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Public versus Private Risk Sharing

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ABSTRACT

In this paper we ask whether public intervention can improve the allocation of risk in an economy in which private financial markets do not provide full insurance. The answer depends crucially on the fundamental friction that limits private risk sharing in the first place. If risk sharing is limited because some insurance markets are missing for model-exogenous reasons (as in Aiyagari, 1994) publicly provided risk sharing (via a progressive tax system, say) always improves on the allocation of risk. If instead insurance markets exist but their use is limited by the absence of complete enforcement (as in Kehoe and Levine, 1993) then the provision of public insurance via a progressive income tax system can crowd out the provision of private insurance to such an extent that total consumption insurance is reduced. The idea is that with progressive taxation the government increases the value of being excluded from insurance markets and hence weakens the enforcement mechanism of private contracts. In order to quantify these effects we fully characterize equilibria in an economy with limited enforcement and a continuum of agents so that our baseline income and consumption distribution can match the respective distributions in US data. We find that for plausible parameter the crowding-out effect of public insurance on private insurance can be substantial.

KEYWORDS: Incomplete Markets, Insurance, Limited Enforcement

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1. Introduction

Should the government provide public insurance against idiosyncratic income uncertainty by implementing a progressive tax system in which households with higher income realizations pay higher average tax rates, thus making the after-tax labor income process less risky than the pre-tax labor income process? The answer that economic theory gives to this question depends on the assumptions about the structure of private insurance markets. If these markets are complete, in that agents can trade a complete set of perfectly enforceable insurance contracts, then complete risk sharing is achieved via private markets and progressive income taxes provide no additional insurance. If, on the other hand, private insurance markets do not provide full risk sharing redistributive taxes might provide additional insurance. As Mirrlees (1974), Varian (1980) and others have pointed out, this beneficial effect of progressive income taxes has to be traded off against the adverse effect on incentives to supply labor and to accumulate capital, leading to a nontrivial optimal taxation problem.²

In this paper we demonstrate that if one models the frictions that lead to incomplete risk sharing explicitly, then the public provision of insurance may adversely affect the way private insurance markets work. Concretely, we show that if private risk sharing is limited because private insurance contracts can only be enforced through exclusion from participating in financial markets in the future, then the provision of public insurance crowds out the provision of private insurance against idiosyncratic uncertainty, potentially more than one for one. That is, by attempting to better insure households against idiosyncratic risk the government achieves exactly the opposite, namely a worse risk allocation of private consumption. This result mirrors Stiglitz' (1981) concern about the lack of modelling the underlying frictions that are the source of market incompleteness:

²A second common justification for redistributive taxation is the social desire to attain a more equal income or wealth (and hence consumption and welfare) distribution. Although we believe that this justification is potentially important we will not address this point in this paper. See the seminal paper by Mirrlees (1971) for an analysis of the trade-off between the equity and the labor supply incentive effect of redistributive taxation.

“Without a clear specification of the information/transactions technology, there is always a danger that any intervention in the economy designed, say, to alleviate problems arising from an absence of risk markets will be either infeasible or so costly to implement that it would not, in fact, constitute a Pareto improvement, for precisely the same reasons that the markets were absent in the first place.” Similarly, for monetary economics, Wallace (2001) advocates, exclusively, the use of models where an explicit modelling of frictions makes money essential. Our paper may be viewed as an example from fiscal policy analysis where the adherence to Stiglitz’ and Wallace’ dictum is crucial.

Our exact modelling approach follows the work by Kehoe and Levine (1993, 2001) and does not impose a priori restrictions on the set of private insurance contracts that can be traded. These contracts, however, can not be fully enforced.³ The only enforcement mechanism is the threat of exclusion from future credit and insurance markets upon default. Tax liabilities, however, are not subject to this enforcement problem as we assume that the penalty for defaulting on tax payments can be made prohibitively large by the government. If agents default on their private debt, they are banned from future credit and insurance markets, but retain their private (labor) endowment which is still subject to income taxation. A change in the tax system changes the severity of punishment from default by altering the utility an agent can attain without access to insurance markets and thus changes the extent of enforcement of private contracts. Since enforcement defines the extent through which private contracts are used, a change in the tax system changes the use of private contracts. The allocative and welfare consequences of a change in the tax system then depend on the relative magnitudes of the change in risk sharing enforced by the new tax system and the change in risk sharing through private insurance markets.

³Another fraction of the literature derives market incompleteness from *informational frictions* underlying the phenomena of adverse selection and moral hazard (see Cole and Kocherlakota (2001) and their review of the literature). In these types of models fiscal policy can affect the degree of market incompleteness, but only under the assumption of informational asymmetry between the government and private agents. However, every policy that leads to the same revelation of information to private agents has the same effect on the degree of market incompleteness. In this sense this approach does not provide a natural link between market incompleteness and redistributive income taxation.

The main methodological contribution of this work is the characterization of the consumption allocation and distribution of a limited commitment model with a *continuum* of agents facing idiosyncratic uncertainty. This model allows us to analyze insurance mechanisms involving the entire population and not only pairwise relationships. We view this as crucial in our analysis of risk sharing arrangements such as progressive taxation since gains from insurance are particularly sizable among a large pool of agents with mostly idiosyncratic (i.e. largely uncorrelated) income uncertainty. We demonstrate this by comparing the consumption allocations in the continuum economy to those arising in a model with only two agents (as studied by Kocherlakota (1996) and Alvarez and Jermann (2000), among others) and show that the allocation of income risk in these two models is *qualitatively* different. In particular, the extent to which publicly provided insurance crowds out private insurance differs substantially among the two models. In addition our model, in contrast to the previous literature, endogenously delivers a rich cross-sectional consumption distribution and thus may be of independent interest for the study of other policy reforms where distributional issues are important. But it is also exactly the rich cross-sectional dimension of the model that leads to considerable theoretical and computational complications in solving it. To this end we adapt the work of Atkeson and Lucas (1992, 1995) who study efficient allocations in an economy with a continuum of agents and private information. We then show, following Kehoe and Levine (1993), how to decentralize efficient allocations as equilibrium allocations in a standard Arrow Debreu equilibrium with individual rationality constraints.

In a related paper Attanasio and Rios-Rull (2000) use a limited commitment model to study the effect of mandatory public insurance programs against aggregate uncertainty on private insurance arrangements against idiosyncratic uncertainty. Although their economy is populated by a large number of (potentially heterogeneous) agents, by assumption agents can only enter pairwise insurance arrangements, not involving any other member of the population. So their underlying

insurance problem is equivalent to the ones studied by Kocherlakota (1996) and Alvarez and Lippi (2000). Similar to our result they show that the extent to which idiosyncratic shocks can be insured depends negatively on the public provision of insurance against aggregate uncertainty. Ligon, Thomas and Worrall (2000, 2002) set up a model with a finite, but potentially large number of agents that can engage in mutual insurance schemes. Once they solve for constrained-efficient insurance contracts numerically, however, they need to restrict attention to economies with either two agents (as in Ligon, Thomas and Worrall (2000), in a model with capital accumulation), or they need to assume that agents engage in contracts with the rest of the population, treating the rest of the population as one agent (as in Ligon, Thomas and Worrall (2002)). This again reduces the problem to a bilateral insurance problem as in the other papers discussed previously.⁴

Finally we demonstrate that the crowding-out effect from the progressive income tax system characterized in this paper may be quantitatively important. To this end we provide an example which uses panel micro-data from the Consumer Expenditure Survey (CEX) to obtain estimates of idiosyncratic income risk and the tax system for the US. One of the advantages of having a continuum of agents is that we can set parameters in our baseline model so as to reproduce key aspects of the US income distribution. Since our main focus is the impact of taxes on risk sharing we construct and compute three measures of risk sharing for the income and consumption distributions: Private risk sharing, which is the reduction of consumption risk below after-tax income risk stemming from private financial markets, public risk sharing which is the reduction in income risk stemming from the progressivity of income taxes and total risk sharing which is the sum of the two. We then conduct various tax experiments changing the progressivity of the tax system, keeping the level of government spending (and the pre-tax income process) fixed. Comparing steady state consumption

⁴The authors have to do this in order to avoid the curse of dimensionality. In their set-up of the problem the cumulative Lagrange multipliers on the enforcement constraints for each agent become (continuous) state variables, in practice ruling out computing allocations for economies with more than two agents.

allocations arising under different tax systems we find that, for a reasonable selection of the structural parameters of our model, making taxes more progressive always increases public risk sharing and always reduces private risk sharing; in some case the reduction of private risk sharing is bigger than the increase in public risk sharing so that making taxes more progressive leads to *less* total risk sharing among individuals. We also show that this never happens in a standard incomplete markets model in the spirit of Huggett (1993) and Aiyagari (1994) in which explicit risk sharing is limited for reasons exogenous to the model. These results demonstrate that, when analyzing a redistributive tax policy reform, it is crucial to take Stiglitz (1981) and Wallace (2001) seriously and model the underlying source of limited risk sharing explicitly.

It is important to note that our quantitative analysis only focuses on the risk sharing effect of taxes and it abstracts from many elements that are important in the design of optimal taxes such as the presence of distortions of labor supply and savings decisions or a society's preference for redistribution. So our findings do not advocate a particular optimal tax schedule but simply suggest that, when computing optimal tax schedules, considering the effects that taxes have on the functioning of private financial markets is something policy makers should consider seriously.

The paper is organized as follows. In Section 2 we lay out the model environment and define equilibrium. In Section 3 we define and characterize efficient allocations. Section 4 discusses the decentralization. Section 5 presents qualitative features of the equilibrium and compares the qualitative features of the consumption allocation in the continuum economy with that arising in a simple economy with two agents and perfectly negatively correlated income shocks. Section 6 provides a quantitative thought experiment of changing the progressivity of the income tax code, both within our model and a standard incomplete markets model. Section 7 concludes; figures and proofs of the main propositions are contained in the appendix.⁵

⁵A separate appendix contains details of some of the more involved technical arguments. It is available at www.wiwi.uni-frankfurt.de/professoren/krueger/theoreticalapp.pdf.

2. The Economy

There is a continuum of consumers of measure 1, who have preferences over consumption streams given by

$$(1) \quad U(\{c_t\}_{t=0}^{\infty}) = (1 - \beta) E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

The period utility function $u : \mathfrak{R}_+ \rightarrow D \subseteq \mathfrak{R}$ is assumed to be strictly increasing, strictly concave, twice differentiable and satisfies the Inada conditions. Its inverse is denoted by $C : D \rightarrow \mathfrak{R}_+$. Hence $C(u)$ is the amount of the consumption good necessary to yield period utility u . Let $\bar{D} = \sup(D)$; note that we do not assume u to be bounded so that $\bar{D} = \infty$ is possible.

An individual has stochastic endowment process $e \in E$, a finite set with cardinality N , that follows a Markov process with transition probabilities $\pi(e'|e)$. In what follows we use the words endowment and income synonymously. For each consumer the transition probabilities are assumed to be the same. We assume a law of large numbers,⁶ so that the fraction of agents facing shock e' tomorrow with shock e today in the population is equal to $\pi(e'|e)$. We assume that $\pi(e'|e)$ has unique invariant measure $\Pi(\cdot)$. We denote by e_t the current period endowment and by $e^t = (e_0, \dots, e_t)$ the history of realizations of endowment shocks; also $\pi(e^t|e_0) = \pi(e_t|e_{t-1}) \cdots \pi(e_1|e_0)$. We use the notation $e^s|e^t$ to mean that e^s is a possible continuation of endowment shock history e^t . We also assume that at date 0 (and hence at every date), the cross-sectional measure over current endowment is given by $\Pi(\cdot)$, so that the aggregate endowment is constant over time. At date 0 agents are distinguished by their initial asset holdings, a_0 (claims to period zero consumption) and by their initial shock e_0 . Let Θ_0 be the joint measure of initial assets and shocks.

The government uses taxes to finance a constant amount of public spending g in every period that yields no utility to consumers. The government specifies a tax policy $\tau(e_t)$ that is constant

⁶Note that we do not require independence of endowment processes across individuals; the assumption of a law of large numbers can then be justified with Feldman and Gilles (1985), proposition 2.

over time. We take government policies $g, \tau(\cdot)$ as exogenously given. For an individual we let $y_t = e_t(1 - \tau(e_t))$ be the after-tax income. Since the function $\tau(\cdot)$ does not depend on time, for a given tax function $\tau(\cdot)$ there is a one-to-one mapping between pre-tax and after-tax endowments. From now on we let $y \in Y \subseteq \mathfrak{R}_{++}$ denote an individual's generic after-tax endowment, following the Markov process π with invariant distribution Π and denote by $y^t = (y_0, \dots, y_t)$ a history of after-tax endowment shocks. We restrict the government policies $g, \tau(\cdot)$ to satisfy

$$(2) \quad g = \int e_t \tau(e_t) d\Pi.$$

With this assumption resource feasibility for this economy states that the sum of all agents' consumption has to be less or equal than the sum over all individuals' after-tax endowment. Therefore, once $g, \tau(\cdot)$ are fixed and hence the after-tax endowment process is specified, we can carry out the subsequent analysis without explicit consideration of the government.

