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PIER Working Paper
20-032

The Long-Term Distributional and Welfare Effects of Covid-19 School Closures

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August 26, 2020

https://ssrn.com/abstract=3682116

The Long-Term Distributional and Welfare Effects of Covid-19 School Closures*

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Abstract

Using a structural life-cycle model, we quantify the long-term impact of school closures during the Corona crisis on children affected at different ages and coming from households with different parental characteristics. In the model, public investment through schooling is combined with parental time and resource investments in the production of child human capital at different stages in the children's development process. We quantitatively characterize both the long-term earnings consequences on children from a Covid-19 induced loss of schooling, as well as the associated welfare losses. Due to self-productivity in the human capital production function, skill attainment at a younger stage of the life cycle raises skill attainment at later stages, and thus younger children are hurt more by the school closures than older children. We find that parental reactions reduce the negative impact of the school closures, but do not fully offset it. The negative impact of the crisis on children's welfare is especially severe for those with parents with low educational attainment and low assets. The school closures themselves are primarily responsible for the negative impact of the Covid-19 shock on the long-run welfare of the children, with the pandemic-induced income shock to parents playing a secondary role.

Keywords: Covid-19, school closures, inequality, intergenerational persistence
J.E.L. Codes: D15, D31, E24, I24

*Fuchs-Schündeln gratefully acknowledges financial support by the European Research Council through Consolidator Grant No. 815378, and by the DFG through a Leibnizpreis. Krueger thanks the National Science Foundation for continued support. Fuchs-Schündeln and Ludwig gratefully acknowledge financial support by NORFACE Dynamics of Inequality across the Life-Course (TRISP) grant: 462-16-120. Ludwig also thanks Universitat Autònoma de Barcelona for its hospitality during his sabbatical year 2019/20.

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1 Introduction

Governments worldwide have reacted to the Covid-19 pandemic by closing schools and child care centers. In many countries, including the US, these closures started in mid March, extended to the summer, and governments and local officials are now wrestling with the question whether to bring children physically back to school in the fall. Whereas the economic consequences of temporary business closures are immediate and drew a lot of media and policy attention, the economic consequences of school and child care closures arise in the longer term and are not easily measured. Given the importance of human capital for individual prosperity and long-term macroeconomic growth, they are however likely substantial (see e.g. Krueger and Lindahl (2001), Manuelli and Seshadri (2014)).

In this paper, we analyze the long-term income-, welfare- and distributional consequences of the school and child care closures on the affected children. The key dimensions of heterogeneity we focus on are the age of a child in 2020 (when the school closures happened), as well as parental socio-economic characteristics, primarily financial resources and education. To do so, we build a heterogeneous agent partial equilibrium model with a human capital production function at its core that takes time and monetary inputs by parents and governmental investment into schooling as inputs. Parents also leave inter vivos transfers to their children, which can be used to finance college and consumption. We model school and child care closures as a reduction in the governmental investment in children corresponding to six fewer months of schooling. In the model, parents endogenously adjust their investment into children and inter vivos transfers in response to the drop in governmental inputs, thereby potentially mitigating the adverse consequences of Covid-19-induced school closures. We use our model as a quantitative laboratory to analyze the long-term aggregate and distributional consequences of the Covid school shock on children's human capital as they progress through their school ages, their high-school graduation and college choice, their labor market earnings, and, ultimately, their welfare. In an extended analysis, in addition to the school closures, we also model a negative income shock to parents due to the Covid-induced economic recession. The reduction in economic resources of parents in turn adversely impacts their ability to invest into their children and endow them with inter vivos transfers to attend college, and therefore aggravates the school closure-induced shock. We view the results of this last thought experiment as our main, model-based predictions of the long-run economic consequences of the Covid-19 crisis on the current generation of school children.

The key quantitative ingredients for the quantitative analysis are the parameters characterizing the human capital production function, which we either take from the literature or calibrate it to data moments on parental investments into children from U.S. household micro data. Once the model is parametrized, we subject children and parents to a one-time, unexpected Covid-19 school closure shock and possibly an associated, recession-induced income shock and document the short- and long run economic consequences. On average (across children aged 4 to 14 when the shock occurs), the model implies an increase in the future share of children without a high school degree of 3.8% and a reduction of the share of children with a college degree of 2.7% when both school closures and the economic recession is taken into account. On average, the earnings losses induced by reduced human capital accumulation and lower educational attainment amount to about -1%. These effects materialize despite a significant endogenous adjustment of parental investments into their children: time inputs rise by 4.29% and monetary inputs by 4.99%. Measured as consumption-equivalent variation, the average welfare loss of children from the (deep but highly temporary) Covid schooling and parental income shock amounts to -0.75%, with 87% of losses being directly attributed to the school closures, and the rest being accounted for by the economic recessions for the children's parents. Given the temporary nature of the shock, assumed to last only half a year, we view these numbers as quite large.

Furthermore, these averages mask substantial heterogeneity by the age and parental socio-economic characteristics of the children at the time of the crisis. Turning first to the age of the child, the adverse impact is most pronounced for younger children of school age (i.e. children of ages 6-10). The welfare losses of children aged 6 at the time of the crisis amount to -1% in terms of consumption-equivalent variation. Even though parents respond strongly to the closure of schools by increasing their time and resource inputs into the child human capital production function, they do not quite fully offset the reduction in public inputs due to schooling. This implies that children arrive at older ages with less human capital due to the Covid crisis, and since the human capital production function features dynamic complementarities, the marginal productivity of private investments at future ages is reduced. Optimizing parents respond to this by investing less into their children at older ages (relative to the pre-Covid scenario), leading to lower human capital at age 16, adverse outcomes on high-school completion and college attendance, future wages and, consequently, welfare. Older children at the time of the Covid crisis, instead, have already accumulated most of their human capital, and therefore the adverse future incentive effect on parental human capital investment due to Covid school closures is less severe.

Although the age of the child is the strongest determinant of the negative impact of the Covid shock on human capital accumulation, wages and welfare, parental background matters as well. Broadly speaking, children with poorer parents suffer more. There are two reasons for this. First, even without any parental adjustments in investments, children from lower income households suffer more from the school closures, since for them a larger part of educational investment comes from the government. Second, as a reaction to the school closures, rich parents increase their investment into children by more than poor parents. They have more financial resources to do so, and their children have on average higher human capital. Given that the human capital production function features dynamic complementarity, rich parents thus have higher incentives to compensate the reduction in government investment than poor parents. Lower college attendance rates are an important driver of welfare losses, and college-educated parents are able to mitigate the adverse effect of school closures better, leading to lower welfare losses for their children. At the other end of the spectrum, college attendance rates are already very low for children from the most disadvantaged backgrounds even before the Covid-19 crisis. Therefore, children of high school educated parents (who have sizeable college completion rates prior to the shock) experience the largest welfare losses, exceeding those for children of high school dropout parents, with and children from college educated parents experiencing the least adverse effects.

1.1 Related Literature

This paper ties into the literature on schooling and human capital formation. Our human capital production function relies on Cunha et al. (2006), Cunha and Heckman (2007) and Cunha et al. (2010), especially on the feature of self-productivity, i.e., higher human capital in one period leads to higher human capital in the next period, and dynamic complementarity, i.e., human capital investment pay out more the higher the human capital.[1] This feature implies that early-childhood education is a crucial determinant of future income (Cunha and Heckman (2007) and Caucutt and Lochner (2020)). Consequently, public schooling is a driver of intergenerational mobility. Relying on quantitative models, Kotera and Seshadri (2017) show that differences in intergenerational mobility across US states can be explained by differences in public school finances. Related, Lee and Seshadri (2019) find that education subsidies can significantly increase intergenerational mobility.

