi_{i,22} \end{pmatrix}\begin{pmatrix} \varepsilon_{i,t-1}^l \\ \varepsilon_{i,t-1}^s \end{pmatrix} + \begin{pmatrix} u_{it}^l \\ u_{it}^s \end{pmatrix}, \tag{7}$$

where the u_{it}^n are disturbances such that $Eu_{it}^n u_{i't'}^{n'} = (\sigma_i^n)^2$ if $i = i'$, $t = t'$ and $n = n'$, and 0 otherwise, $n = l, s$. We also assume the shocks to the global factors U_t^n and the shocks to the country-specific factors $u_{it}^{n'}$ are orthogonal: $EU_t^n u_{i,t-s}^{n'} = 0$, for all n, n', i, and s.

Many variations, extensions and specializations of this basic model are of course possible. For example, a useful specialization to facilitate tractable estimation would restrict the dynamic matrices in (5) and (7) to be diagonal. (We shall do this.) As another example, an interesting extension would include not only global factors, but also regional factors, in which case country factors could depend on regional factors, which in turn could depend on global factors. We shall not consider such extensions in this paper; instead, we now implement empirically our basic model sketched thus far.

[1] This follows Sargent and Sims (1977) and Stock and Watson (1989).

Fig. 1. Yield curves across countries and time. (Notes to figure: All yield data are monthly, 1985.09 through 2005.08.)

3. Data construction, data description and preliminary analysis

In this section, prior to fitting the full global yield model, we discuss and describe the data. We perform several preliminary analyses that provide background, motivation and a foundation for the subsequent analysis.

3.1. Data construction

Our data, generously supplied by Michael Brennan and Yihong Xia for 1985.09–2002.05 and extended by us to 2005.08, consist of government bond prices, coupon rates, and coupon structures, as well as issue and redemption dates, in local currency terms for the US, Germany, Japan, and the UK.

We calculate zero-coupon bond yields using the unsmoothed Fama–Bliss (Fama and Bliss, 1987) approach.[2] We measure the bond yields on the second day of each month. We also apply several data filters designed to enhance data quality and focus attention on maturities with good liquidity. First, we exclude floating rate bonds, callable bonds and bonds extended beyond the original redemption date. Second, we exclude outlying bond prices less than 50 or greater than 130 because their price discounts/premium are too high and imply thin trading, and we exclude yields that differ greatly from yields at nearby maturities. Finally, we use only bonds with maturity greater than one month and less than fifteen years, because other bonds are not actively traded. To simplify our subsequent estimation, using linear interpolation we pool the bond yields into constant maturities of 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108 and 120 months, where a month is defined as 30.4375 days.

3.2. Data description

In Fig. 1 we show the government bond yield curves across countries and time. It is apparent that each yield curve displays substantial level movements. Cross-country comparison of the yield curves, moreover, reveals clear commonality in level movements. Yield curve slopes vary less, although they do of course vary, and Fig. 1 suggests that they may also display some cross-country commonality in movements.

In Table 1 we report summary statistics for bond yields at representative maturities. Japanese yields are lowest on average, typically around two or three percent. All yield curves are upward-sloping, and yield volatility decreases with maturity. In addition,

[2] Our zero-coupon bond yields are highly correlated with those obtained by Brennan and Xia (2006), who use a cubic spline and maturities of 3, 6, 12, 24, 36, 60, 84, 96, 108 and 120 months.