Consumers can trade a full set of state-contingent commodities. A consumption allocation $c = \{c_t(a_0, y^t)\}$ specifies how much an agent of type (a_0, y_0) consumes who experienced a history of endowment shocks y^t . Individuals, at any point in time, have the option to renege on existing contracts. The only punishment for doing so, and hence the only enforcement mechanism for contracts, is that agents that default on their contracts are banned from future insurance markets. They are, however, allowed to self-insure by saving (but not borrowing) at an exogenous constant interest rate r .⁷ The expected continuation utility for an agent who defaults after history y^t is given by $U^{Aut}(y_t; r) = U(0, y_t)$, where U is the solution to the functional equation

$$(3) \quad U(a, y) = \max_{0 \leq a' \leq y + (1+r)a} (1 - \beta)u(y + (1 + r)a - a') + \beta \sum_{y'} \pi(y'|y)U(a', y')$$

⁷This assumption is motivated by current US bankruptcy laws. Agents filing for bankruptcy under Chapter 7 must surrender all their assets above certain exemption levels; the receipts from selling these assets are used to repay the consumer's debt. Remaining debt is discharged. In most cases of Chapter 7 bankruptcy debtors have no non-exempt assets (see White (1998)), so the consequences of filing for bankruptcy only entail restrictions on future credit. Individuals that declared personal bankruptcy are usually denied credit for seven years from major banks and credit card agencies. We view our assumption of being banned forever as a first (and easily tractable) approximation, keeping in mind that it may overstate the punishment from default.

with $a_0 = 0$ given. It is obvious that $U^{Aut}(y_t; r)$ is strictly increasing in y_t , as long as the income shocks are uncorrelated or positively correlated over time.

Individuals have no incentive to default on a consumption allocation c , at any point in time and any contingency, if and only if an allocation satisfies following continuing participation or debt constraints

$$(4) \quad U_t(a_0, y^t, c) \equiv (1-\beta) \left(u(c(a_0, y^t)) + \sum_{s>t} \sum_{y^s|y^t} \beta^{s-t} \pi(y^s|y^t) u(c(a_0, y^s)) \right) \geq U^{Aut}(y_t; r) \quad \forall y^t$$

i.e. if the continuation utility from c is at least as big as the continuation utility from defaulting on c , for all histories y^t . Since there is no private information and markets are complete, exclusion will not happen in equilibrium as nobody would offer a contract to an individual for a contingency at which this individual would later default with certainty.

Notice that our specification of the debt constraint is more general than the one introduced by Kehoe and Levine (1993) in which agents who default are not allowed to save. If $r = -1$, our model is equivalent to theirs and the right hand side of the debt constraint reduces to

$$(5) \quad U^{Aut}(y_t; -1) = (1 - \beta) \left(u(y_t) + \sum_{s>t} \sum_{y^s|y^t} \beta^{s-t} \pi(y^s|y^t) u(y_s) \right)$$

From now on, whenever there is no danger of ambiguity, we omit the dependence of U_t^{Aut} on r .

A. Equilibrium

We now define a competitive equilibrium for the economy described above. We will follow the approach of Kehoe and Levine (1993). Consider an agent with period zero endowment of y_0 and initial wealth of a_0 . Wealth is measured as entitlement to the period 0 consumption good. Let Θ_0 be the joint distribution over (a_0, y_0) and denote by $p_t(y^t)$ the date zero price⁸ of a contract

⁸Note that in standard Arrow Debreu equilibrium theory with finitely many consumers, a complete description of the state of the economy would be *everybody's* endowment shock history, and all prices would be contingent on this complete state. With atomistic individuals, the assumed law of large number and no aggregate uncertainty, attention can be restricted to equilibria in which prices (and quantities) depend only on own personal histories.

that specifies delivery of one unit of the consumption good at period t to/from a person who has experienced endowment shock history y^t . For each contingency $c_t(a_0, y^t) - y_t$ is the net trade of individual (a_0, y_0) for that contingency. In period 0 there is no uncertainty, so normalize the price of the consumption good at period 0 to 1.

A household of type (a_0, y_0) chooses an allocation $\{c_t(a_0, y^t)\}$ to solve

$$\begin{aligned}
(6) \quad & \max U_0(a_0, y_0, c) \\
(7) \quad & \text{s.t. } c_0(a_0, y_0) + \sum_{t=1}^{\infty} \sum_{y^t|y_0} p_t(y^t) c_t(a_0, y^t) \leq a_0 + y_0 + \sum_{t=1}^{\infty} \sum_{y^t|y_0} p_t(y^t) y_t \\
(8) \quad & U_t(a_0, y^t, c) \geq U^{Aut}(y_t)
\end{aligned}$$

Note that, as in Kehoe and Levine (1993), the continuing participation constraints enter the individual consumption sets directly.

DEFINITION 1. *An equilibrium consists of prices $\{p_t(y^t)\}_{t=0}^{\infty}$ and allocations $\{c_t(a_0, y^t)\}_{t=0}^{\infty}$ such that*

given prices, the allocation solves household's problem for almost all (a_0, y_0)

markets clear, i.e. for all t ,

$$(9) \quad \int \sum_{y^t} c_t(a_0, y^t) \pi(y^t|y_0) d\Theta_0 = \int \sum_{y^t} y_t \pi(y^t|y_0) d\Theta_0.$$

As is clear from the equilibrium definition our economy does not include physical capital accumulation or government debt, so assets are in zero net supply and the aggregate asset to income ratio is identically equal to zero. While this may seem unrealistic, we deliberately chose to abstract from both types of assets. In a closed economy with incomplete markets and precautionary savings motives an increase in income uncertainty leads to higher precautionary saving, hence higher investment, a higher steady state capital stock and thus higher steady state production (see Aiyagari

(1994)). In our economy relaxed borrowing constraints drive the interest rate up and thus, in a version of the model with capital, the aggregate capital stock down. Since in this paper we want to focus on the risk sharing properties of different taxation schemes rather than the effects of taxation and income uncertainty on capital accumulation, we compromise on realism to more clearly isolate the *potential* quantitative importance of the crowding-out mechanism introduced in this paper.

With respect to government debt, the government budget constraint would mandate that, for the same amount of outstanding government debt, the amount of taxes levied to finance the interest payments on the debt would vary across steady states, due to changes in the interest rate. Since the comparison of private households' welfare across economies with different tax burdens seems problematic, we also abstract from government debt in this paper.

3. Efficient Allocations

The next step in our analysis is to characterize and compute equilibrium allocations. Unfortunately this is hard to do by tackling the equilibrium directly. In particular, the presence of the infinite number of dynamic constraints (8) restricting the choice of state contingent claims does not allow to solve the household's problem as a standard dynamic programming problem. Therefore in this section we will follow Atkeson and Lucas (1992, 1995) to first characterize efficient allocations and then argue in the next section that they can be decentralized as competitive equilibrium allocations. As shown by Atkeson and Lucas solving for efficient allocations *does* reduce to solving a standard dynamic programming problem which makes their approach so useful for our problem. As they, however, we also have to restrict our analysis to stationary allocations, i.e. to allocations for which the cross-sectional consumption and wealth distribution is constant over time.

The key insight of Atkeson and Lucas is to analyze the problem of finding efficient allocations in terms of state contingent *utility* promises rather than state contingent consumption. Instead of being indexed by initial assets and endowment shock, now individuals are indexed by initial

entitlements to expected discounted utility at period 0, w_0 and initial endowment shocks, y_0 . We will discuss the connection between initial asset positions and initial utility promises in Section 4. Let Φ_0 be the period 0 joint measure over (w_0, y_0) . An allocation is then a sequence $\{h_t(w_0, y^t)\}_{t=0}^\infty$ that maps initial entitlements w_0 and sequences of shocks y^t into levels of current utility in period t . Here $h_t(w_0, y^t)$ is the current period utility that an agent of type (w_0, y_0) receives if she experienced a history of endowment shocks y^t . Note that $c_t(a_0, y^t) = C(h_t(w_0, y^t))$ for an agent whose utility entitlement w_0 corresponds to initial asset holdings a_0 , where C is the inverse of the period utility function as defined in Section 2. For any allocation $h = \{h_t(w_0, y^t)\}_{t=0}^\infty$ define

$$(10) \quad U_t(w_0, y^t, h) = (1 - \beta) \left(h_t(w_0, y^t) + \sum_{s>t} \sum_{y^s|y^t} \beta^{s-t} \pi(y^s|y^t) h_t(w_0, y^s) \right)$$

Equation (10) defines the continuation utility from an allocation h of agent of type (w_0, y_0) from date t and shock history y^t onwards.

DEFINITION 2. *An allocation $\{h_t(w_0, y^t)\}_{t=0}^\infty$ is constrained feasible with respect to a joint distribution over utility entitlements and initial endowments, Φ_0 , if for almost all $(w_0, y_0) \in \text{supp}(\Phi_0)$*

$$(11) \quad w_0 = U_0(w_0, y_0, h)$$

$$(12) \quad U_t(w_0, y^t, h) \geq U^{Aut}(y_t) \quad \forall y^t$$

$$(13) \quad \lim_{t \rightarrow \infty} \beta^t \sup_{y^t} U_t(w_0, y^t, h) = 0$$

$$(14) \quad \sum_{y^t} \int \left(C(h_t(w_0, y^t)) - y_t \right) \pi(y^t|y_0) d\Phi_0 \leq 0. \quad \forall t$$

An allocation $\{h_t(w_0, y^t)\}_{t=0}^\infty$ is efficient with respect to Φ_0 if it is constrained feasible with respect to Φ_0 and there does not exist another allocation $\{\hat{h}_t(w_0, y^t)\}_{t=0}^\infty$ that is constrained-feasible with respect to Φ_0 and such that

$$(15) \quad \sum_{y^t} \int C(\hat{h}_t(w_0, y^t)) \pi(y^t|y_0) d\Phi_0 < \sum_{y^t} \int C(h_t(w_0, y^t)) \pi(y^t|y_0) d\Phi_0 \text{ for some } t$$

We call equation (11) the promise keeping constraint: the allocation delivers utility w_0 to agents entitled to w_0 . Equations (12) are the continuing participation constraints.⁹ Equation (13) is a boundedness condition that assures that continuation utility goes to zero in the time limit. Equation (14) is the resource feasibility condition, requiring aggregate consumption in every period to be less or equal than aggregate endowment in that period. The definition basically says that a utility allocation is efficient if it attains the utility promises made by Φ_0 in an individually rational and resource feasible way and there is no other allocation that does so with less resources. In order to use recursive techniques, however, we have to restrict ourselves to stationary allocations. Define Φ_t to be the joint measure over endowment shocks y_t and continuation utilities $U_t(w_0, y^t, h)$ for a given allocation. An allocation is stationary if $\Phi_t = \Phi_0 = \Phi$. In the next subsections we will show that such an allocation exists, characterize it and demonstrate how to compute it.

A. Recursive Formulation

In order to solve for stationary efficient allocations we consider the problem of a planner that is responsible of allocating resources to a given individual and who can trade resources at a fixed intertemporal price $\frac{1}{R}$. In this subsection we discuss such a planners' recursive problem and in the next subsection its solution. We then show that the planners' policy functions induce a Markov process over utility promises and income shocks which has a unique invariant distribution, and finally we demonstrate that there exists an R^* at which the resources needed to deliver utility promises dictated by the stationary distribution equal the aggregate endowment in the economy.

For constant $R \in (1, \frac{1}{\beta}]$, consider the following functional equation (*FE*) problem. Individual state variables are the promise to expected discounted utility that an agent enters the period with, w , and the current income shock y . The planner chooses how much current period utility to give to

⁹Note that a Φ_0 that puts positive mass on (w_0, y_0) and satisfies $w_0 < U^{Aut}(y_0)$ does not permit a constraint feasible allocation as promise keeping and debt constraints are mutually exclusive. We restrict attention to Φ_0 with the property that only initial utility entitlements at least as big as the utility from autarky have positive mass.

the individual, h , and how much to promise her for the future, $g_{y'}$, conditional on her next periods endowment realization y' . We now make the following assumptions on the individual endowment process¹⁰

Assumption 1: $\pi(y'|y) = \pi(y')$ for every $y', y \in Y$

Assumption 2: $\pi(y) > 0$, for all $y \in Y$

The operator T_R defining the functional equation of the planner's problem is:

$$(16) \quad T_R V(w) = \min_{h, \{g_{y'}\}_{y' \in Y} \in D} \left\{ \left(1 - \frac{1}{R}\right) C(h) + \frac{1}{R} \sum_{y' \in Y} \pi(y') V(g_{y'}) \right\}$$

$$(17) \quad \text{s.t. } w = (1 - \beta)h + \beta \sum_{y' \in Y} \pi(y') g_{y'}$$

$$(18) \quad g_{y'} \geq U^{Aut}(y') \quad \forall y' \in Y$$

where $V(w)$ is the resource cost for the planner to provide an individual with expected utility w when the intertemporal shadow price of resources for the planner is $\frac{1}{R}$. The cost consists of the cost for utility delivered today, $(1 - \frac{1}{R})C(h)$, and expected cost from tomorrow on, $\sum_{y'} \pi(y') V(g_{y'})$, discounted to today. Atkeson and Lucas (1992, 1995) show that a stationary allocation $\{h_t(w_0, y^t)\}_{t=0}^{\infty}$ is efficient if it is induced by an optimal policy from the functional equation above with $R > 1$ and satisfies the resource constraint with equality.¹¹

Equation (17) is the promise-keeping constraint: an individual that is entitled to w in fact receives utility w through the allocation rules $\{h(\cdot), g_{y'}(\cdot)\}_{y' \in Y}$. The continuing participation constraints in equation (18) state that the social planner for each state tomorrow has to guarantee individuals an expected utility promise at least as high as obtained with the autarkic allocation.

¹⁰For the quantitative analysis we will relax these assumptions; however, we could not prove some of our theoretical results without these assumptions.

¹¹A policy $(h, \{g_{y'}\})$ induces an allocation, for all (w_0, y_0) , in the following way: $h_0(w_0, y_0) = h(w_0, y_0)$, $w_1(w_0, y^1) = g_{y_1}(w_0, y_0)$ and recursively $w_t(w_0, y^t) = g_{y_t}(w_{t-1}(w_0, y^{t-1}), y_t)$ and $h_t(w_0, y^t) = h(w_t(w_0, y^t), y_t)$. Adaptations of their proofs to our environment are contained in a separate theoretical appendix, available from the authors upon request.