Whereas the quantitative literature focuses on the effects of public schooling investment on children's outcomes and intergenerational mobility, there exists an empirical literature that analyzes specifically the link between school instruction time and outcomes of children. Lavy (2015) exploits international differences in school instruction time, caused by differences in the length of an average school day and usual school weeks, and finds that school instruction time in core subjects significantly affects testing outcomes of children. Similarly, Carlsson et al. (2015) exploit exogenous variation in testing dates in Sweden and report that extra 10 days of school instruction raise scores on intelligence tests by 1% of a standard deviation.[2] There are few studies investigating longer-term outcomes of school instruction time on children. Cortes et al. (2015) find a positive effect of math instruction time on the probability of attending college for low-

[1] Our human capital production function does not feature innate ability, only innate human capital, and therefore by construction does not include complementarity between ability and human capital investment.

[2] Other papers studying the link between school instruction time and test scores are Rivkin and Schimann (2015) and Fitzpatrick et al. (2011).

performing students. Pischke (2007) exploits short school years associated with a shift in the school starting date in Germany in the 1960s, and finds no significant effects on employment and earnings of affected children. Jaume and Willén (2019) find that half a year less instruction time during primary school caused by school strikes in Argentina lowers the long-term earnings by 3.2% for men and 1.9% for women.

The remainder of this paper unfolds as follows. Section 2 presents the model, and Section 3 its calibration. Sections 4, 5, and 6 then discuss the results, first in terms of aggregate effects, then in terms of its distribution, before inspecting the mechanisms that give rise to them. Section 7 analyzes the scenarios of longer school closures and a longer recession, both lasting one year instead of half a year. Last, section 8 concludes, and the Appendix contains further details about the data used, theoretical properties of the model, as well as additional quantitative results.

2 A Quantitative Model of Education During the Epidemic

We now describe the quantitative life cycle model that we will use to quantify the short- and long-run consequences of school closures during the COVID-19 pandemic. After setting out the fundamentals of the economy (demographics, time, risk, endowments, preferences and government policy) we immediately focus on the recursive formulation of the model, since this is the representation we will compute.

2.1 Individual State Variables, Risk, and Economic Decisions

Time in the model is discrete and the current period is denoted by t. We model the life cycle of one adult and one children generation in partial equilibrium. The timing and events of this life cycle are summarized in Figure 1 below.

Agents are heterogeneous with respect to the generation they belong to $k \in \{ch, pa\}$, either being part of the child or parental generation, and they differ by their marital status $m \in \{si, ma\}$ for single and married, their age $j \in \{0, \ldots, J < \infty\}$, their asset position a, their current human capital h, their education level $s \in \{no, hs, co\}$ for no higher education (no high school completion), $h$igh $s$chool attendance and completion, college attendance and completion, and idiosyncratic productivity risk $\eta \in \{\eta_l, \eta_h\}$, where η_l is low and η_h is high labor productivity. The individual state variables and the range of values they can take are summarized in Table 1.

We assume that parents give birth to children at the age of j_f and denote the fertility rate of households by $\xi(m, s)$, which differs by marital status and education groups. Notice that $\xi(m, s)$

Table 1: State Variables

State Var.	Values	Interpretation
k	$k \in \{ch, pa\}$	Generation
m	$m \in \{si, ma\}$	Marital Status
j	$j \in \{0, 1, \ldots, J\}$	Model Age
a	$\geq -\underline{a}(j, s, k)$	Assets
h	$h > 0$	Human Capital
s	$s \in \{no, hi, co\}$	Education
η	$\eta \in \{\eta_l, \eta_h\}$	Productivity Shock

Notes: List of state variables of the economic model.

is also the number of children per household. There is no survival risk and all households live until age J and thus the cohort size within each generation is constant (and normalized to 1). We now describe in detail how life unfolds for parents, and then for children, as summarized in Figure 1 below.

2.1.1 Life of the Parental Generation

Parental households become economically active at age j_f just before their children are born. They start their economic life in marital status m and with education level s, an initial idiosyncratic productivity state η and initial assets a. These initial states are exogenously given to the household, and drawn from the population distribution $\Phi(s, m, \eta, a)$ which will be informed directly by the data.

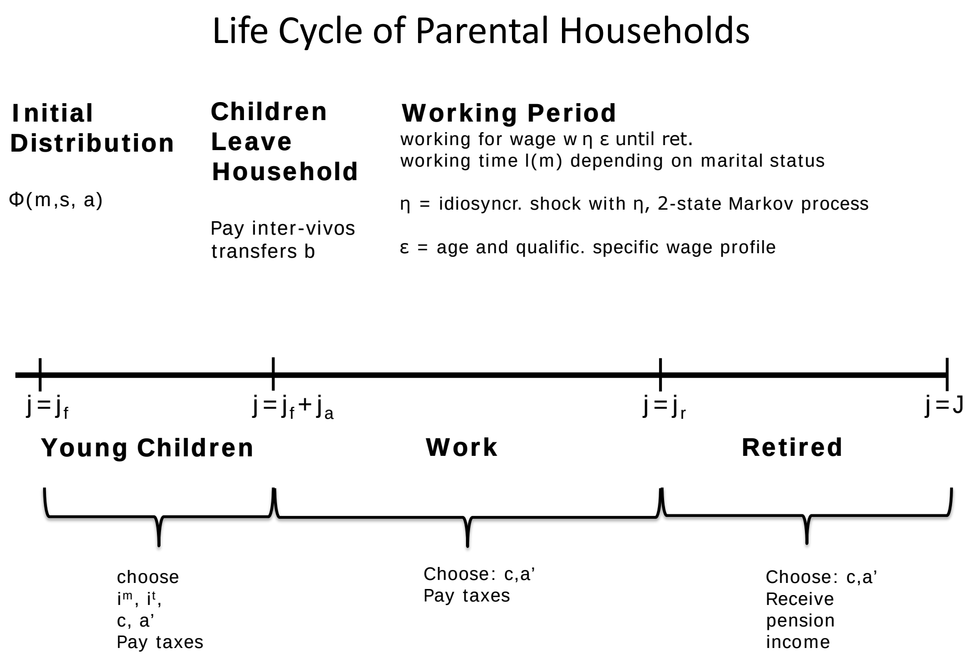
Parents then observe the innate ability (initial human capital) $h = h_0$ of their children which is drawn from an initial distribution $\Psi(h_0|s, m)$ that depends on parental education s and marital status m. Children live with their parents until age j_a (parental age $(j_f + j_a)$) at which point they leave the parental household to form their own household.

During the parental part of the life cycle in which children live with parents (parental age $j \in \{j_f, ..., j_f + j_a\}$) they invest money i^m and time i^t into the accumulation of human capital h of their children, taking as given public investment into schooling i^g. As a result, the human capital of the child evolves according to

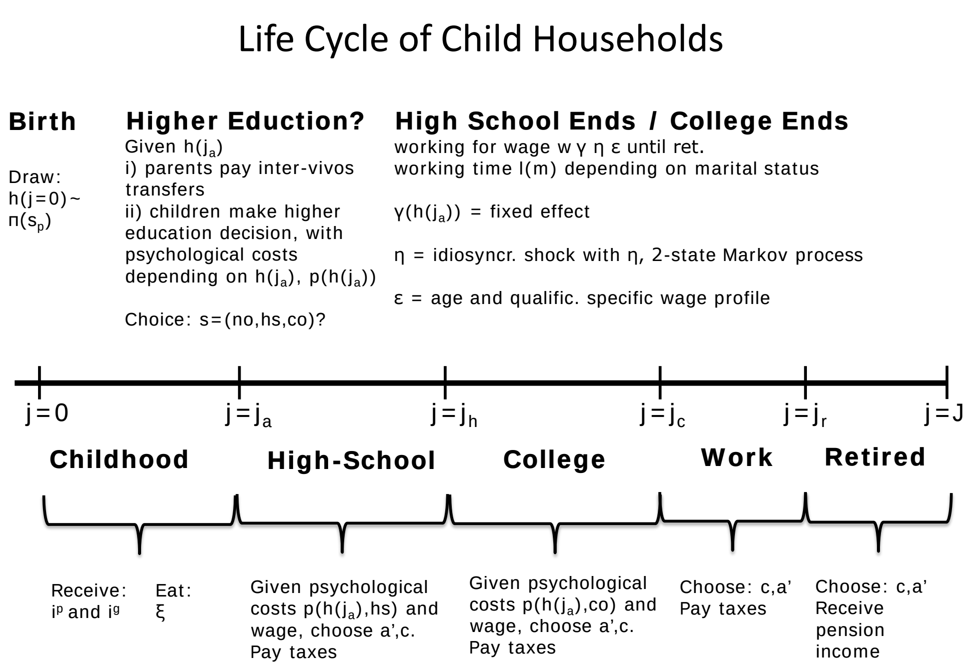
$$h' = g\left(j, h, i^m, i^t, i^g\right), \tag{1}$$

where g is a function of the child's age (to reflect differences in the relative weights of education inputs at different ages), and depends positively on the three inputs (parental time, parental resources and public education).

