

University of Pennsylvania – Department of Economics
Economics 897 – Summer 2004

July 19 – August 27

Course Objective

Economics 897 covers the basic principles of mathematical analysis. It is the first course of a two-course sequence; the second part, Economics 898, is offered in the Fall. The primary objective of the sequence is to provide incoming graduate students with the mathematical foundations necessary for the first-year sequence of theory and econometric courses

Prior Preparation

It is highly recommended that the incoming student have a very strong background in calculus. The textbook by Simon & Blume (Mathematics for Economists, Norton, 1994) covers most of the material in Economics 897, but at a more basic level than the course aims at. If your only background in formal analysis is one or two years of college level calculus, we strongly recommend that you review Simon & Blume thoroughly before you come. The course moves very quickly and this prior preparation will help you keep up with the classes. Any other reviews you are able to do of material relevant to the course will only be to your advantage. We have included a syllabus and some past exams to help you with your preparation.

Who should take this course?

The textbooks we plan to use are Walter Rudin's Principles of Mathematical Analysis (McGraw-Hill, 1976) and Rangarajan K. Sundaram's A First Course in Optimization Theory (Cambridge University Press, 1996). If you are **really** comfortable with the material in these two books, then you should take the waiver exam offered at the end of August. All others should and must (we strongly urge you to) take the course.

Course Schedule

The course begins on Monday, July 19 and ends on Friday, August 27 (Room and location will be announced). It meets daily from 10:00 -- 12:00 and from 1:30 -- 3:30. New material will be covered during the sessions on Monday through Thursday. The morning session and part of the afternoon session on Friday serve as a review, questioning, and clarification opportunity.

There will be a short quiz every Friday, evaluating you on material taught during the previous four days. The final exam for the course is on Tuesday, August 31. It is the same exam as the waiver for the course. It will be comprehensive, and will examine material taught throughout the course.

General Aspects

This course is designed on the presumption that the students have been exposed to most of the material in their previous studies and therefore covers a fairly ambitious schedule in a relatively short period of time. Students will be regularly assigned problem sets. **Passing this course, or its waiver exam, is a requirement for the doctoral program.**

Instructors

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Text Books:

Required:

Rudin, W., *Principles of Mathematical Analysis*. McGraw-Hill, 1976.

Sundaram, R. K., *A First Course in Optimization Theory*. Cambridge University Press, 1996.

Recommended:

Apostol, T. Mathematical Analysis. Addison Wesley, 1976, covers almost all the material that Rudin does, and is less concise. Some of you may find it easier to read the first time around, when you are newly introduced to this material. We plan to put this book on reserve in the library so that those of you who want to consult it, can do so. At the same time, we believe you would benefit from becoming familiar with Rudin. In addition to being a required text for Economics 897, Rudin is also a required text for Economics 701, one of the two core Microeconomics courses you will take in the fall, and a great reference book during later years of research.

Lang, Serge, Linear Algebra. Springer-Verlag, 1987, covers Linear Algebra at a higher level than Simon & Blume. We will put this book also on reserve at the library, if you would like to consult it.

Shiryayev, A. N., Probability (Graduate Texts in Mathematics 95), Springer Verlag, 1991, is a comprehensive introduction to Probability Theory. It will be used as reference and will also be put on reserve at the library.

Comment:

Apart from Rudin's and Apostol's book, for the first part we will be also using parts of "Analysis, with an Introduction to Proof" by S. Lay (Prentice Hall). This book contains nice explanations and examples for the entire first part of the course and will be the source material for the first classes on logic and the structure of proofs. The level is very accessible but the scope is limited to the first third of the course. If you feel comfortable enough with Rudin, you can skip it. Anyway, the lecture notes will contain the necessary material from this book.

Course Outline

Module I

Section 1: Metric Spaces, Sequences

1. Preliminaries
 - I. Logic
 - II. Elementary Set Theory
 - III. The Structure of Proofs
 - IV. Relations and Functions
2. The Real Number System
3. Metric Spaces.
4. Properties of Metric Spaces
5. Sequences and Series
6. Convergence and Limits

Section 3: One-Variable Calculus

1. Continuity
2. Differentiation
3. Properties of Continuous Functions
4. Properties of Differentiable Functions
5. Linear Approximation, Taylor's Theorem
6. Optimization in R
7. Concave functions

Module II

Section 1: Topology of R^n

1. Balls and bounded sets
2. Sequences in R^n
3. Continuous maps
4. Limits
5. Open and closed sets
6. Compactness
7. Connectedness

Section 2: Linear Algebra (Review)

1. Vector spaces; subspaces
2. Linear maps
3. Kernel and image
4. Direct sum and projection

Section 3: Differentiable Functions

1. Partial derivatives; directional derivatives
2. Differentiability
3. Inverse Function Theorem
4. Local structure of immersions and submersions
5. Implicit Function Theorem

Section 4: Riemann Integral

1. Definition and properties
2. Measure zero sets
3. Jordan measurable sets
4. Lebesgue Theorem
5. Iterated integrals
6. Change of Variables

Module III

Section 1: Optimization

1. Convexity and Separation Theorems
2. Concave Functions
3. Existence of Solutions
4. Unconstrained Optimization
5. Equality Constraints: The Theorem of Lagrange
6. Inequality Constraints: The Theorem of Kuhn – Tucker

Section 2: Probability Theory

1. Probability Spaces, Events, and Random Variables
2. Integration and Expectation
3. Independence, the CLT, and the SLLN
4. Conditional Expectation