The utility in autarky is given as the solution to the functional equation in (3).

B. Existence and Characterization of Policy Functions for Fixed R

We first prove the existence of optimal allocation rules in the problem with the additional constraints $g_{y'} \leq \bar{w}$ in (18), where \bar{w} is an upper bound on future utility promises. We then characterize the solution of this problem and show that the additional constraints are not binding so that the solution to the problem with additional constraints is also solution to the original problem.¹² The modified Bellman equation is defined on $C(W)$, that is, the space of continuous and bounded functions on W , where $W = \{w \in \mathfrak{R} | \underline{w} \leq w \leq \bar{w}\} \subseteq D$ is a compact subset of \mathfrak{R} and $\underline{w} := \min_y U^{Aut}(y)$. This gives us a standard *bounded* dynamic programming problem. From now on we will denote by T_R the operator defined above, but *including* the additional constraints.

Note that with the additional constraints on future utility promises, (17) and (18) imply that for every w in W possible choices h for current utility satisfy

$$(19) \quad \underline{h}(w) := \frac{w - \beta \bar{w}}{(1 - \beta)} \leq h \leq \frac{w - \beta \sum \pi(y') U^{Aut}(y')}{(1 - \beta)} =: \bar{h}(w)$$

Accordingly define $\underline{h} := \underline{h}(\underline{w})$ and $\bar{h} := \bar{h}(\bar{w})$. We will show below that we can choose $\bar{w} = \max_y U^{Aut}(y) + \varepsilon$, for $\varepsilon > 0$ arbitrarily small, without the constraints $g_{y'}(w) \leq \bar{w}$ binding at the optimal solution, for all $w \in W$. In order to assure that the constraint set of our dynamic programming problem is compact, for all $w \in W$ we need (since D need not be compact)

Assumption 3: $[\underline{h}, \bar{h}] \subseteq D$.

Assumption 3 is an assumption purely on the fundamentals (u, π, Y, r) of the economy and hence straightforward to check. In particular, for $r = -1$ (the case studied by Kehoe and Levine

¹²Note that if we had assumed that u and hence C are bounded functions this complication is avoided as the upper bound on u serves as upper bound \bar{w} . The results to follow do *not* require boundedness of u .

(1993)) we have $\bar{h}(\bar{w}) = u(y_{\max}) \in D$ and $\underline{h}(\underline{w}) = u(y_{\min}) - \beta[u(y_{\max}) - Eu(y)] \in D$ as long as $\frac{y_{\max}}{y_{\min}}$ is sufficiently small and/or β is sufficiently small.¹³

Using standard theory of dynamic programming with bounded returns it is easy to show that the operator T_R has a unique fixed point $V_R \in C(W)$ and that for all $v_0 \in C(W)$, $\|T_R^n v_0 - V_R\| \leq \frac{1}{R^n} \|v_0 - V_R\|$, with the norm being the sup-norm. Also V_R is strictly increasing, strictly convex and continuously differentiable and the optimal policies $h(w), g_{y'}(w)$ are continuous, single-valued functions.¹⁴

We will now use the first order conditions to characterize optimal policies.

$$\begin{aligned}
 C'(h) &\leq \frac{1-\beta}{\beta(R-1)} V'(g_{y'}) \\
 (20) \quad &= \frac{1-\beta}{\beta(R-1)} V'(g_{y'}) \quad \text{if} \quad g_{y'} > U^{Aut}(y') \\
 w &= (1-\beta)h + \beta \sum_{y' \in Y} \pi(y') g_{y'}
 \end{aligned}$$

The envelope condition is:

$$(21) \quad V'(w) = \frac{(R-1)}{R(1-\beta)} C'(h)$$

First we characterize the behavior of h and $g_{y'}$ with respect to w . The planner reacts to a higher utility promise w today by increasing current and expected future utility, i.e. by smoothing the cost over time and across states. The continuing participation constraints, though, prevent complete cost smoothing across different states: some agents have to be promised more than otherwise optimal in certain states to be prevented from defaulting in that state. This is exactly the reason why complete risk sharing may not be constrained feasible.

¹³For CRRA utility with coefficient of relative risk aversion $\sigma \geq 1$ and $r = -1$ assumption 3 is always satisfied.

¹⁴The proofs of these results are again adaptations of proofs by Atkeson and Lucas (1995). They are available from the authors upon request.

LEMMA 1. Let assumptions 1-3 be satisfied. The optimal policy h , associated with the minimization problem in (16) is strictly increasing in w . The optimal policies $g_{y'}$, are constant in w and equal to $U^{Aut}(y')$ or strictly increasing in w , for all $y' \in Y$. Furthermore

$$g_{y'}(w) > U^{Aut}(y') \text{ and } g_{\bar{y}'}(w) > U^{Aut}(\bar{y}') \text{ imply } g_{y'}(w) = g_{\bar{y}'}(w)$$

$$g_{y'}(w) > U^{Aut}(y') \text{ and } g_{\bar{y}'}(w) = U^{Aut}(\bar{y}') \text{ imply } g_{y'}(w) \leq g_{\bar{y}'}(w) \text{ and } y' < \bar{y}'$$

Proof. See Appendix

The last part of the lemma states that future promises are equalized across states whenever the continuing participation constraints permit it. Promises are increased in those states in which the constraints bind.

Now we state a result that is central for the existence of an upper bound \bar{w} of utility promises. For promises that are sufficiently high it is optimal to deliver most of it in terms of current period utility, and promise less for the future than the current promises. This puts an upper bound on optimal promises in the long run, the main result in this section, stated in Theorem 1.

LEMMA 2. Let assumptions 1-3 be satisfied. For every $(w, y') \in W \times Y$, if $g_{y'}(w) > U^{Aut}(y')$, then $g_{y'}(w) < w$. Furthermore, for each y' , there exists a unique $w_{y'}$ such that $g_{y'}(w_{y'}) = w_{y'} = U^{Aut}(y')$.

Proof. See Appendix

THEOREM 1. *Let assumptions 1-3 be satisfied. There exists a \bar{w} such that $g_{y'}(w) < w$ for every $w \geq \bar{w}$ and every $y' \in Y$.*

Proof. See Appendix

Note that the preceding theorem implies that whenever $w \in [\underline{w}, \bar{w}] = W$, then for all $y' \in Y$, the constraint $g_{y'}(w) \leq \bar{w}$ is never binding; since the constraint set in the original dynamic program-

ming problem without the additional constraints is convex, the policy functions characterized in this section are also the optimal policies for the original problem for all $w \in W$. For any $(w_0, y_0) \in W \times Y$ these policies then induce efficient sequential allocations as described in Section A.

The policy functions $g_{y'}$ together with the transition matrix π induce a Markov process on $W \times Y$. In the next subsection we will show that this Markov process has a unique invariant measure, the long-run cross sectional distribution of utility promises (and hence welfare) and income, for any given fixed $R \in (1, \frac{1}{\beta})$.

C. Existence and Uniqueness of an Invariant Probability Measure

Let $\mathcal{B}(W)$ and $\mathcal{P}(Y)$ the set of Borel sets of W and the power set of Y . The function $g_{y'}(w)$, together with the transition function π for the endowment process, defines a Markov transition function on income shock realizations and utility promises $Q : (W \times Y) \times (\mathcal{B}(W) \times \mathcal{P}(Y)) \rightarrow [0, 1]$ as follows:

$$(22) \quad Q(w, y, \mathcal{W}, \mathcal{Y}) = \sum_{y' \in \mathcal{Y}} \begin{cases} \pi(y') & \text{if } g_{y'}(w) \in \mathcal{W} \\ 0 & \text{else} \end{cases}$$

Given this transition function, we define the operator T^* on the space of probability measures $\Lambda((W \times Y), (\mathcal{B}(W) \times \mathcal{P}(Y)))$ as

$$(23) \quad (T^*\lambda)(\mathcal{W}, \mathcal{Y}) = \int Q(w, y, \mathcal{W}, \mathcal{Y}) d\lambda = \sum_{y' \in \mathcal{Y}} \pi(y') \int_{\{w \in W | g_{y'}(w) \in \mathcal{W}\}} d\lambda$$

for all $(\mathcal{W}, \mathcal{Y}) \in \mathcal{B}(W) \times \mathcal{P}(Y)$. Note that T^* maps Λ into itself (see Stokey et. al. (1989), Theorem 8.2). An invariant probability measure associated with Q is defined to be a fixed point of T^* . We now show that such a probability measure exists and is unique.

THEOREM 2. *Let assumptions 1-3 be satisfied. Then there exists a unique invariant probability measure Φ associated with the transition function Q defined above. For all $\Phi_0 \in \Lambda((W \times Y), (\mathcal{B}(W) \times \mathcal{P}(Y)))$, $(T^*\Phi_0)^n$ converges to Φ in total variation norm.*

Proof. See Appendix

Note that Lemma 2. and Theorem 1. above imply that any ergodic set of the Markov process associated with Q must lie within $[U^{Aut}(y_{\min}), U^{Aut}(y_{\max})] \times Y$ and that the support of the unique invariant probability measure is a subset of this set.

So far we proved that, for a fixed intertemporal price R , policy functions $(h, g_{y'})$, cost functions V and invariant probability measures Φ exist and are unique. From now on we will index $(h, g_{y'})$, V and Φ by R to make clear that these functions and measures were derived for a fixed R . In the next section we will discuss how to find the intertemporal price R^* associated with an *efficient* stationary allocation. Remember from Subsection A that this requires the allocation to satisfy the aggregate resource constraint with equality, a constraint that we have not yet imposed and will do so in the next subsection in order to solve for R^* .

D. Determination of the “Market Clearing” R

In this section we will analyze how the resource requirements imposed by the cross-sectional distribution of utility promises Φ_R vary with R . We will provide conditions under which an R^* exists for which these resource requirements exactly equal the aggregate endowment in the economy.

In the previous section we showed that for a fixed $R \in (1, \frac{1}{\beta})$ there exists a unique stationary joint distribution Φ_R over (w, y) . Define the “excess demand function” $d : (1, \frac{1}{\beta}) \rightarrow \Re$ as

$$(24) \quad d(R) = \int V_R(w) d\Phi_R - \int y d\Phi_R$$

In this section we discuss the qualitative features of the function $d(\cdot)$. Since by assumption $\bar{y} := \int y d\Phi_R$ does not vary with R , the behavior of d depends on how V_R and Φ_R vary with R . The behavior of Φ_R with respect to R in turn depends on the behavior of $g_{y'}^R$ with respect to R as $g_{y'}^R$ determines the Markov process to which Φ_R is the invariant probability measure. Following Atkeson

and Lucas (1995) we can show that $d(R)$ is continuous and increasing on $(1, \frac{1}{\beta})$.¹⁵

Thus, if one can show that

$$(25) \quad \lim_{R \searrow 1} d(R) < 0$$

$$(26) \quad \lim_{R \nearrow \frac{1}{\beta}} d(R) > 0$$

then the existence of a resource-clearing R^* follows.¹⁶

The Case $R = \frac{1}{\beta}$

In this subsection we characterize optimal policies of the planner for $R = \frac{1}{\beta}$ and provide a sufficient condition for (26) to hold. Note that for $R = \frac{1}{\beta}$

$$(27) \quad g_{y'}(w) = \begin{cases} w & \text{if } w \geq U^{Aut}(y') \\ U^{Aut}(y') & \text{if } w < U^{Aut}(y') \end{cases}$$

from the first order conditions of the recursive planners' problem (which still has a unique solution as all the results of Section B go through). Now there is a continuum of invariant measures associated with the Markov chain induced by the optimal policies. From (27) it is clear that any such measure $\Phi_{\frac{1}{\beta}}$ satisfies $w \notin \text{supp}\left(\Phi_{\frac{1}{\beta}}\right)$ for all $w < U^{Aut}(y_{\max})$ as the probability of leaving such a w is at least $\pi(y_{\max})$ and the probability of coming back (into a small enough neighborhood) is 0. Therefore all w in the support of any possible invariant measure satisfy $g_{y'}(w) = w$. From the promise-keeping constraint $h(w) = w$ follows, and each individuals' consumption is constant over time: for $R = \frac{1}{\beta}$ there is complete risk sharing.

¹⁵Again the arguments are adaptations of Atkeson and Lucas' (1995) results and available upon request. For continuity of $d(R)$ one shows that V_R is uniformly continuous in R and that $g_{y'}^R$ is continuous as a function of R so that Φ_R is continuous in R (in the sense of weak convergence). For monotonicity of $d(R)$ the key results are that $g_{y'}^R$ is increasing in R so that $\Phi_R(\cdot, y)$ is increasing in R (in the sense of stochastic dominance), which, together with the fact that V_R is increasing in w proves that $d(R)$ is an increasing function.

¹⁶Also note that, given our previous theoretical results, it is straightforward to search for R^* numerically: fix an R^0 , solve the planners' dynamic programming problem (which we proved to have a unique solution), determine the induced invariant measure over promises (whose existence and uniqueness we proved), and compute $d(R^0)$. If $d(R^0) > 0$, reduce the guess for R , otherwise increase it. We have included details of our computational algorithm in the separate theoretical appendix, available upon request.