Figure 1: Life-Cycle of Child and Parental Households



(a)



(b)

When children leave the household at parental age $j_f + j_a$, parents may transfer additional monetary resources as inter-vivos transfers b to their children. After this transfer parents and children separate and there are no further interactions between the two generations.

Throughout the work life parental households spend an exogenous amount of time $\ell(m) > 0$ on market work which differs by marital status. Labor productivity and thus individual wages are determined by an exogenous productivity profile $\epsilon(j, s, m)$ that depends on household age, education and marital status, as well as by a stochastic shock η. This shock η follows a first-order Markov chain with state space $\{\eta_l, \eta_h\}$, transition matrix $\pi(\eta' \mid \eta)$ and initial distribution Π. Current labor income of parents with characteristics j, s, m is then given by

$$y = w \cdot \epsilon(j, s, m) \cdot \eta \cdot \ell(m). \tag{2}$$

Parents work until retirement at age j_r, when they receive lump-sum (but earnings history dependent) retirement benefits $b^p > 0$, and live until age J. In addition to making human capital investment decisions for their children when these are present in the household, parents in each period make a standard consumption-saving choice, where household asset choices are subject to a potentially binding borrowing constraint $a' \geq -\underline{a}(j, s, k)$, which will be parameterized such that the model replicates well household debt at the age at which households have children j_f. The borrowing limit is assumed to decline linearly to zero over the life cycle towards the last period of work at age $j_r - 1$. Table 2 summarizes the choices of parents (and children, as described in the next subsection).

Table 2: Per Period Decision Variables

State Var.	Values	Decision Period	Interpretation
c	$c > 0$	$j \geq j_a$	Consumption
a'	$a' \geq -\underline{a}(j, s, k)$	$j \geq j_a$	Asset Accumulation
i^t	$i^t \geq 0$	$j \in \{j_f, ..., j_f + j_a\}$	Time Investments
i^m	$i^m \geq 0$	$j \in \{j_f, ..., j_f + j_a\}$	Monetary Investments
b	$b \geq 0$	$j = j_f + j_a$	Monetary Inter-vivos Transfer
s	$s \in \{no, hi, co\}$	$j = j_a$	(Higher) Education

Notes: List of control variables of the economic model.

2.1.2 Life of the Children Generation

Children are born at age $j = 0$, but for the first $j_a - 1$ periods of their life do not make economic decisions. Their human capital during these periods evolves as the outcome of parental investment decisions (i^m, i^t) described above. At the beginning of age j_a, and based on both the level of human capital as well as the financial transfer b from their parents (which determines their initial wealth $a = b$), children make a discrete higher education decision $s \in \{no, hs, co\}$, where $s = no$ stands in for the choice not to complete high school, hs for high school completion, and co for college completion, respectively. For simplicity, children are stand-in bachelor households through their entire life-cycle.

Acquiring a high school or college degree comes at a cost (capturing either a time- or psychological cost) $p(s, s_p, h)$ which is decreasing in the child's acquired human capital h and also depends on parental education s_p. In addition, college education requires a monetary cost $\iota \geq 0$. Children may finance some of their college expenses by borrowing, subject to a credit limit by $-\underline{a}(j, s, k)$ which is zero for $s \in \{no, hs\}$, i.e. for individuals not going to college. As was the case for parents, this limit decreases linearly with age and converges to zero at the age of retirement j_r, requiring the children generation to pay off their student loans prior to their retirement.

Youngsters who decide not to complete high school, $s = no$, enter the labor market immediately at age $j_w(s = no) = j_a$. Those who decide to complete high school, but not to attend college, do so at age $j_w(s = hs) = j_h > j_a$. While at high school, $\{j_a, ..., j_h - 1\}$, they work part-time at wages of education group $s = no$. Youngsters who decide to attend college enter the labor market at $j_w(s = co) = j_c$ and also work part-time at wages of education group $s = no$ during their high-school and college years $\{j_a, j_c - 1\}$.

At time of labor market entry, $j_w(s)$, the acquired human capital of a worker is mapped into an idiosyncratic permanent productivity state $\gamma(s, h)$. When starting to work, children also draw stochastic productivity η, which follows the same first-order Markov chain as for the parental generation. Labor income of children during the working period is then given by

$$w \cdot \gamma(s, h) \cdot \epsilon(j, s, si) \cdot \eta \cdot \ell(si).$$

Since the children generation does not have any offspring of their own, the remaining decision problem of the child generation amounts to a simple life-cycle consumption-saving problem.

2.2 Decision Problems

Since we focus on a single parent and children generation, we can solve the model backward, starting from the dynamic programming problem of the children.

2.2.1 Children

The children generation makes their first meaningful economic decision at age j_a, when it leaves the parental household with the receipt of inter-vivos transfers, which constitute initial assets a, as well as with human capital h.

The Education Decision Denoting by $V(j_a, s, s_p; a, h)$ the lifetime utility of a child of age j_a with initial assets and human capital (a, h), with parents with educational attainment s_p and who chooses education level s, we obtain as post-education decision skill state

$$s = \begin{cases} no & \text{if } V(j_a, s = no; a, h) \geq \max\{V(j_a, s = hs, s_p; a, h), V(j_a, s = co, s_p; a, h)\} \\ hs & \text{if } V(j_a, s = hs; a, h, s_p) \geq \max\{V(j_a, s = no; a, h), V(j_a, s = co; s_p, a, h)\} \\ co & \text{if } V(j_a, s = co; a, h, s_p) \geq \max\{V(j_a, s = no; a, h), V(j_a, s = hs, s_p; a, h)\}. \end{cases} \tag{3}$$

The pre-education decision value function is then given as

$$V(j_a, a, h, s_p) = \max_{s \in \{no, hs, co\}} \{V(j_a, s; a, h, s_p)\}. \tag{4}$$

In the computational implementation, we additionally apply extreme value type I (Gumbel) distributed taste shocks to smooth the decision problem.[3] Accordingly, decisions for the three education alternatives are probabilistic and governed by the choice probabilities $\pi(j_a, s, s_p; a, h)$.

Choices During Working Life After having made the education decision $s \in \{no, hs, co\}$, which determines their permanent productivity in the labor market $\gamma(s, h)$, children draw the stochastic component of their labor productivity $\eta \sim \Pi(\eta)$ which then evolves according to the Markov transition matrix $\pi(\eta'|\eta)$. The state variables of the newly formed household consequently are (j, s, η, a, h) and the continuation value functions $V(j, s, \eta; a, h)$ are determined by simple life-cycle consumption-saving problems, whose details vary by the chosen education level s.