For complete risk sharing to be efficient it has to satisfy the resource constraint. Since the cost function V_R is strictly increasing in w , the one of the continuum of invariant measures with lowest cost is

$$(28) \quad \Phi_{\frac{1}{\beta}}(w, y) = \begin{cases} \pi(y) & \text{if } w = U^{Aut}(y_{\max}) \\ 0 & \text{if } w \neq U^{Aut}(y_{\max}) \end{cases}$$

All individuals receive utility promises $w = U^{Aut}(y_{\max})$ and hence the same current utility $h(U^{Aut}(y_{\max})) = U^{Aut}(y_{\max})$. This allocation has per-period resource cost $C(U^{Aut}(y_{\max}))$ and is resource feasible if and only if $C(U^{Aut}(y_{\max})) \leq \bar{y}$, or applying the strictly increasing period utility function u to both sides, if and only if $U^{Aut}(y_{\max}) \leq u(\bar{y})$. Let the net resource cost of this allocation be denoted by

$$(29) \quad d\left(\frac{1}{\beta}\right) = C(U^{Aut}(y_{\max})) - \bar{y}$$

We summarize the discussion in the following

LEMMA 3. Let assumptions 1-3 be satisfied. For $R = \frac{1}{\beta}$ any solution to the recursive social planners' problem exhibits complete risk sharing. There exists an efficient stationary allocation with complete risk sharing if and only if $U^{Aut}(y_{\max}) \leq u(\bar{y})$.

Intuitively, the lemma states that it is constrained efficient to share resources equally among the population in this economy if the agents with the highest incentive to renege on this sharing rule, namely the agents with currently high income, find it in their interest to accept constant consumption at $c = \bar{y}$ and lifetime utility $u(\bar{y})$, rather than to leave and obtain lifetime utility $U^{Aut}(y_{\max})$.

Using arguments similar to showing continuity of $d(R)$ on $(1, \frac{1}{\beta})$ one can show that $\lim_{R \nearrow \frac{1}{\beta}} d(R) = d\left(\frac{1}{\beta}\right)$, where $d\left(\frac{1}{\beta}\right)$ is defined as in (29). In order to rule out complete risk sharing¹⁷ we now make

¹⁷If there is complete risk sharing under a particular tax system (remember that the tax system maps a given pre-tax income process into a particular after-tax income process), then a small tax reform has no effect on the extent of risk sharing since the resulting allocation is the complete risk sharing allocation: our crowding-out effect is absent.

Assumption 4: $U^{Aut}(y_{\max}) > u(\bar{y})$

Note that this assumption is satisfied if the time discount factor β is sufficiently small, agents are not too risk-averse or the largest endowment shock is sufficiently large. We obtain

LEMMA 4. Let assumptions 1-4 be satisfied. Then $\lim_{R \nearrow \frac{1}{\beta}} d(R) > 0$.

Proof. Applying the strictly increasing cost function C to the inequality of assumption 4 gives

$$d\left(\frac{1}{\beta}\right) = C\left(U^{Aut}(y_{\max})\right) - \bar{y}$$

$\dot{\dot{0}}$

The Case of R Approaching 1

In this subsection we provide necessary and sufficient conditions for autarky (all agents consume their endowment in each period) to be an efficient allocation and characterize policies for R approaching 1 from above.

If agents are very impatient and/or the risk of future low endowments is low, then it is not efficient for the planner to persuade currently rich agents to give up resources today in exchange for insurance tomorrow. For $r = -1$ (no saving after default, as in Kehoe and Levine (1993)) this result can be stated and proved formally in the next

LEMMA 5. Let $r = -1$ and let assumptions 1-3 be satisfied. Autarky is efficient if and only if

$$(30) \quad \beta \frac{u'(y_{\min})}{u'(y_{\max})} < 1$$

Proof. For the if-part we note that the autarkic allocation satisfies the first order conditions for some $R > 1$ if (30) holds. Since autarky is constrained feasible, it is efficient.¹⁸ The only-if part is proved in the appendix.

¹⁸This is in fact true for arbitrary $r \geq -1$.

The previous lemma provides a condition under which $d(R) = 0$ as R approaches 1, with autarky as the (efficient) allocation. In order to assure that autarky is not efficient¹⁹ we make

Assumption 5:

$$(31) \quad \beta \frac{u'(y_{\min})}{u'(y_{\max})} \geq 1$$

With assumption 5, as R approaches 1, the resulting allocation features some, but (as long as assumption 4 holds) not complete risk sharing. We state the following conjecture, which we were able to prove for CRRA utility, $r = -1$ and $Y = \{y_l, y_h\}$ but not for the general case considered here.²⁰

CONJECTURE 3. *Let assumptions 1-5 be satisfied. Then there exists $R > 1$ such that $d(R) < 0$.*

We then can conclude our theoretical analysis of stationary efficient allocations with the following theorem, whose proof follows directly from the previous lemmas and conjecture.²¹

THEOREM 4. *Let assumptions 1-5 be satisfied. There exists $R^* \in (1, \frac{1}{\beta})$ such that $d(R^*) = 0$. The allocation induced by $(h^{R^*}, g_y^{R^*})$ is efficient and has some, but not complete risk sharing.*

As indicated above, some of our results and proof strategies resemble Atkeson and Lucas (1995). The basic strategy to prove existence of a stationary general equilibrium (as we will show in the next section stationary efficient allocations induce stationary equilibrium allocations) also exhibits similarities to existence proofs for standard incomplete markets models as in Huggett (1993) and Aiyagari (1994).²² The main difference is that the authors, due to the simple asset structure

¹⁹If the efficient allocation is autarkic a small change in the tax system changes the allocation on a one to one basis with after tax incomes. No private insurance is crowded out since no private insurance takes place.

²⁰Given our other theoretical results, we can check whether $d(R) < 0$ for R sufficiently close to 1 numerically. In all our quantitative experiments this was the case.

²¹No claim of uniqueness can be made. In all our numerical exercises $d(R)$ was *strictly* increasing, however, yielding a unique R^* and associated unique stationary efficient allocation.

²²We will contrast the quantitative findings from our model with the Huggett (1993) version of the standard exogenous incomplete markets model in Section 10.

in their models, can tackle the equilibrium directly. As we do, they first, for a fixed and constant interest rate, solve a simple dynamic programming problem²³ (they for the single household, with assets as state variable, we for the planners, with utility promises as state variables). Then they let the optimal policies induce a Markov process to which a unique invariant distribution is shown to exist.²⁴ Finally the market clearing interest rate is determined from the goods or asset market clearing condition.²⁵ These similarities in the theoretical analysis also suggest similar computational algorithms when solving both models numerically.

4. Decentralization

In this section we describe how we decentralize a stationary efficient allocation $h = \{h_t(w_0, y^t)\}_{t=0}^{\infty}$ induced by the optimal policies from the recursive planners' problem as a competitive equilibrium as defined in Section 3. Let $\beta^t \pi(y^t|y_0) \mu(a_0, y^t) \geq 0$ be the Lagrange multiplier associated with the continuing participation constraint at history y^t and $P(y^t) = \{y^\tau | \pi(y^t|y^\tau) > 0\}$ be the set of all endowment shock histories that can have y^t as its continuation. Using the first order necessary conditions of the household's maximization problem (6) one obtains

$$(32) \quad \beta \frac{u'(c_t(a_0, y^{t+1})) \pi(y^{t+1}|y_0)}{u'(c_t(a_0, y^t)) \pi(y^t|y_0)} = \frac{p_{t+1}(y^{t+1})}{p_t(y^t)} \frac{1 + \sum_{y^\tau \in P(y^t)} \mu(a_0, y^\tau)}{1 + \sum_{y^\tau \in P(y^{t+1})} \mu(a_0, y^\tau)}$$

Obviously, an agent whose participation constraint does not bind at contingency y^{t+1} , following history y^t , faces the standard complete markets Euler equation (as $\mu(a_0, y^{t+1}) = 0$).

²³As in our model, boundedness of the state space for assets from above has to be assured. Huggett assumes that income can only take two values, but doesn't need the stochastic process to be *iid* over time nor any assumption on the period utility function. Aiyagari assumes *iid* income and u to be bounded and to have bounded relative risk aversion -see his working paper. We do not require any boundedness assumption on u , but need the *iid* assumption.

²⁴The theorems invoked for the existence of a unique invariant measure are similar in spirit; in particular they all require a "mixing condition" that assures that there is a unique ergodic set. In their setting agents with bad income shocks run down their assets, and good income shocks induce upward jumps in the asset position; in our setting agents with bad shocks move down in the entitlement distribution towards $U^{Aut}(y_{\min})$, with good shocks inducing jumps towards higher w , due to binding participation constraints.

²⁵Huggett provides no theoretical properties of the excess asset demand function, in Aiyagari the presence of physical capital, which makes the supply of assets interest-elastic, assures (together with continuity of the asset demand function) the existence of a market-clearing interest rate.

Now consider the efficient allocation of utilities $\{h_t(w_0, y^t)\}$ as found in the previous section.

Combining the first order condition and the envelope condition from the planners problem we have

for an agent that is unconstrained²⁶ (see (20) and (21)):

$$(33) \quad \frac{1}{R} = \beta \frac{C'(h_t(w_0, y^t))}{C'(h_{t+1}(w_0, y^{t+1}))} \equiv \beta \frac{u'(c_{t+1}(w_0, y^{t+1}))}{u'(c_t(w_0, y^t))}$$

This suggests that the equilibrium prices satisfy (with normalization of $p_0 = 1$)

$$(34) \quad p_t(y^t) = \frac{\pi(y^t|y_0)}{R^t} = p_t \pi(y^t|y_0).$$

with $p_t = R^{-t}$. That is, the price for a commodity delivered contingent on personal histories is composed of two components, an aggregate intertemporal price $p_t = R^{-t}$ and an individual specific, history dependent component, equal to the probability that the personal history occurs.

Given prices, the initial wealth level that makes the efficient consumption allocation affordable for an agent of type (w_0, y_0) is given by

$$(35) \quad a_0 = c_0(w_0, y_0) - y_0 + \sum_{t=1}^{\infty} \sum_{y^t|y_0} \frac{\pi(y^t|y_0)}{R^t} (c_t(w_0, y^t) - y_t) = a_0(w_0, y_0) < \infty$$

where the last inequality follows from the fact that the efficient consumption allocation is bounded from above.²⁷ Finally, the equilibrium consumption allocation corresponding to the efficient allocation is given by²⁸

$$(36) \quad c_t(a_0, y^t) = c_t(a_0^{-1}(w_0, y_0), y^t) = C(h_t(w_0, y^t)).$$

The preceding discussion can be summarized in the following

²⁶If no agent is unconstrained we are in autarky and can take $\frac{1}{R} = \beta \frac{u'(y_{\min})}{u'(y_{\max})}$.

²⁷Therefore, to decentralize a particular stationary efficient consumption allocation we require a very particular initial distribution over initial assets. In this sense one of the primitives of our model, Θ_0 , can't be chosen arbitrarily, which is true in all steady state analyses.

²⁸Given that the optimal recursive policy function $h(\cdot, y)$ is a strictly increasing function in w , the $h_t(\cdot, y^t)$ and hence the $c_t(\cdot, y^t)$ are strictly increasing in w_0 . Therefore $a_0(\cdot, y_0)$ is strictly increasing and thus invertible. We denote its inverse by a_0^{-1} .

THEOREM 5. Suppose that $\{h_t(w_0, y^t)\}_{t=0}^\infty$ is a stationary efficient allocation (with associated shadow interest rate $R > \frac{1}{\beta}$). Then prices $\{p_t(y^t)\}$ and allocations $\{c_t(a_0, y^t)\}$, as defined in (34) and (36) are an equilibrium for initial wealth distribution Θ_0 derived from Φ_0 and (35).

Proof. See Appendix

So far we have shown the existence of a stationary equilibrium of our economy and characterized some of its properties. In the next section we illustrate some of its qualitative features.

5. Qualitative Features of the Efficient Allocation

In this section we illustrate some of the qualitative features of the efficient allocation characterized in the section above. To do so we consider a simple numerical example of our economy in which the endowment process is *iid* can take only two values, $0 \leq y_l < y_h \leq 2$, which are equally likely and satisfy

$$(37) \quad 0.5(y_l + y_h) = 1$$

so that $y_l = 2 - y_h$. We compare the qualitative features of this economy to a similar economy with only two agents, each of which has endowment $y^i \in \{y_l, y_h\}$. We assume that in the two agent economy incomes are perfectly negatively correlated²⁹, so that if agent 1 has income y_l , agent 2 has income y_h and vice versa. Consequently, as in the continuum economy, average income in the economy is nonstochastic and equal to 1. In accordance with the continuum economy we also assume that the income process in the two-agent economy is *iid* over time, with equal probability of each agent being rich in every period.

For both economies we assume that the outside option is characterized by $r = -1$, that is, no saving in autarky is permitted. Then it is straightforward to compute that for both economies

²⁹Similar, although less clean results can be derived for the two-agent economy where shocks are not perfectly negatively correlated, in which case aggregate (average) income is stochastic.

the utilities from autarky are given by

$$\begin{aligned} U^{Aut}(y_l) &= \left(1 - \frac{1}{2}\beta\right) u(2 - y_h) + \frac{1}{2}\beta u(y_h) \\ U^{Aut}(y_h) &= \frac{1}{2}\beta u(2 - y_h) + \left(1 - \frac{1}{2}\beta\right) u(y_h) \end{aligned}$$

We note that the size of y_h is a measure of income risk, with higher y_h associated with more income risk. It is straightforward to show that $U^{Aut}(y_l)$ is strictly declining in y_h , whereas $U^{Aut}(y_h)$ is strictly increasing in y_h at $y_h = 1$, and strictly concave with unique maximum $y_h^* \in (1, 2)$ (see Krueger and Perri, 2005). The maximum satisfies

$$\frac{1}{2}\beta u'(2 - y_h) = \left(1 - \frac{1}{2}\beta\right) u'(y_h)$$

We now want to compare symmetric constrained efficient stationary consumption distributions across the two models; we are particularly interested in the potential qualitative and quantitative differences of comparative statics results with respect to changes in after-tax income inequality y_h (that is, changes of the progressivity of the tax code). Note that, for a given pre-tax income process an increase in y_h stands for a less progressive tax system. Thus all comparative statics results with respect to y_h to follow can be interpreted as a change in the progressivity of the tax code; with higher progressivity representing a lower value of y_h .³⁰

³⁰As a simple example, consider the situation with $g = 0$, pre-tax income realizations e_l, e_h that are *iid* and equally likely and

$$0.5(e_l + e_h) = 1$$

The tax system is proportional with deduction:

$$T(e) = \tau e - d$$

where τ is a proportional income tax and d is a deduction. The government budget constraint reads as

$$d = \tau \sum_{l,h} 0.5(e_l + e_h) = \tau$$

so that

$$\begin{aligned} y_l &= e_l - T(e_l) = (1 - \tau)e_l + \tau \\ y_h &= (1 - \tau)e_h + \tau \end{aligned}$$

A. Three Risk Sharing Regimes

In both economies the constrained-efficient consumption allocation is either autarky (everybody consumes its income in all states), perfect risk sharing (everybody consumes average income in all states) or partial, but not perfect risk sharing. Keeping fixed all other fundamentals of the economy (u, β, ϕ) , one can define, for both economies, critical income values for y_h ,

$$1 \leq y_2^{aut} \leq y_2^f \leq 2$$

$$1 \leq y_c^{aut} \leq y_c^f \leq 2$$

so that if $y_h \in [1, y_2^{aut}]$, the two agent economy's efficient consumption distribution is autarky, if $y_h \in [y_2^f, 2]$, it is characterized by perfect risk sharing, and if $y_h \in (y_2^{aut}, y_2^f)$ it is characterized by partial risk sharing. Similar notation applies for the continuum economy.

Perfect Risk Sharing

Perfect risk sharing entails consuming average income in the population in each state. For this allocation to be constrained efficient it must satisfy all enforcement constraints. It is straightforward to show that this requires

$$(38) \quad u(1) \geq \frac{\beta}{2} u(2 - y_h) + \left(1 - \frac{\beta}{2}\right) u(y_h)$$

in both economies

The two critical values $y_2^f > 1$ and $y_c^f > 1$ satisfy the above equations with equality, unless even at $y_h = 2$, the equations hold with strict inequality. Perfect risk sharing occurs for exactly the same set of parameter values (and thus for the same range of tax progressivity) in the continuum

The higher is τ , the more progressive is the tax system, since the average tax rate equals

$$t(e) = \tau \left(1 - \frac{1}{e}\right)$$

which is increasing in e , the more the higher is τ . Similarly, the higher is τ , the lower is y_h , and thus a more progressive tax system corresponds one to one with a lower y_h .

economy and the two-agent economy. As long as there is perfect risk sharing, a marginal change in y_h (equivalently, a marginal change in the progressivity of the income tax) has no effect on the consumption allocation in either economy; there is exactly a one-for-one crowding out of private risk sharing from public risk sharing.

Autarky

Autarky may be the only feasible allocation, and thus the constrained efficient allocation.

For the continuum economy autarky is efficient if and only if

$$u'(y_h) \geq \beta u'(2 - y_h)$$

and for the two agent economy it is efficient if and only if

$$u'(y_h) \geq \frac{\beta}{2 - \beta} u'(2 - y_h)$$

Thus we conclude that $y_c^{aut} \in (1, 2)$ and $y_2^{aut} \in (1, 2)$ and that $y_c^{aut} < y_2^{aut}$. Thus the set of values of income (risk) y_h for which the constrained efficient allocation is autarky is strictly bigger in the two agent economy than in the continuum economy. In this sense, there is more risk sharing possible in a continuum economy than in the two-agent economy. Furthermore note that, for the two agent economy,

$$(39) \quad y_2^{aut} = y_h^*$$

so that

$$1 < y_c^{aut} < y_2^{aut} = y_h^*$$

In this region of the parameter space, a small change in y_h (equivalently in the tax system) changes the consumption distribution one-for one with the income distribution. There is no crowding-out effect induced by a change in the tax system. Again, the absence of a crowding out effect occurs for a wider set of parameter values in the two agent economy, relative to the continuum economy.

Partial Risk Sharing

For all $y_h \in (y_c^{aut}, y_c^f)$ (respectively, for all $y_h \in (y_2^{aut}, y_2^f)$) in the two agent economy the stationary constrained efficient consumption distribution is characterized by partial risk sharing. In the next subsection we will characterize this distribution further in both economies, with particular focus on how it changes with the measure of inequality y_h , and thus with the degree of tax progressivity.

Consumption Dynamics with Partial Risk Sharing in the Two Agent Economy: Characterization and Comparative Statics In Krueger and Perri (2005), building on results by Kehoe and Levine (2001) we show that the constrained efficient consumption allocation in the case $y_h \in (y_2^{aut}, y_2^f)$ is characterized by one number, c_h , the consumption level of households with currently high income. This number is the smallest solution of the equation

$$U^{Aut}(y_h) = \frac{1}{2}\beta u(2 - c_h) + \left(1 - \frac{1}{2}\beta\right) u(c_h) = \Gamma(c_h)$$

Note that since the right hand side of the equation is nothing else but the value of autarky with c_h replacing y_h , we know from Krueger and Perri (2005) that the right hand side is continuous and strictly concave in c_h . Furthermore we have $\Gamma(c_h = 1) < U^{Aut}(y_h)$, since we are not in the perfect risk sharing case (see equation (38) above) and $\Gamma(c_h = y_h) = U^{Aut}(y_h)$. Thus there exists a unique consumption level $c_h \in (1, y_h)$ characterizing the efficient consumption allocation.

Furthermore note from (39) that, within the range $y_h \in (y_2^{aut}, y_2^f)$, an increase in y_h reduces the value of $U^{Aut}(y_h)$ and thus reduces c_h ; the consumption distribution fans in an consumption unambiguously becomes less risky. In other words, an increase in public risk sharing (a reduction of y_h) unambiguously increases consumption risk of individuals; in the two agent economy, as long as there is partial insurance, public insurance crowds out private insurance more than one-for-one.

Consumption Dynamics with Partial Risk Sharing in the Continuum Economy: Characterization and Comparative Statics

For the continuum economy, under partial risk sharing (that is, for For all $y_h \in (y_c^{aut}, y_c^f)$), the consumption dynamics and distribution is more complex. The policy functions imply that, in the stationary distribution, an agent with high income receives promised utility equal to the value of autarky regardless of her w . Hence, when an agent receives a high income shock, history is erased in that present utility entitlements w , which summarize the history of past endowment shocks, do not matter for future utility entitlements. For agents with low income history matters. An agent with $w = U^{Aut}(y_h)$ that receives $y' = y_l$ drops to $g_{y'}(U^{Aut}(y_h)) < U^{Aut}(y_h)$, and, upon further bad shocks, works herself downwards through the entitlement distribution. In a countable number of steps an agent with a string of bad shocks arrives at $w = U^{Aut}(y_l)$, with any good shock putting her immediately back to $U^{Aut}(y_h)$.

The properties of the entitlement distribution translate into properties of the stationary consumption measure. Since $h(\cdot)$ and $C(\cdot)$ are strictly increasing functions, consumption reacts to endowment shocks in the same qualitative fashion as utility entitlements. This can be seen from Figure 1. All agents with a high endowment shock consume the same, regardless of their history. Hence the consumption distribution has a mass point at this consumption level equal to fraction of agents with the high income. Another mass point occurs at the minimum of the support of the consumption distribution, provided that $g_{y_l}(w) = U^{Aut}(y_l)$ for some $w > U^{Aut}(y_l)$. In between these two mass points there is a finite number of other mass points consisting of individuals with currently bad shock who will either fall toward the bottom (with an additional bad shock) or jump again to the top (with a good shock), but who have not yet reached the lower bound of the distribution.

Thus the constrained-efficient stationary consumption distribution is formally characterized as follows:³¹

³¹Krueger and Uhlig (2005) prove a similar result, in a model where the consequence of default is not financial autarky, but the best insurance contract a competing financial intermediary offers. The proof of the result here is

THEOREM 6. *The constrained efficient stationary consumption allocation is characterized by a number $n > 2$, an interest rate R and consumption levels c_1, c_2, \dots, c_n , lifetime utility levels w_1, w_2, \dots, w_n and associated probabilities $\pi_1, \pi_2, \dots, \pi_n$ such that*

1. *The stationary consumption and utility distribution is given by*

$$\begin{aligned}\pi_1 &= 0.5^{n-1} \\ \pi_j &= 0.5^{n-j+1} \text{ for } j = 2, \dots, n\end{aligned}$$

2. *The consumption and utility levels satisfy*

$$\begin{aligned}w_1 &= U^{Aut}(y_l) \\ w_n &= U^{Aut}(y_h) \\ (40) \quad w_j &= (1 - \beta)u(c_j) + 0.5\beta \left(w_{\max\{j-1, 1\}} + w_n \right) \text{ for } j = 1, \dots, n\end{aligned}$$

and

$$(41) \quad u'(c_j) = \beta R u'(c_{j-1}) \text{ for } j = 3, \dots, n$$

$$(42) \quad u'(c_2) \geq \beta R u'(c_1)$$

as well as

$$(43) \quad \sum_{j=1}^n \pi_j c_j = 1$$

For a given n , equations (40), (41) and (43) form a system of $2n+1$ equations in the unknowns $c_1, c_2, \dots, c_n, w_1, w_2, \dots, w_n, R$. If for a given n , equation (42) is satisfied and there is no larger n such that this is true, we have found the optimal step size n . Note that this simple algorithm, of course, is

identical to theirs.

only applicable in the *iid* case with two shocks. For a more general endowment process the method theoretically developed above and described in detail in the appendix needs to be used to compute stationary constrained-efficient allocations.

Despite the fact that it is in general hard to analytically solve for the consumption distribution, qualitative features of it can be readily deduced.

COROLLARY 1. In the continuum economy we have (from (40)) that $c_1 = y_l$ and $c_n < y_h$.

Thus it is immediately clear that, in stark contrast to the two-agent economy, an increase in y_h and thus a decrease in y_l reduces the lower end of the support of the consumption distribution. Thus an increase in tax progressivity, while leading to more consumption risk and lower minimum consumption in the two-agent model, leads to an *increase* of minimum consumption in the continuum model.

In the remainder of the paper we parameterize our economy using data from the US economy. This will allow us to analyze the *quantitative* properties of equilibria of our economy as well as to quantify the size of the crowding-out effect.

6. A Quantitative Example³²

In this section we describe the policy experiments we conduct. It is our goal to investigate how different tax systems affect equilibrium risk sharing. In particular we use the model to measure the extent to which public risk sharing mechanisms (i.e. progressive taxes) and private risk sharing mechanisms (i.e. financial markets) help to insulate private consumption from random income fluctuations.

³²THIS PART WILL BE REVISED. NEW RESULTS WILL BE PRESENTED FOR AN INCOME PROCESS WITH A DETERMINISTIC AND A STOCHASTIC PART. ALSO THE BASELINE MODELS (BOTH THE LIMITED ENFORCEMENT AND THE EXOGENOUS INCOMPLETE) WILL BE PARAMETRIZED SO TO MATCH THE JOINT DISTRIBUTION OF CONSUMPTION AND INCOME IN THE US PRE-TAX CHANGES. IN ORDER TO DO SO AN ASSET IN POSITIVE NET SUPPLY (BONDS WITH A FIXED INTEREST RATE) WILL BE ADDED

We first use CEX data to measure a tax system that we identify with the current (progressive) tax system in the US. Keeping the pre-tax income process unchanged we then study how a change in the tax system affects steady state allocations.

It is important to stress that we have to restrict our analysis to a comparison of steady states. An unexpected change in government policies changes the possible distribution of lifetime expected discounted utilities this economy can attain with given aggregate resources (which remain unchanged). Thus, for a particular agent the promised utility w she entered the period with is not a valid description of her state after the change in fiscal policy anymore. Consequently a method that employs promised expected utility as a state variable cannot be employed to compute transitional dynamics induced by unexpected policy innovations.³³

A. Parameterization

First we describe how we set the parameters governing the individual endowment process, together with the government policies (spending and taxes). We then discuss our selection of the preference parameters β , the subjective time discount factor, and σ , the coefficient of relative risk aversion, as we assume that the period utility function is of CRRA-form.

Income Process

To characterize the Markov chain governing the individual endowment process in the model we need to set the N possible values the endowment e_t can take and estimate the transition matrix $\pi(e_{t+1}|e_t)$. In order to do so we use household level data from the Consumer Expenditure Survey (CEX) for the years 1986-1998. We use CEX income data, whose quality is supposedly inferior to PSID data because the CEX reports also taxes paid by the household members and transfers

³³Technically speaking, after the unexpected change in fiscal policy either some of the promise-keeping constraints are not binding anymore or some of the debt constraints are violated. Any transition analysis in this economy has to tackle the competitive equilibrium directly, where the equilibrium is formulated as a sequential markets equilibrium in the spirit of Alvarez and Jermann (2000).

received, such as welfare and unemployment insurance payments. We try to reduce measurement error by excluding from our sample households classified as incomplete income respondents, as suggested by Nelson (1994). For the same reason we exclude households that report negative total consumption expenditure. Also, since we interpret our pre-tax endowment concept from the model as labor income we exclude households which are solely composed of members that are older than 64 years and families without labor income earners.

The CEX quantity we interpret as e_t , household endowment before taxes, is labor earnings. In the data we measure this entity by the sum of labor earnings, plus a fraction³⁴ of business and farm income earned by all the members of the household, all divided by the number of adult equivalents³⁵ in the household.