Starting with children who have decided to drop out of high-school, $s = no$, a typical single household solves the following dynamic problem during working ages $j \in \{j_a, ..., j_r - 1\}$

$$V(j, no, \eta; a, h) = \max_{c, a'} \left\{ u(c) - v(\ell(si)) + \beta \sum_{\eta'} \pi(\eta' \mid \eta) V(j + 1, no, \eta'; a', h) \right\}$$

[3] Given this structure, the set of individuals exactly indifferent between two education choices is of measure zero and thus it is inconsequential how we break the indifference.

subject to

$$
\begin{aligned}
a' + c(1+\tau^c) &= a(1+r(1-\tau^k)) + y(1-\tau^p) - T(y(1-0.5\tau^p)) \\
y &= w\gamma(no,h)\epsilon(no,j,si)\eta\ell(si) \\
a' &\geq 0
\end{aligned}
$$

where the per period utility function $u(\cdot)$ with household consumption c as its argument satisfies standard properties. Since labor supply is exogenous, the disutility of work $v(\cdot)$ does not affect optimal choices of children, but impacts the child value functions which enter the parental transfer decision problem. In the budget constraint, recall that $\gamma(s,h)$ is permanent labor productivity which depends on acquired human capital and the chosen level of education $s = no$. The function $T(\cdot)$ represents a progressive labor income tax code and $(1-0.5\tau^p)y$ is taxable labor income, where τ^p is the social security contribution rate (and employer contributions to social security are non-taxable income). Accordingly, we can write the education-specific expected value functions $V(j,no,a,h)$ in (3) as

$$
V(j,no;a,h) = \sum_{\eta} \Pi(\eta) \cdot V(j,no,\eta;a,h).
$$

Children who continue in high school but do not attend or complete college, i.e., education group $s = hs$, solve the following decision problem at age $j = j_a$

$$
V(j,hs,s_p;a,h) = \max_{c,a'} \Big\{ u(c) - v(\chi(hs)\ell(si)) - p(s,s_p,h) + \beta \sum_{\eta'} \Pi(\eta')V(j+1,hs,\eta',a',h) \Big\}
$$

subject to

$$
\begin{aligned}
a' + c(1+\tau^c) &= a(1+r(1-\tau^k)) + y(1-\tau^p) - T(y(1-0.5\tau^p)) \\
y &= w\gamma(no,h)\epsilon(no,j,si)\chi(hs)\ell(si) \\
a' &\geq 0.
\end{aligned}
$$

That is, high-school students work for high-school dropout wages $\gamma(no,h) \cdot \epsilon(no,j,si)$ during a fraction $\chi(hs)$ of their time $\ell(si)$. The term $p(s,s_p,h)$ represents a utility cost associated with attending high school which is decreasing in the amount of human capital h previously acquired by the student. They have to form expectation over stochastic labor market productivity upon graduating in the subsequent period. Upon graduating, for their remaining working life-cycle $j \in \{j_h, ..., j_r - 1\}$ these individuals solve a dynamic problem analogous to the one of high-school dropouts described above, but with earnings process $w\gamma(hs, h)\epsilon(hs, j, si)\eta\ell(si)$.

Finally, children that decide, at age j_a, to attend (and by assumption, to complete) college have education indicator $s = co$, and solve, during ages $\{j_a, ..., j_{h-1}\}$ the same problem as group $s = hs$, with the modification that the continuation value differs at age j_{h-1} through the value function $V(j_h, co, s_p; a, h)$. For ages $j \in \{j_h, ..., j_c - 2\}$ they solve

$$V(j, co, s_p; a, h) = \max_{c,a'} \{u(c) - v(\chi(co)\ell(si)) - p(co, s_p, h) + \beta V(j + 1, co, s_p; a', h)\}$$

subject to

$$\begin{aligned} a' + c(1 + \tau^c) &= a(1 + r(1 - \tau^k)) + y(1 - \tau^p) - T(y(1 - 0.5\tau^p)) - \iota \\ y &= w\gamma(hs, h)\epsilon(hs, j, ma)\chi(co)\ell(si) \\ a' &\geq -\underline{a}(j, co, ch). \end{aligned}$$

At age $j = j_c - 1$ their continuation value (but not their budget set) changes to reflect entry into the labor market in the subsequent period, taking expectations over the stochastic component of productivity η' next period.

$$V(j, s, s_p; a, h) = \max_{c,a'} \Big\{u(c) - v(\chi(co)\ell(si)) - p(s, s_p, h) + \beta \sum_{\eta'} \Pi(\eta') \cdot V(j + 1, co, \eta'; a', h)\Big\}.$$

For the remaining working phase of the life-cycle $j \in \{j_c, ..., j_r - 1\}$ these individuals solve a dynamic problem analogous to the one of high-school dropouts described above, but with earnings process $w\gamma(co, h)\epsilon(co, j)\eta\ell(si)$.

The Retirement Phase During retirement, at ages $\{j_r, ..., J\}$, all three education groups of the children generation solve a standard consumption-saving problem of the form:

$$V(j, s, \eta; a) = \max_{c,a'} \{u(c) + \beta V(j + 1, s, \eta; a')\}$$

subject to

$$
\begin{aligned}
a' + c(1+\tau^c) &= a(1+r(1-\tau^k)) + y - T(y) \\
y &= pen(s, si, \eta_{j_r-1}, h) \\
a' &\geq -0 \\
\eta &= \eta_{j_r-1},
\end{aligned}
$$

where $pen(s, si, \eta_{j_r-1}, h)$ is retirement income, whose dependence on η_{j_r-1}, s and h serves to proxy for the progressive nature of the social security system.

2.2.2 Parents

Given the focus of the paper, we model parental households as becoming economically active at the beginning of age $j_f > j_a$ when they give birth to children. Parents are endowed with initial assets a, education s, an initial idiosyncratic productivity state η, and are distinguished by their marital status m. Children live with adult households until they form their own households as described above. Thus, for parental ages $\{j_f, ..., j_f + j_a - 1\}$ children are present in the parental household. Parents derive utility from per capita consumption of all household members and leisure. During the age bracket $\{j_f, j_f + j_a - 1\}$ they solve the dynamic problem

$$
\begin{aligned}
V(j, s, m, \eta; a, h) = \max_{c, i^m, i^t, a', h'} \Bigg\{ & u\left(\frac{c}{1 + \zeta_c \xi(m,s) + \mathbf{1}_{m=ma}\zeta_a}\right) - v\left(\frac{\ell(m) + \kappa \cdot \xi(m,s) \cdot i^t}{1 + \mathbf{1}_{m=ma}}\right) \\
& + \beta \sum_{\eta'} \pi(\eta'|\eta) V(j, s, m, \eta'; a', h') \Bigg\}
\end{aligned}
$$

subject to

$$
\begin{aligned}
a' + c(1+\tau^c) + \xi(m,s) i^m &= a(1 + r(1-\tau^k)) + y(1-\tau^p) - T(y(1-0.5\tau^p)) \\
y &= w\epsilon(s, j, m)\eta\ell(m) \\
a' &\geq -\underline{a}(j, s, k) \\
h' &= g(j, h, i(i^m, i^t, i^g))
\end{aligned}
$$

where h is the human capital of the number $\xi(m, s)$ of children in the household characterized by parental education s and marital status m. We express monetary investments i^m and time investments i^t on a per-child basis. Notice that the sum of hours worked and weighted time investment in children in the disutility function $v(\cdot)$ is divided by the number of household members. Parameter κ is a weight on time investments into children, and reflects the possibility that reading to children carries a different disutility of time than answering emails at work.