We pick $N = 5$. The transition matrix $\pi(e_{t+1}|e_t)$ is computed as follows. For any quarter t in the CEX sample we group households into 5 relative earning classes delimited by 4 uniformly spaced quintiles: the first class is composed by the bottom 20% of the earning distribution in that quarter, the second class by the following 20% and so on. We then search for all households for which we have earning observations at t and $t + 4$ (e_t and e_{t+4}) and compute which relative class they belong to in period $t + 4$. Note that the class delimiters are time-dependent in order to take into account of aggregate growth. We repeat this for every quarter in the sample. Then the probability of transiting from class i to class j is given by the number of households transiting from i to j , divided by the total number of households starting in class i for the entire sample. To set e_1, \dots, e_N we first compute, in each quarter, the median earning within each earnings class. We detrend it by taking its ratio to the overall median earning for that quarter and then set e_1, \dots, e_N equal to the average across quarters of the median (detrended) earnings in each earning class. Tables 1 and 2

³⁴The fraction of business and farm income we impute to labor income is 0.864 as reported in Diaz Jimenez, Quadrini and Rios Rull (1997).

³⁵The number of adult equivalents is defined as in Deaton and Paxson (1994) as the number of households members over age 16 plus .5 times the number of members below age 16.

show the results of this exercise.

Table 1. Endowment values. Ratio to median. (1986.1-1998.4, 89504 obs.)

e_1	e_2	e_3	e_4	e_5
0.18	0.60	1.00	1.51	2.55

Table 2. Transition Matrix (1986.1-1998.4, 27633 obs.)

Quintile at t	Quintile at $t + 4$				
	1	2	3	4	5
1	.67	.18	.05	.02	.02
2	.22	.57	.18	.05	.02
3	.06	.19	.56	.18	.03
4	.03	.05	.18	.60	.15
5	.02	.02	.03	.15	.78

Fiscal Policy

To characterize current fiscal policy we need to measure the values of $\tau(e_j), j = 1, \dots, N$. In the CEX households are asked to report federal, state and local taxes deducted from their last paycheck separately from any additional (not deducted from paycheck) federal, state and local taxes paid. We add to taxes social security contributions and subtract transfers (welfare, unemployment compensation and food stamps). We then set $\tau(e_j)$ equal to the ratio between the total sum of federal, state and local taxes and social security deducted from paycheck, net of transfers, in the j -th earnings class and the total labor income as measured above in the same class. Once the tax policy is set we can compute the implied level of government spending (net of transfers) such that the budget is balanced in every period. The tax policy we will use in our experiments is the average of the tax policies measured in the last four years of the sample (1995-1998) and is reported below.

Table 3. Tax Rates, (1995.1-1998.4, 26333 obs.)

$\tau(e_1)$	$\tau(e_2)$	$\tau(e_3)$	$\tau(e_4)$	$\tau(e_5)$
-40%	+8.5%	+14%	+16.5%	+19%

The implied government spending, net of transfers, is equal to $g = 14.9\%$ of total endowment.

Preference Parameters

We calibrate the preference parameters (σ, β) so that the solution for the benchmark model delivers an interest rate of 2.9% per year. This was the average real return on high grade municipal bonds³⁶ over the period 1986-1998. We use this interest since returns on municipal bonds are usually tax-exempt and hence marginal tax rates for interest income need not be specified.

We set σ equal to 2 and then choose β so to match the interest rate. The non-standard part of this exercise is that in a debt constrained economy for a given σ there might be multiple β that deliver the same interest rate. Figure 2 shows the relation between the time discount factor and the interest rate in our calibrated economy: the figure shows that there are three values of β that generate an equilibrium interest rate of 2.9%. To understand the non-monotonic relationship between R and β note that the real interest rate is given by the marginal rate of substitution of an unconstrained agent

$$\frac{1}{R} = \beta \max_{w_0, y^{t+1}} \frac{u'(c_{t+1}(w_0, y^{t+1}))}{u'(c_t(w_0, y^t))}$$

For any value of σ if the time discount factor β is sufficiently close to 1 the efficient allocation involves complete risk sharing of idiosyncratic risk, individual consumption is constant, $\frac{u'(c_{t+1}(w_0, y^{t+1}))}{u'(c_t(w_0, y^t))}$ is always equal to 1 and the gross interest rate R equals $\frac{1}{\beta}$, hence is decreasing in β .

As β is reduced, complete risk sharing is no longer an equilibrium and there are two effects on the equilibrium interest rate. On one hand there is the standard effect: with a lower β individuals

³⁶The series for the returns is available on the Economic Report of the President, (2000), Table B-71.

care less about the future, want to save less and an increase in interest rate is needed to restore equilibrium in the credit market. On the other hand a lower β reduces possible risk sharing³⁷ and increases consumption risk faced by agents; this effect induces consumers to save more and thus a reduction in the interest rate is needed to restore equilibrium.

Figure 3 shows that when β is relatively close to 1 the first effect dominates and when β is relatively close to 0 the second effect dominates. Finally if β is sufficiently close to 0 then autarky is the efficient allocation, the interest rate is given by $\frac{1}{R} = \beta \frac{u'(y_{\min})}{u'(y_{\max})}$, and there are no longer changes in the consumption allocation, implying that the interest rate is again decreasing in β .

For a value of $\sigma = 2$ there are three possible values of β ($\beta = 0.02, \beta = 0.024, \beta = 0.967$) consistent with an interest rate of 2.9%. Since the allocation arising under $\beta = 0.02$ is the autarkic allocation³⁸ we will focus only on the allocation arising under $\beta = 0.024$ and $\beta = 0.967$.

Finally we specify the consequences of default as exclusion from insurance and credit markets. Individuals, however can privately save at interest rate r even after default. We set $r = 2.9\%$, i.e. individuals can save (but not borrow) at the equilibrium interest rate after default.

B. Quantitative Results

Before we present our numerical results we define different measures of risk sharing. We define *Total Intermediation* (TI) of risk as one minus the ratio between the standard deviations of log-consumption to log-pre-tax income:

$$TI = 1 - \frac{std(\log(c))}{std(\log(e))}.$$

Note that when $std(\log(c)) = 0$, $TI = 1$; consumption does not vary at all across individuals and the economy exhibits complete risk sharing. If $std(\log(c)) = std(\log(e))$, $TI = 0$ and consumption

³⁷Households with high income realizations discount the future possibility of having low income heavily and thus large transfers from these agents to low-income agents violate the continuing participation constraints.

³⁸The quantitative feature of the autarkic allocation are very similar to those of the allocation arising with a discount factor of $\beta = 0.024$

varies one to one with pre-tax endowments For $0 < TI < 1$ there is some, but not complete risk sharing, with higher TI indicating higher risk sharing.

We can decompose TI into two components reflecting risk intermediation enforced by the government (GI) via the tax system and risk intermediation achieved in addition by private insurance contracts, (PI). We define

$$GI = 1 - \frac{std(\log(y))}{std(\log(e))} \quad PI = 1 - \frac{std(\log(c))}{std(\log(y))}$$

When $std(\log(y)) = std(\log(e))$, $GI = 0$. After-tax income is as variable as pre-tax income, which is the case if the tax system is proportional. If $std(\log(y)) < std(\log(e))$, $GI > 0$ and the tax system is progressive, with the extreme of complete redistribution via the tax system, $std(\log(y)) = 0$ and $GI = 1$. On the other hand, if $std(\log(y)) > std(\log(e))$, $GI < 0$ and the tax system is regressive. The interpretation of PI is similar: if $std(\log(c)) = 0$, $PI = 0$ and there is complete risk sharing achieved through private markets. If, on the other hand $std(\log(c)) = std(\log(y))$, $PI = 0$ and private markets do not achieve any risk sharing over and above that achieved by the tax system. Note that $TI = GI + (1 - GI) * PI$. Hence total intermediation of risk equals government intermediation of risk plus private intermediation of that part of risk that is not already removed by the tax system. In particular, under a proportional tax system $GI = 0$ and $TI = PI$. We will report the measures TI, GI and PI for all our policy simulations.

As described above we consider the following policy experiment: change the tax system from the progressive system found in the calibration section to a proportional system (constant average tax rates), keeping the level of government spending constant. We compare interest rates and measures of intermediation of risk in the steady states arising under the two tax systems. The results of our policy experiments are summarized in Table 4. For the high value of $\beta = 0.967$ a switch from a progressive to a proportional tax system leads to a reduction of publicly enforced risk sharing from 19.7% to 0. But risk sharing achieved via private arrangements increases with the

proportional system since the value of autarky falls and hence the debt constraints are relaxed. This effect is sufficiently strong to raise total risk sharing with proportional taxes over the level achieved with progressive taxes. . In particular households with low income now face less binding borrowing constraints and can increase their consumption borrowing more. In summary, the government, in trying to help households to share risk by making the tax system more progressive, achieves exactly the opposite *-lower* risk sharing. Again, this result is obtained in a model in which progressive taxes do not distort the labor-leisure decision.

Table 4. Results: **High Risk Sharing ($\beta = 0.967$)** **Low Risk Sharing ($\beta = .024$)**

Variable	Progressive	Proportional	Progressive	Proportional
Real Rate $R - 1$	2.9%	3.1%	2.9%	117%
Govt. Intermed. GI	19.7%	0%	19.7%	0%
Priv. Intermed. PI	92.3%	94.5%	0.2%	0.6%
Total Intermed. TI	93.8%	94.5%	19.9%	0.6%

Also note that the interest rate rises with the switch to the proportional system. This is because proportional taxes lower the value of default, hence relaxing the debt constraints. Households can now borrow more and the interest rate has to rise to bring credit markets back into equilibrium.

The results from our policy experiment are drastically different for the case of low time discount factors. If agents discount the future very heavily, the extent of risk sharing achievable with self-enforceable private contracts is very limited, as the threat of future exclusion from credit markets is not severe for very impatient agents. We see from the table that the low risk sharing allocation is significantly different from the high risk sharing allocation. The effect of the tax reform on government intermediated risk sharing is similar to the case with high discount factor (by construction). Now, however, private markets are almost completely ineffective in providing risk

sharing for both tax systems. Hence total intermediation of risk consists (almost) exclusively of GI .

Again the interest rate rises (this time very sharply) with the proportional system. Comparing the high risk sharing with the low risk sharing case we conclude that when private markets are very effective in providing risk sharing contracts, then the attempt of the government to do even more may be counterproductive. If, on the other hand, private markets do not work because it is difficult to enforce private contracts then public risk sharing provided by redistributive taxation leads to potentially large welfare gains.

Sensitivity Analysis

In this section we explore the sensitivity of our quantitative findings to changes in the parameter values. In particular we consider changes in the risk aversion coefficient, in the persistence of the income process and alternative changes in the tax system. Throughout this section we focus on the high risk sharing case, corresponding to a high $\beta = 0.967$.

Risk Aversion and Persistence We first compute the impact on risk sharing (measured as total intermediation) of the same change in the taxation schemes analyzed in the section above. The first two rows of table 5 we consider economies with a risk aversion parameter of 2.5 (High Risk Aversion) and 1.5 (Low Risk Aversion). In the third and fourth rows we analyze economies with different persistence of the income process. In particular we consider a modified transition matrix for the endowment process $\tilde{\Pi} = (1 - \alpha)\Pi + \alpha I$ where I is the identity matrix³⁹. We consider two cases: $\alpha = 0.1$ (High persistence) and $\alpha = -0.1$ (Low Persistence).

³⁹We adopt this way of modifying the persistence of the income process from Alvarez and Jermann (2000).

Table 5. Changes Total Intermediation in Different Economies

	Progressive	Proportional
High Risk Aversion	97.4%	100%
Low Risk Aversion	89.7%	90.5%
High Persistence	92.4%	93.4%
Low Persistence	94.6%	95%
Benchmark Parameters	93.8%	94.5%

The results in the table show how the higher the risk aversion and the lower the persistence of the income process the more risk sharing there is in the economy, because the value of autarky is reduced. Note, however, that in all cases reported a change from proportional to progressive increases total risk sharing of the economy, meaning that the increase in private intermediation more than offsets the reduction in public risk sharing. The analysis also suggests that if risk aversion is sufficiently high, exclusion from credit markets is a harsh punishment and complete risk sharing through private financial markets is achieved. independent of the tax system. In this case, obviously, a change in the tax system would have no effect on consumption allocations.

Tax Policies Instead of focusing on a discrete change in the tax system we now analyze how total risk sharing evolves with a marginal change in progressivity. We restrict our discussion to a tax system of the form

$$\tau(e_j) = a - \frac{b}{e_j} \quad j = 1, \dots, 5$$

i.e. to a system with constant marginal tax rate a and a fixed deduction b . We estimate a and b by running a regression on our five data points for taxes derived from CEX data. We find estimates $\hat{a} = 25\%$ and $\hat{b} = 9.6\%$ The R^2 of this regression equals a very high 0.995, so that the progressive tax system used in the last section is almost perfectly approximated by a tax system with a constant

marginal tax rate of 25% and a fixed deduction of 9.6% of mean per adult earnings (see Figure 3). The implied level of government spending equals 15.5% of average pre-tax earnings, which is similar to the level we used in the previous section. We now vary the degree of progressivity marginally by marginal changes in the deduction b , with varying a correspondingly to assure that all tax systems generate the same revenue⁴⁰. Note that for $b = 0$ we are back in the proportional tax system from the last section. In Figure 4 we plot total risk sharing against the deduction b , where total risk sharing is measured as TI . Complete risk sharing in this economy can be achieved with two rather extreme policies. One policy that obviously achieves the first best allocation is to tax all income differences away, i.e. making the system extremely progressive and equalize after-tax income of all agents. This is achieved by taxing income at a 100% rate ($a = 1$) and by setting the deduction equal to the average income ($b = (1 - g) = \bar{y}$). On the other hand, making the tax system sufficiently regressive makes the punishment from default sufficiently harsh and allows private financial markets to achieve perfect risk sharing. This occurs for $b < 0.1$, i.e. for a poll tax of at least 10% of mean pre-tax earnings. In between these extreme cases the effect on total risk sharing of a marginal increase in tax progressivity depends on the relative magnitudes of the two effects at work: the direct effect of reducing the variability of after-tax income and the indirect crowding-out effect.