At parental age $j_f + j_a$ children form own adult households and this is the only period in which parents can make inter-vivos transfers b. These transfers immediately (that is, within the period) become assets of their children, and thus generate utility for their parents.[4] The dynamic program then reads as

$$V(j_a + j_f, s, m, \eta; a, h) = \max_{c,b,a'} \left\{ u\left(\frac{c}{1 + \mathbf{1}_{m=ma}\zeta_a}\right) - v\left(\frac{\ell(m)}{1 + \mathbf{1}_{m=ma}\zeta_a}\right) \right.$$
$$\left. + \beta \sum_{\eta'} \pi(\eta'|\eta) V(j_a + j_f + 1, s, m, \eta'; a') + \nu V\left(j_a, \frac{b}{1 + r(1 - \tau^k)}, h, s\right) \right\}$$

subject to

$$\begin{aligned} a' + c(1 + \tau^c) + \xi(m, s)b &= a(1 + r(1 - \tau^k)) + y(1 - \tau^p) - T(y(1 - \tau^p)) \\ y &= w\epsilon(s, j, m)\eta\ell(m) \\ a' &\geq 0. \end{aligned}$$

After children have left the household, the parent generation solves, at age $j \in \{j_a + j_f + 1, ..., j_r - 1\}$

$$V(j, s, m, \eta, a) = \max_{c,a'} \left\{ u\left(\frac{c}{1 + \mathbf{1}_{m=ma}\zeta_a}\right) - v\left(\frac{\ell(m)}{1 + \mathbf{1}_{m=ma}\zeta_a}\right) + \beta \sum_{\eta'} \pi(\eta'|\eta) V(j + 1, s, m, \eta', a') \right\}$$

subject to

$$\begin{aligned} a' + c(1 + \tau^c) &= a(1 + r(1 - \tau^k)) + y(1 - \tau^p) - T(y(1 - \tau^p)) \\ y &= w\epsilon(s, j, m)\eta\ell(m) \\ a' &\geq 0 \end{aligned}$$

Finally, in retirement ages $j \in \{j_r, ..., J\}$, all three education groups of parents solve a standard consumption-saving problem analogous to the one of the children generation described in the previous section.

[4] Note that since assets in the value function enter the budget constraint as being multiplied by the gross, after-tax interest rate $1 + r(1 - \tau^k)$, and since inter-vivos transfers are received in the same period in which they are made and thus do not accrue interest, these transfers b have to be divided by $1 + r(1 - \tau^k)$ in the Bellman equation of the parent.

2.3 Government

The government runs a PAYGO pension system with a balanced budget. It also finances exogenous government spending, expressed as a share of aggregate output G/Y, and aggregate education subsidies through consumption taxes, capital income taxes and a progressive labor income tax code. In the initial scenario without the COVID-19 shock and the ensuing recession, the government budget clears by adjustment of the consumption tax rate. In the thought experiments we hold all tax parameters constant, therefore implicitly assuming that the shortfalls or surpluses generated by a change in the environment are absorbed by government debt serviced by future generations.

2.4 Thought Experiment

We compute an initial stationary partial equilibrium with exogenous wages and returns prior to model period $t = 0$. In period $t = 0$, the COVID-19 shock unexpectedly occurs, and from that point on unfolds deterministically. That is, factor prices and fiscal policies are fixed, and households get surprised once by the shock, after which they have perfect foresight with respect to aggregate economic conditions. The COVID-19 crisis impacts the economy through two channels:

1. An education crisis: the government closes schools, represented in the model by a temporary (6 month) reduction in public investment i^g into child human capital production.

2. An aggregate income recession: a decline in parental incomes induced by an assumed decline of hours worked and an increase in the probability of drawing a low labor productivity $\pi(\eta' = \eta_l \mid \eta \in \{\eta_l, \eta_h\})$, which also increases the probability weight on η_l in the stationary invariant distribution $\Pi(\eta)$.

We then trace out the impact of these temporary shocks on parental human capital inputs (both time and money) and intergenerational transfer decisions, as well as on the education choices, future labor market outcomes, and welfare of the children generation, both in terms of its aggregates as well as in terms of its distribution. Since children differ by age at the time of the shock (as well as in terms of parental characteristics), so will the long-run impact on educational attainment, future wages, and welfare. We will place special emphasis on this heterogeneity.

3 Calibration

A subsect of parameters is calibrated exogenously not using the model. These first stage parameters are summarized in Table 3. The second stage parameters are those that are calibrated endogenously by matching moments in the data and are summarized in Table 4. We next describe in detail our choice and sources of first stage parameters and the moments we match to calibrate the second stage parameters.

Table 3: First Stage Calibration Parameters

Parameter	Interpretation	Value
	Population	
$j=0$	Age at economic birth (age 4)	0
j_a	Age at beginning of econ life (age 16)	6
j_h	Age at finishing HS (age 18)	7
j_c	Age at finishing CL (age 22)	9
j_f	Fertility Age (age 32)	14
j_r	Retirement Age (age 66)	31
J	Max. Lifetime (age 80)	38
$\xi(m,s)$	Fertility rates	see main text
$\Phi(j_f,m,s)$	Distribution of parents by martial status and education, age j_f	PSID
	Preferences	
θ	Relative risk aversion parameter	1
φ	Curvature of labor disutility	0.5
ζ_a	Adult equivalence parameter	0.5 (OECD)
	Labor Productivity	
$\{\epsilon(j,s,m)\}$	Age Profile	PSID
$[\eta_l,\eta_h]$	States of Markov process	[0.8439, 1.1561]
π_{hl}	Transition probability of Markov process	0.0324
$\pi_{hl}^{ld}(s)$	Transition probability of Markov process after lockdown	[0.5078,0.4537,0.2081]
χ^s	Hours worked for students, as a fraction of full time (HS and CL)	$\{0.2, 0.5\}$
$\gamma(s,h)$	Ability gradient of earnings	Estimated on NLSY79 data from Abbott et al. (2019)
	Endowments	
$l(m)$	Average hours worked by marital status	PSID
$\Phi(a\|j_f,m,s)$	Asset distr-n of parents by martial status and education, age j_f	PSID
$\underline{a}(j_f,s,pa)$	Borrowing limit for parents at age j_f	PSID
	Ability/Human Capital and Education	
ι	College tuition costs (annual)	14756.43762$
$\underline{a}(j\in[j_h,j_c-1],co,ch)$	College borrowing limit	45000$
σ^h	Elast of subst b/w human capital and CES inv. aggr.	1
σ^g	Elast of subst b/w public inv. and CES aggr. of private inv.	2.43
σ^m	Elast of subst b/w monetary and time inv.	1
κ_3^m	CES share parameter of monetary and time inv. (age bin 6-8)	0.5
$\kappa_j^g, j>0$	Share of government input for ages 6 and older	0.676
$\Phi(h(j=0)\|s_p,y_p,a_p)$	Innate ability dist-n of children by parental char-s	PSID-CDS 2014
$\underline{h}(j=0)$	Normalization parameter of initial dist-n of initial ability	0.1248
	Government policy	
ξ	Public CL education subsidy	38.8%
i_j^g	Public early education spending by age	≈ 5000$ (UNESCO)
τ_c	Consumption Tax Rate	5.0%
τ_k	Capital Income Tax Rate	28.3%
τ^p	Soc Sec Payroll Tax	12.4%

Notes: First stage parameters calibrated exogenously by reference to other studies and data.

3.1 Age Brackets

The model is calibrated at a biannual frequency. We initialize the parental economic life-cycle when their children are of age 4, which is model age $j=0$. The reason for this initialization

Table 4: Second Stage Calibration Parameters

Parameter	Interpretation	Value
	Preferences	
β	Time discount rate (target: asset to income ratio, age 25-60)	0.9812
ν	Altruism parameter (target: average IVT transfers to total wealth ratio)	0.7853
ϕ	Weight on labor disutility (target: fraction of group $s = hs$)	0.1139
	Labor Productivity	
$\rho_0(s)$	Normalization parameter (target: $\mathbb{E}\gamma(s, h) = 1$)	$[0.2261, 0.0429, -0.1815]$
	Human Capital and Education	
κ	Utility weight on time inv. (target: average time inv.)	0.7216
κ_j^h	Share of human capital (target: average monetary inv. & slope of time inv.)	cf. Figure 2
κ_j^m	Share of monetary input (target: slope of money inv.)	cf. Figure 2
κ_0^g	Share of government input for age bin 4-6 (target: average time inv. age bin 4-6)	0.4055
$\bar{A}$	Investment scale parameter (target: average HK at age j_a)	1.2110
$\bar{A}^{HS}$	Investment scale parameter in HS (target: average HK at age j_{a+1})	1.0740
$\varrho(s^p = no)$	psychological costs (target: fraction of group $s = co$)	-0.0001
$\varrho(s^p = co)$	psychological costs (target: conditional fraction of group $s = co$)	-0.3300
	Government policy	
λ	Level parameter of HSV tax function (balance gvt budget)	0.8644
ρ^p	Pension replacement rate (balance socsec budget)	0.1893

Notes: Second stage parameters calibrated endogenously by targeting selected data moments.

age is the calibration of the initial human capital endowment $h(j = 0)$, which is informed by data on test scores measures at child biological ages 3 to 5, as described below. Thus, children are irrelevant to the economic model for the first 3 years of their biological lives. Parental age at the economic "birth" of children is $j_f = 14$, which we also refer to as "fertility" age. This corresponds to a biological age of 32, when children are of biological age 4.[5] Children make the higher eduction decision at biological age 16, model age $j_a = 6$. Children who complete high school stay in school for one additional model period, thus high school is completed at $j_h = 7$. Children who attend college stay in college for two model periods, thus college is completed at $j_c = 9$. Retirement is at the exogenous age $j_r = 31$, corresponding to biological age 66. Households live at most until age $J = 38$.

3.2 Preferences

The per period subutility function $u(x)$ is of the standard iso-elastic power form

$$u(x) = \frac{1}{1-\theta}\left(x^{1-\theta} - 1\right).$$

[5]Thus, children are biologically born at parental age 28.

In our baseline specification, we set $\theta = 1$ (logarithmic utility), and consequently child and adult equivalence scale parameters are irrelevant for the problem. In the parental household's problem, the per period subutility function $v(x)$ is

$$v(x) = x^{1+\frac{1}{\varphi}}$$

so that if $x = \ell$, parameter φ can be interpreted as a Frisch elasticity of labor supply. In our model of exogenous labor supply this interpretation of course seizes to be relevant, but it provides us with a direct way of calibrating the power term of the utility function. We set $\varphi = 0.5$ based on standard estimates of the Frisch elasticity.

When children live in the parental household, we have $x = \frac{\ell(m)+\kappa \cdot \xi(m,s) \cdot i^t}{1+\mathbf{1}_{m=ma}}$. $\ell(m)$ are hours worked by marital status, which we estimate from the data, giving annual hours of $\ell(si) = 1868$ and $\ell(ma) = 3810$. The time cost parameter κ is calibrated to match average time investments by parents into the education of children, giving $\kappa = 0.72$ (with further details described below as part of the calibration of the human capital technology). The adult equivalence parameter ζ_a is set to $\zeta_a = 0.5$ according to standard OECD adult equivalence scales.

When children attend high school or college, they experience psychological costs for $s \in \{hs, co\}$ according to the cost function

$$p(s, s^p; h) = \phi(1 + \varrho(s^p)\mathbf{1}_{j \in [j_h, j_c - 1]}\mathbf{1}_{s=co})\frac{1}{h}.$$

We calibrate the parameters of the cost function to match education shares in the data for the three groups $s \in \{no, hs, co\}$. We measure these shares for adults older than age 18 for the PSID waves 2011, 2013, 2015 and 2017, and assume that children have the same education shares as the couples of the parental generation. Parameter ϕ is calibrated to match the fraction of children with a high school degree and no college of 55.65%, giving $\phi = 0.11$. The additional shifter $\varrho(s^p)$ only applies to children who attend college and reflects that psychological costs of college attendance depend on parental education. We set $\varrho(s^p = no) = \varrho(s^p = hs)$ and calibrate it to match the fraction of children with a college degree of 28.02%, giving $\varrho(s^p = no) = \varrho(s^p = hs) = -0.0001$. In light of the monetary costs of college attendance this parameter can also be interpreted as consumption value of college attendance and, thus, can take either positive or negative values. In turn, $\varrho(s^p = co)$ is calibrated to match the fraction of children in college conditional on parents having a college degree of 63.3% (cf. Krueger and Ludwig (2016)). This gives $\varrho(s = co) = -0.33$ reflecting that utility benefits for children of households with a college degree are even higher.

Households discount utility at rate β. We follow Busch and Ludwig (2020) and calibrate it to match the assets to income ratio in the PSID for ages 25 to 60 giving an annual discount factor of $\beta = 0.98$.

Utility of future generations is additionally discounted at rate ν. Parameter ν is chosen so that in equilibrium total inter-vivos transfers in the economy account for 0.82% of wealth as in the 1986 SCF. Our target is based on Gale and Scholz (1994), Table 4, as the sum of inter-vivos transfers for college expenses and other inter-vivos transfers. This gives $\nu = 0.79$.

3.3 Initial Distribution of Parents

3.3.1 Marital Status

We measure marital status by the legal status from the PSID years 2013 to 2017 in the sample of parents. This gives a share of singles of 51.7% and a share of married households of 48.3%.

3.3.2 Education Categories

We group the data by years of education. Less than high school, $s = no$, is for less than 12 years of formal education. High school completion (but no college) is for more than 12 but less than 16 years of education. College is at least 16 years of education. The population shares of parents in the three education categories by their marital status are summarized in Table 5.[6] In the initial steady state pre shock solution of the model we calibrate the model so that the education shares of children are equal to those of the parental generation.

Table 5: Fraction of Households by Education for each Marital Status

Education s/Marital Status m	si	ma
no	0.2194	0.1621
hs	0.6064	0.5577
co	0.1742	0.2802

Notes: Fraction with education $s \in \{no, hs, co\}$ by marital status.

3.3.3 Demographics

The number of children by marital status and education of parents $\xi(m, s)$ computed as the average number of children living in households with household heads aged 25-35 is summarized in Table 6.

[6] The educational distribution is consistent with many other studies based on the PSID, cf., e.g., Heathcote et al. (2010).

Table 6: Number of Children by Marital Status and Education

Education s/Marital Status m	si	ma
no	2.36	2.33
hs	1.86	2.15
co	1.77	1.96

Notes: Number of children by marital status and education.

3.3.4 Assets

Conditional on the initial distribution of parents by marital status and education, we measure as averages of biological ages 25-35 the distribution of assets according to asset quintiles, which gives the initial distribution $\Phi(a \mid j_f, m, s)$. We set the borrowing constraint of parents according to asset holdings in the lowest quintile of the asset distribution giving $\underline{a}(j_f, s, pa)$. For all ages $j > j_f$ we let

$$\underline{a}(j, s, pa) = \underline{a}(j-1, s, pa)(1+r) - rp$$

and compute rp such that the terminal condition $\underline{a}(j_r, s, pa) = 0$ is met.

3.3.5 Income

We draw initial income shocks assuming independence of the asset position according to the stationary invariant distribution of the 2-state Makov process, thus $\Pi(\eta_h) = 0.5$.