For a fixed deduction of $b < 0.39$ the crowding-out effect dominates; more progressivity reduces total risk sharing. Notice that this is a large range of tax policies; furthermore the experiment considered in the previous section falls into this class. For $b > 0.39$ the direct effect dominates and more progressive taxes lead to more total risk sharing among individuals.

⁴⁰Normalizing the endowments so that $\sum_{j=1}^5 \Pi(e_j)e_j = 1$ one obtains that to keep the tax revenues constant requires to set $a = g + b$.

C. Standard Incomplete Markets

In this section we contrast our findings with the effects of the same reform in a standard incomplete markets model. In this economy agents are only allowed to trade a single uncontingent bond and they face an exogenously specified constant borrowing limit. There are no enforcement problems in this economy. The economy we consider is similar to the one studied by Huggett (1993).

The household problem in recursive formulation reads as

$$v(a, y) = \max_{-b \leq a' \leq y + Ra} (1 - \beta)u(y + Ra - a') + \beta \sum_{y'} v(a', y')\pi(y'|y)$$

where a are holdings of the one-period bond and $R - 1$ is the interest rate on these bonds. Again we compare stationary equilibria under different tax systems.

To enable comparison with the debt constrained economy we calibrate this economy to the same observations. In particular, the endowment and tax processes are kept the same. The coefficient of relative risk aversion is again chosen to equal $\sigma = 2$. We then identify pairs of borrowing limits \underline{b} and time discount factors β such that the equilibrium interest rate equals 2.9%. Table 6 presents the results for our policy experiments for a selection⁴¹ of (\underline{b}, β) -pairs

Table 6. Results for Incomplete Markets

Variable	$\underline{b} = 1, \beta = 0.85$		$\underline{b} = 4, \beta = 0.93$		$\underline{b} = 7, \beta = 0.95$	
	Progr.	Prop.	Progr.	Prop.	Progr.	Prop.
Real Interest rate $R - 1$	2.9%	-1%	2.9%	1.8%	2.9%	2.0%
Govt. Interm. GI	19.7%	0%	19.7%	0%	19.7%	0%
Priv. Interm. PI	15.5%	24.8%	28.4%	38.4%	31%	42.2%
Total Interm. TI	32.1%	24.8%	42.5%	38.4%	44.6%	42.2%

⁴¹There is a limit as to how negative one can chose \underline{b} . This limit is given by the constraint that agents with $y = y_{\min}$ and $a = -\underline{b}$ can attain nonnegative consumption by setting $a' = -\underline{b}$ at the equilibrium interest rate (for both tax systems). This limit on \underline{b} turns out be approximately equal to 7. Since the average before tax income is normalized to 1 \underline{b} can be interpreted as the ratio between the maximum borrowing limit and the average annual pre-tax earning.

Note that a switch from a progressive to a proportional tax system induce significant falls in total intermediation. Redistributive taxes act as a partial substitute for private insurance markets that are exogenously assumed to be missing from this economy. This is true regardless of how tight the borrowing limit is specified. In contrast to the debt-constrained economy a tax reform (by assumption) does not change the assets that can be traded nor the extent to which they can be traded. Therefore the crowding-out effect that was crucial in the previous sections cannot occur in this economy.

Finally note that while in the debt-constrained economy a shift from progressive to proportional taxes causes an increase in the interest rate, in this economy the interest rate falls. Remember that the tax change increases income risk. In this economy agents thus increase precautionary saving and this excess saving reduces the interest rate.

7. Conclusion

In this paper we present a model that highlights a new channel through which income taxation affects private financial markets. Although in our model taxes do not directly affect labor-leisure and wealth accumulation decisions they affect the functioning of private financial markets by changing the incentives to default on private contracts. We show that when private insurance markets are active, public risk sharing provided via taxes crowds out private risk sharing. In order to gain some insights into the magnitude of this effect we calibrate our very simple model to US income and tax data in order to quantify the effects of changes in the progressivity of taxation. We find that the magnitude of the crowding-out effect can be substantial and making taxes more proportional can even reduce the total amount of risk sharing in the economy.

In contrast, if private insurance markets are assumed to be missing for model-exogenous reasons, a tax reform that reduces the variance of after-tax income serves as a partial substitute for private insurance markets and always increase the amount of risk sharing in the economy. This

indicates that the assumption about the structure of private capital markets is important when analyzing redistributive government policies.

In order to isolate the effect of the tax system on private insurance markets and on risk sharing as clearly as possible we abstracted from several features of actual economies that are potentially important. A comprehensive analysis of redistributive taxation should include the effects we highlight in this work, together with the effects of a tax reform on incentives to work, on physical capital accumulation and on the redistribution of wealth (as opposed to risky income). We view this as interesting issue for future research.

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Appendix

Proof of Lemma 1.:

We want to show that for all $\underline{w} \leq w < \hat{w} \leq \bar{w}$, $h(w) < h(\hat{w})$. Suppose not. Then from (20) $V'_R(g_{y'}(w)) \geq V'_R(g_{y'}(\hat{w}))$ for all y' such that $g_{y'}(\hat{w}) > U^{Aut}(y')$, and hence $U^{Aut}(y') < g_{y'}(\hat{w}) \leq g_{y'}(w)$ for all those y' by strict convexity of V_R . From promise keeping there must exist \bar{y}' such that $g_{\bar{y}'}(w) < g_{\bar{y}'}(\hat{w}) = U^{Aut}(\bar{y}')$, a violation of the debt constraint.

Now, since h is strictly increasing in w , $C'(h(w)) < C'(h(\hat{w}))$. Suppose that $g_{y'}(w) > U^{Aut}(y')$. Then from (20) we have $V'_R(g_{y'}(w)) < V'_R(g_{y'}(\hat{w}))$ and from the strict convexity of V_R it follows that $g_{y'}(\hat{w}) > g_{y'}(w)$. Obviously, if $g_{y'}(w) = U^{Aut}(y')$ then $g_{y'}(\hat{w}) \geq g_{y'}(w)$.

Thus we conclude that either $g_{y'}(\hat{w}) > g_{y'}(w)$ or $g_{y'}(w) = g_{y'}(\hat{w}) = U^{Aut}(y')$ ■

Proof of Lemma 2.:

V_R is strictly convex and differentiable. By assumption $g_{y'}(w) > U^{Aut}(y')$. Combining (20) and (21) we obtain $\beta R V'_R(w) = V'_R(g_{y'}(w))$. Since $R < \frac{1}{\beta}$ we have $V'_R(w) > V'_R(g_{y'}(w))$. By strict convexity of V_R the first result follows. Hence $g_{y'}(\cdot)$ are always strictly below the 45° line in its strictly increasing part. But $g_{y'}(w) \geq U^{Aut}(y')$ for all w . Hence for $w < U^{Aut}(y')$ it follows that $g_{y'}(w) = U^{Aut}(y') > w$. By continuity of $g_{y'}(\cdot)$ we obtain that $g_{y'}(U^{Aut}(y')) = U^{Aut}(y')$, and from the first result it follows that $g_{y'}(w) < w$ for all $w > U^{Aut}(y')$ ■

Proof of Theorem 1.:

Take $\bar{w} = \max_y U^{Aut}(y) + \varepsilon$, for $\varepsilon > 0$. If $g_{y'}(w) > U^{Aut}(y')$, then the previous Lemma yields the result. If $g_{y'}(w) = U^{Aut}(y')$, then $g_{y'}(w) = U^{Aut}(y') \leq \max_y U^{Aut}(y) < \bar{w}$

Proof of Theorem 2.:

We first prove that there exists $w^* \in W$ such that $w^* > U^{Aut}(y_{\max})$ and $g_{y_{\max}}(w^*) = U^{Aut}(y_{\max})$, from which it follows that $g_{y_{\max}}(w^*) = U^{Aut}(y)$ for all $w \leq w^*$.

Suppose, to obtain a contradiction, that $g_{y_{\max}}(w) > U^{Aut}(y_{\max})$ for all $w \in W, w > U^{Aut}(y_{\max})$. Then by Lemma 1. we have $g_{y'}(w) = g_{y_{\max}}(w)$, for all $y' \in Y$ and all $w > U^{Aut}(y_{\max})$. By continuity of $g_{y'}$ and Lemma 2. we conclude that $g_{y'}(U^{Aut}(y_{\max})) = U^{Aut}(y_{\max})$, for all $y' \in Y$. But since $U^{Aut}(y_{\max}) > U^{Aut}(y')$ for all $y' \neq y_{\max}$, by Lemma 2. it follows that $g_{y'}(U^{Aut}(y_{\max})) < U^{Aut}(y_{\max})$ for all $y' \neq y_{\max}$, a contradiction.

We now can apply Stokey et al., Theorem 11.12. For this it is sufficient to prove there exists an $\varepsilon > 0$ and an $N \geq 1$ such that for all $(w, y) \in (W, Y)$ we have $Q^N((w, y, U^{Aut}(y_{\max}), y_{\max})) \geq \varepsilon$.

If $w^* \geq \bar{w}$ this is immediate, as then for all $(w, y) \in (W, Y)$, $Q((w, y, U^{Aut}(y_{\max}), y_{\max})) \geq \pi(y_{\max})$, since $g_{y_{\max}}(w) = U^{Aut}(y_{\max})$ for all $w \in W$. So suppose $w^* < \bar{w}$. Define

$$(A1) \quad d = \min_{w \in [w^*, \bar{w}]} \{w - g_{y_{\max}}(w)\}$$

Note that d is well-defined as $g_{y_{\max}}$ is a continuous function and that $d > 0$ from Lemma 2. Define

$$(A2) \quad N = \min\{n \in \mathbb{N} | \bar{w} - nd \leq w^*\}$$

and $\varepsilon = \pi(y_{\max})^N$. Suppose an individual receives y_{\max} for N times in a row, an event that occurs with probability ε . For (w, y) such that $w \leq w^*$ the result is immediate as for those w , $g_{y_{\max}}(w) = U^{Aut}(y_{\max})$ and $g_{y_{\max}}(U^{Aut}(y_{\max})) = U^{Aut}(y_{\max})$. For any $w \in (w^*, \bar{w}]$ we have $g_{y_{\max}}(w) \leq w - d$, $g_{y_{\max}}(g_{y_{\max}}(w)) \leq w - 2d$, etc. The result then follows by construction of (N, ε) ■

Proof of Lemma 5.: We first show that there is an allocation attaining a distribution of utility that stochastically dominates the utility distribution in autarky and requires no more resources. It is then immediate that autarky is not efficient. In autarky the measure over utility entitlements and endowment shocks is given by

$$(A3) \quad \Phi^{Aut}(\{U^{Aut}(y), y\}) = \pi(y)$$

We show that there exist allocations that attain the joint measure $\hat{\Phi}$ defined as

$$\begin{aligned}
\hat{\Phi}(\{U^{Aut}(y), y\}) &= \pi(y) \quad \text{all } y \neq y_{\min} \\
\hat{\Phi}(\{U^{Aut}(y_{\min}), y_{\min}\}) &= \pi(y_{\min})(1 - \pi(y_{\max})) \\
\hat{\Phi}(\{\tilde{w}, y_{\min}\}) &= \pi(y_{\min})\pi(y_{\max})
\end{aligned}
\tag{A4}$$

where $\tilde{w} = U^{Aut}(y_{\min}) + \varepsilon$ for small $\varepsilon > 0$. Define δ_{\max} and δ_{\min} implicitly by

$$\begin{aligned}
\tilde{w} &= (1 - \beta)(u(y_{\min}) + \delta_{\min}) + \beta \sum_y \pi(y)U^{Aut}(y) \\
U^{Aut}(y_{\max}) &= (1 - \beta)(u(y_{\max}) - \delta_{\max}) + \beta \sum_{y \neq y_{\min}} \pi(y)U^{Aut}(y) + \beta\pi(y_{\min})\tilde{w}
\end{aligned}
\tag{A5}$$

Since $\tilde{w} = U^{Aut}(y_{\min}) + \varepsilon$, we have

$$\begin{aligned}
\delta_{\max} &= \frac{\beta\pi(y_{\min})}{(1 - \beta)}\varepsilon \\
\delta_{\min} &= \frac{\varepsilon}{(1 - \beta)}
\end{aligned}
\tag{A6}$$

The autarkic allocation exhausts all resources. The new allocation reduces consumption for the $\pi(y_{\max})$ agents with y_{\max} by δ_{\max} and increases consumption for $\pi(y_{\max})\pi(y_{\min})$ agents by δ_{\min} . Hence, compared to the autarkic allocation the change in resource requirements is given by

$$\begin{aligned}
\Delta &= -\pi(y_{\max})C'(u(y_{\max}))\delta_{\max} + \pi(y_{\max})\pi(y_{\min})C'(u(y_{\min}))\delta_{\min} \\
&= \frac{\pi(y_{\min})\pi(y_{\min})\varepsilon}{(1 - \beta)} \left(\frac{-\beta}{u'(y_{\max})} + \frac{1}{u'(y_{\min})} \right)
\end{aligned}
\tag{A7}$$

Therefore $\Delta \leq 0$ if and only if

$$\beta \frac{u'(y_{\min})}{u'(y_{\max})} \geq 1
\tag{A8}$$

Under this condition the new allocation is resource feasible, incentive feasible and attains $\hat{\Phi}$, a distribution that dominates Φ^{Aut} . It is straightforward to construct the sequential allocation h induced by the recursive policies supporting $\hat{\Phi}$. By reducing $h_0(U^{Aut}(y_{\min}), y_{\min})$ so that the agents receiving discounted utility \tilde{w} under $\hat{\Phi}$ receive $U^{Aut}(y_{\min})$ the new allocation attains Φ^{Aut} but with less resources, a contradiction to the assumption that autarky is constrained efficient. ■

Proof of Theorem 5.