3.4 Productivity

We use PSID data to regress log wages measured at the household level on a cubic in age of the household head, time dummies, family size, a dummy for marital status, and person fixed effects. Predicting the age polynomial (and shifting it by marital status) gives our estimates of $\epsilon(m, s, j)$. We next compute log residuals and estimate moments of the earnings process by GMM and pool thos across education categories and marital status.[7] We assume a standard process of the log residuals according to a permanent and transitory shock specification, i.e., we decompose log residual wages y_t as

$$y_t = z_t + \epsilon_t$$
$$z_t = \rho z_{t-1} + \nu_t$$

[7]We thank Zhao Jin for sharing her code with us.

and estimate this process pooled across education and marital status. To approximate the process in our model, we first translate it into an AR(1) process as

$$y_t = \rho y_{t-1} + \nu_t + \epsilon_t - \rho \epsilon_{t-1}$$

with variance of the total error term $\sigma_\nu^2 + (1-\rho)^2\sigma_\epsilon^2$, and then approximate the resulting process as a 2-state Markov process. The estimates are reported in Table 7.

Table 7: Stochastic Wage Process

	Estimates					Markov Chain	
Parameter	ρ	σ_ν^2	σ_ϵ^2	$\sigma_\nu^2 + (1-\rho)^2\sigma_\epsilon^2$	σ_y^2	$\pi_{hh} = \pi_{ll}$	$[\eta_l, \eta_h]$
Estimate	0.9559	0.0168	0.0566	0.0217	0.2516	0.9676	[0.8439, 1.1561]

Notes: Estimated moments of residual log wage process.

We set the fraction of time working during high school to $\chi(hs) = 0.2$, which can be interpreted as a maximum time of work of one day of a regular work week. In college, students may work for longer hours and we accordingly set $\chi(co) = 0.5$.

The mapping of acquired human capital into earnings according to $\gamma(s, h)$ is based on Abbott et al. (2019). We use their data—the NLSY79, which includes both wages and test scores of the Armed Forces Qualification Test (AFQT)—to measure residual wages $\omega(s)$ of education group s (after controlling for an education specific age polynomial) and run the regression

$$\ln(\omega(s)) = \rho_1(s) \cdot \ln\left(\frac{e}{\bar{e}}\right) + \epsilon(s),$$

where $\epsilon(s)$ is an education group specific error term and $\bar{e}$ are average test scores. We denote the education group specific coefficient estimate by $\hat{\rho}_1(s)$, see Table 8. The estimated ability gradient is increasing in education reflecting complementarity between ability and education. In the model, we correspondingly let

$$\ln(\gamma(s, h)) = \rho_0(s) + \hat{\rho}_1(s) \cdot \ln\left(\frac{h}{\bar{h}}\right),$$

where $\bar{h}$ is average acquired human capital at $j = j_a$ (biological age 16) and $\rho_0(s)$ is an education group s specific normalization parameter, chosen such that

$$\int \exp\left(\rho_0(s) + \hat{\rho}_1(s) \cdot \ln\left(\frac{h}{\bar{h}}\right)\right) \Phi(dh, s) = 1.$$

The normalization—which gives $\rho_0(s) = 0.23, 0.04, -0.18$, for $s \in \{no, hs, co\}$, respectively—implies that the average education premia are all reflected in $\epsilon(s, j, ma)$, which in turn are directly estimated on PSID data.

Table 8: Ability Gradient by Education Level

Education Level	Ability Gradient
HS-	0.351 (0.0407)
(HS & CL-)	0.564 (0.0233)
(CL & CL+)	0.793 (0.0731)

Notes: Estimated ability gradient $\hat{\rho}_1(s)$, using NLSY79 as provided in replication files for Abbott et al. (2019). Standard errors in parentheses.

3.5 Human Capital Production Function

At birth at age $j = 0$, children draw their innate ability (initial human capital) $h = h_0$ conditional on the distribution of parents by parental characteristics s_p, m_p, thus $h_0 \sim \Psi(h(j = 0) \mid s_p, m_p)$. We calibrate the distribution from the test score distribution in the PSID Child Development Supplement (CDS) survey I-III, and match it to parental characteristics by merging the survey waves with the PSID. We consider children for whom both caregivers correspond to the household head and spouse in a PSID household[8], and for whom at least one of the caregivers is the biological parent. This leaves us with 4393 observations (2419 children) for the three waves of the survey. Table 9 reports the joint distribution of average test scores of the children by parental education and marital status. We use this test score distribution as a proxy for the initial human capital distribution of children conditional on parental education and marital status.[9] We base the calibration of the initial ability distribution of children on this data by drawing six different types of children depending on the combination of marital status (2) and parental education (3). Children's initial human capital is normalized as the test score of that m^p, s^p-group relative to the average test score. We further scale the resulting number by the calibration parameter $\bar{h}_0$ and, thus, initial human capital of the children is a multiple of $\bar{h}_0$. The calibration of $\bar{h}_0$ is described next as part of the calibration of the overall human capital technology. Initial abilities relative to average abilities and the corresponding multiples of $\bar{h}_0$ for the six types are contained in Table 9.

At ages $j_0, \ldots, j_a - 1$ children receive parents' education investments through money and time $i^m(j), i^t(j)$ and governmental time investments i^g, respectively. Education investments of

[8] In case of singles, only the household head is the primary caregiver.

[9] Importantly, by correlating the test score distribution with these parental characteristics, we do not pose a causal link between parental education and children's characteristics. The test scores just give us a convenient way to proxy the initial joint distribution.

Table 9: Initial Ability by Parental Education

Marital Status and Educ of HH Head	Avg. Score	Fraction of $\bar{h}_0$
Single Low	35	0.843
Single Medium	38	0.906
Single High	46	1.107
Married Low	39	0.945
Married Medium	41	0.984
Married High	45	1.085

Notes: Estimated initial ability of children as measured by the letter word test in the Child Development Supplement Surveys 1-3 (years 1997, 2002, 2007) of the PSID.

the government are certain, known by parents, and equal across children. Human capital is acquired given a multi-layer human capital production function

$$h'(j) = \left(\kappa_j^h h^{1-\frac{1}{\sigma^h}} + (1-\kappa_j^h) i(j)^{1-\frac{1}{\sigma^h}}\right)^{\frac{1}{1-\frac{1}{\sigma^h}}} \tag{5a}$$

$$i(j) = \bar{A}\left(\kappa_j^g \left(\frac{i^g}{\bar{i}^g}\right)^{1-\frac{1}{\sigma^g}} + (1-\kappa_j^g)\left(\frac{i^p(j)}{\bar{i}^p}\right)^{1-\frac{1}{\sigma^g}}\right)^{\frac{1}{1-\frac{1}{\sigma^g}}} \tag{5b}$$

$$i^p(j) = \left(\kappa_j^m \left(\frac{i^m(j)}{\bar{i}^{m,d}}\right)^{1-\frac{1}{\sigma^m}} + (1-\kappa_j^m)\left(\frac{i^t(j)}{\bar{i}^{t,d}}\right)^{1-\frac{1}{\sigma^m}}\right)^{\frac{1}{1-\frac{1}{\sigma^m}}}, \tag{5c}$$

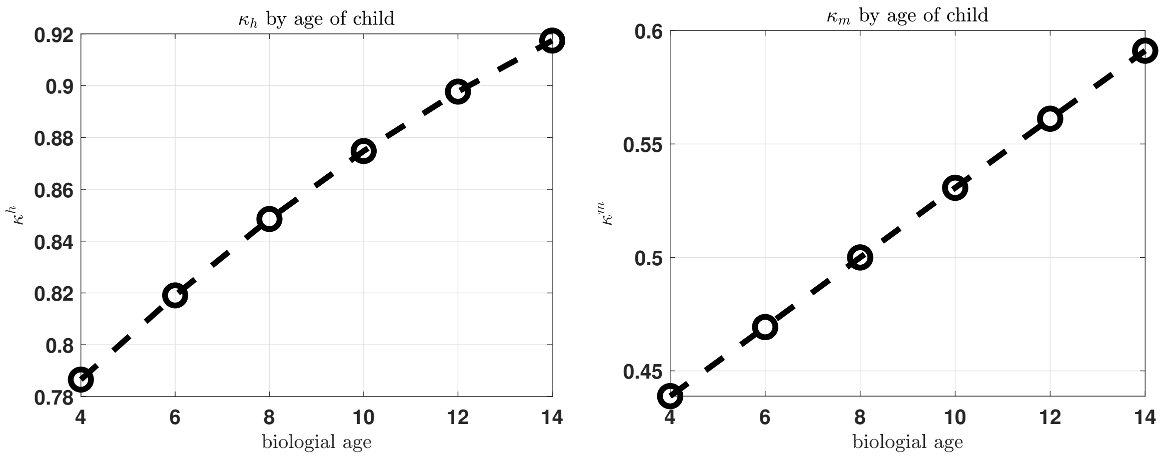
which partially features age dependent parameters for calibration purposes. We also divide the exogenous investments by the government i^g and the endogenous age dependent per child monetary and time investments by the parents $i^m(j), i^t(j)$, as well as the CES aggregate of these (normalized) investments, $i^p(j)$, by their respective unconditional means through which we achieve unit independence.