The allocation satisfies the resource constraint (9) since the efficient allocation does and Θ_0 is derived from Φ_0 . Also the allocation satisfies the continuing participation constraints, and, by construction of $a_0(w_0, y_0)$, the budget constraint. It remains to be shown that $\{c_t(a_0, y^t)\}$ is utility maximizing among the allocations satisfying the budget and the continuing participation constraints. The first order conditions

$$(1 - \beta)\beta^t \pi(y^t|y_0) u'(c_t(a_0, y^t)) \left(1 + \sum_{y^\tau \in P(y^t)} \mu(a_0, y^\tau) \right) = \lambda(a_0, y_0) p(y^t)
\tag{A9}$$

are sufficient for consumer optimality.⁴² Define Lagrange multipliers $\mu(a_0, y_0) = 0$, $\lambda(a_0, y_0) =$

⁴²The consumer maximization problem is a strictly convex programming problem (the constraint set with the debt

$(1 - \beta)u'(c_0(a_0, y_0))$ and recursively

$$(A10) \quad 1 + \sum_{y^\tau | y^t} \mu(a_0, y^\tau) = \frac{u'(c_0(a_0, y_0))}{(\beta R)^t u'(c_t(a_0, y^t))}$$

Note that the allocation by construction (see 33) satisfies $\frac{u'(c_t(a_0, y_0))}{\beta R u'(c_{t+1}(a_0, y^{t+1}))} \geq 1$, with equality if the limited enforcement constraint is not binding. Hence $\mu(a_0, y^{t+1}) \geq 0$ and $\mu(a_0, y^{t+1}) = 0$ if the constraint is not binding. By construction the allocation and multipliers satisfy the first order conditions. ■

constraints remains convex). Note that since the efficient consumption allocation is bounded from above, the expected continuation utility from any history y^T onward, discounted at market prices R^{-T} goes to zero as $T \rightarrow \infty$ (i.e. the relevant transversality condition is satisfied). For details see the separate theoretical appendix, available upon request by the authors.

Figure 1. Consumption Distribution

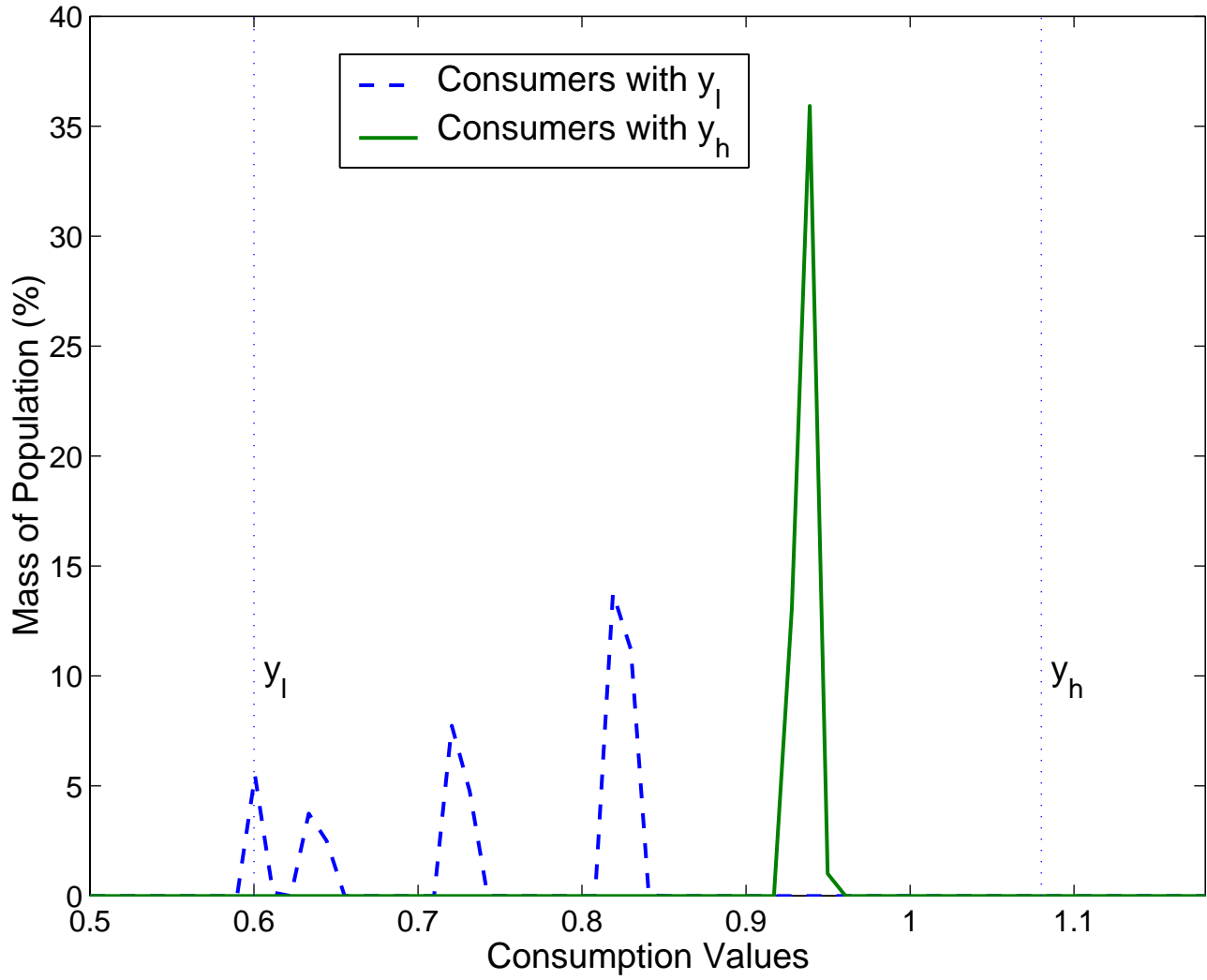


Figure 2. Relation between β and Real Interest Rate

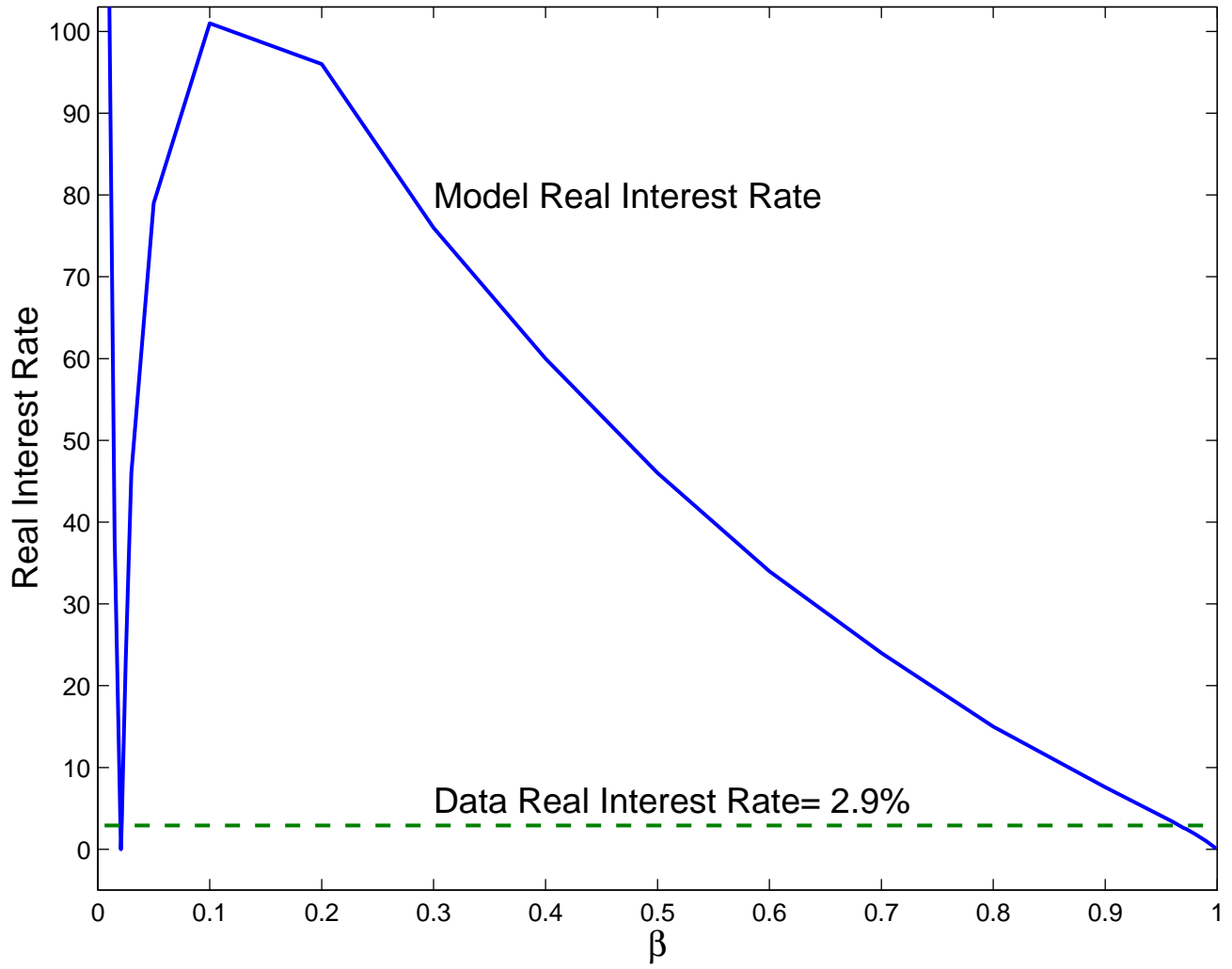


Figure 3. Average 1995–98 Tax Rates (Computed from CEX data)

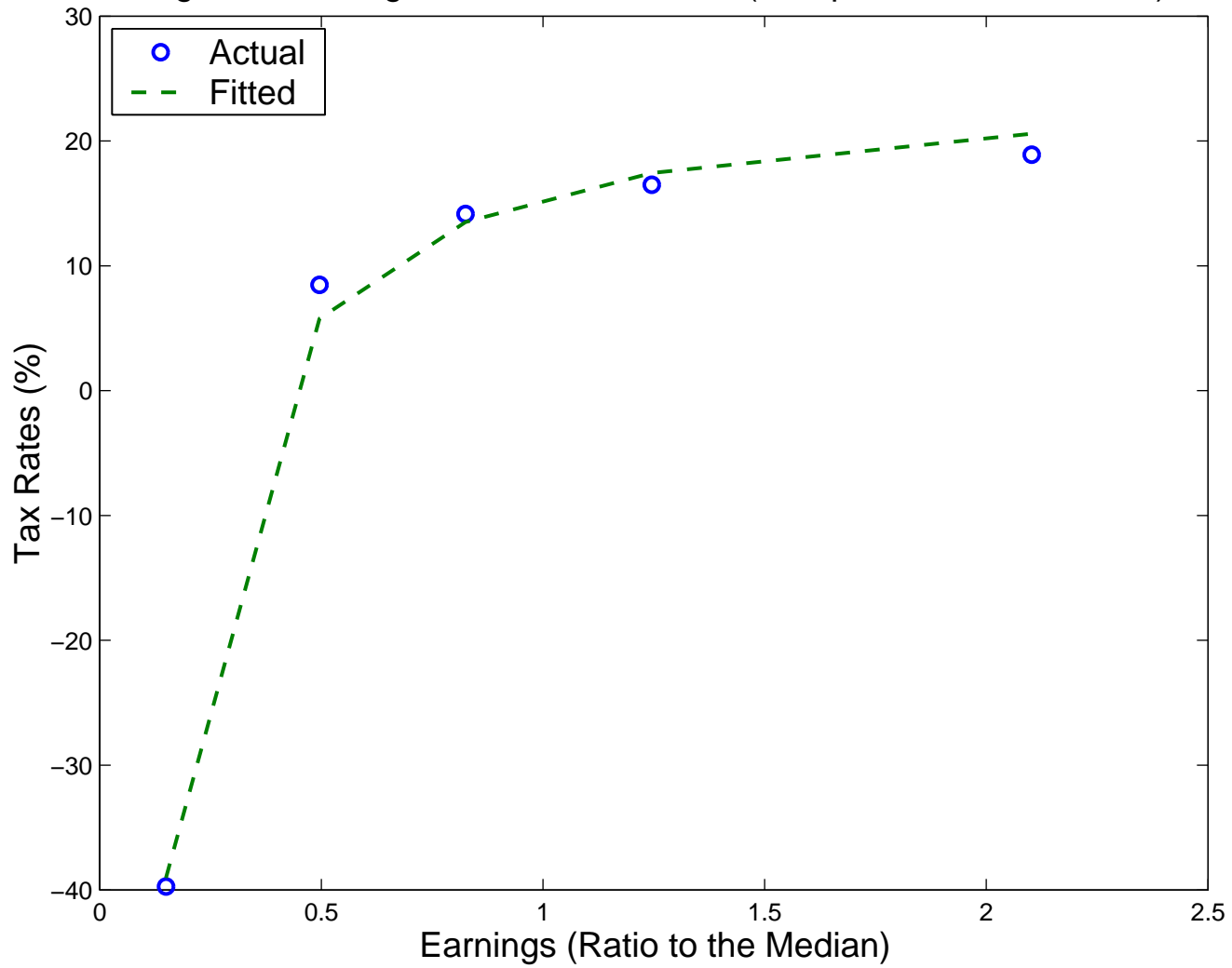


Figure 4. Risk Sharing as a Function of Progressivity of the Tax System