The outermost nest (first nest) augments human capital and total investments according to a CES aggregate with age-specific parameter κ_j^h and age-independent σ^h. We set $\sigma^h = 1$ and calibrate κ_j^h to match (per child) time investments by age of the child. We model age dependency as

$$\ln\left(\frac{1-\kappa_j^h}{\kappa_j^h}\right) = \alpha_0^{\kappa^h} + \alpha_1^{\kappa^h} \cdot j + \alpha_1^{\kappa^h} \cdot j^2 \tag{6}$$

and determine $\alpha_1^{\kappa^h}, \alpha_2^{\kappa^h}$ by an indirect inference approach such that the age pattern of log per child time investments in the data equals the pattern in the model for biological ages 6 to 14 of the child. Recall that we in turn match the average level of time investments at biological ages 6 to 14 by calibrating the utility cost parameter κ. Time investments at biological age 4 are matched differently with details described below. The intercept term $\alpha_0^{\kappa^h}$ is calibrated to match average monetary investments. Panel (a) of Figure 2 displays the resulting age profile. Consistent with Cunha et al. (2010), we find that the weight on acquired human capital at age j is increasing in j, so that investments become less important in the course of the life-cycle. While our model is not directly comparable to their empirical analysis,[10] also the magnitude of κ_j^h is similar.

Figure 2: Age Dependent Parameters κ_j^h, κ_j^m over Child Age



(a) Weight κ_j^h

(b) Weight κ_j^m

Notes: Age-specific weight parameters κ_j^h and κ_j^m calibrated endogenously to match time and money investment profiles.

In the second nest, division of time investments by the government through its mean implies that in the initial equilibrium $\frac{i^g}{\bar{i}^g} = 1$. We restrict $\kappa_j^g = \bar{\kappa}^g$ for $j > 0$ and calibrate it exogenously according to the estimates for the US by Kotera and Seshadri (2017)—who estimate the parameters of a CES nesting of private and public education investments similar to ours—giving $\bar{\kappa}^g = 0.676$.

At biological age 4 of the child, children are still in kindergarten. To take into account this structural break in the process of education according to the institutional setting, we separately calibrate κ_0^g to match the average time investments by parents into their children at biological age 4 of the child. This gives $\kappa_0^g = 0.41$.

The calibration of the substitution elasticity σ^g is again by reference to Kotera and Seshadri (2017) who estimate an elasticity of substitution between private and government investment of $\sigma^g = 2.43$. Thus, parental and government investments are gross substitutes but substitution across these education inputs is far from perfect.

[10] Total Investments in our model in the first nest include government investments from the second nest, and we do not distinguish explicitly between cognitive and non-cognitive skills.

$\bar{A}$ is a computational normalization parameter which we choose such that average acquired human capital is equal to 1, sufficiently below the maximum human capital gridpoint, giving $\bar{A} = 1.21$. Parameter $\bar{h}_0$—which we introduced above as part of our discussion of the initial ability distribution in Table 9—is calibrated such that the ratio of average human capital at $j = 5$ (biological age 14) to $j = 0$ (biological age 4) is equal to the ratio of test scores for age groups 14-15 to age group 3-5, which gives $\bar{h}_0 = 0.12$.

The third nest augments the endogenous age specific per child monetary and time investments. As in Lee and Seshadri (2019) we restrict $\sigma^m = 1$. The age dependency of κ_j^m is specified as

$$\ln\left(\frac{1-\kappa_j^m}{\kappa_j^m}\right) = \alpha_0^{\kappa^m} + \alpha_1^{\kappa^m} \cdot j$$

We calibrate $\alpha_0^{\kappa^m}$ to achieve the normalization $\kappa_2^m = 0.5$, and $\alpha_1^{\kappa^m}$ is calibrated to match the monetary investment profile, which is relatively flat in the data. The resulting age profile of κ_j^m is displayed in Panel (b) of Figure 2.

At age j_a the human capital process is extended to the high school period (i.e., for all children with education $s = hs$ and $s = co$). Time and monetary investments by parents in this phase of the life-cycle are zero because children have already left the parental household and the human capital production function at $j = j_a, s \in \{hs, co\}$ is

$$h'(j) = \tilde{A}\left(\kappa_6^h h^{1-\frac{1}{\sigma^h}} + (1-\kappa_6^h)\left(\frac{i^g}{\bar{i}^g}\right)^{1-\frac{1}{\sigma^h}}\right)^{\frac{1}{1-\frac{1}{\sigma^h}}}. \qquad (7)$$

We compute κ_6^h as a predicted value from the above described regression in (6) and calibrate the additional scaling parameter $\tilde{A}$ such that the ratio of average human capital at $j = 6$ (biological age 16) to average human capital at age $j = 5$ is equal to the ratio of test scores of ages $16-17$ to age $14-15$ of 1.05. This gives $\tilde{A} = 1.07$.

The production function in (7) is an approximation as it ignores parental inputs entirely,[11] reflecting that parental inputs may not be that effective at that age. The specification also ignores that children may invest into the human capital formation themselves, which may be of particular relevance for our main experiment of school closures. We thus regard our model of biological age 16 children as a crude approximation and will accordingly not put a key emphasis on those children when discussing our results. However, it is important for parental decisions at younger child ages that parents do foresee that the human capital process for age 16 children

[11] It would not be possible in our setup to model parental inputs at that age because children have already left the household.

continues when children have left the household, which is our main motivation for extending the human capital accumulation process beyond that age.

3.6 College Tuition Costs & Borrowing Constraint of Children

We base the calibration of college tuition costs and borrowing constraints for college youngsters on Krueger and Ludwig (2016). The net price (tuition, fees, room and board net of grants and education subsidies) for one year of college in constant 2005 dollars is 13,213\$. In 2008 dollars, the maximum amount of publicly provided students loans per year is given by $11,250\$$. which is the borrowing limit in the model for $s = co$ and $j \in [j_h, j_c - 1]$. For all ages $j \geq j_c$ we let

$$\underline{a}(j, co, ch) = \underline{a}(j-1, co, ch)(1+r) - rp$$

and compute rp such that the terminal condition $\underline{a}(j_r, co, ch) = 0$ is met.

3.7 Government

The government side features the budget of the general tax and transfer system and a separate budget of the pension system. In the general budget the revenue side is represented by consumption, capital income and labor income taxes. The respective values for the first two are as follows: $\tau_c = 0.05$ and $\tau_k = 0.2$. The labor income tax code is approximated by the following two-parameter function, as in, e.g., Benabou (2002) and Heathcote et al. (2017):

$$T(y) = y - \lambda y^{1-\tau},$$

where τ is the progressivity parameter and λ determines the average tax rate. We set $\tau = 0.18$ as suggested by estimates of Heathcote et al. (2017) and calibrate λ endogenously to close the government budget, giving $\lambda = 0.86$.

The expenditure side of the government budget (beyond the transfers implied by the tax function above) is given by the government spending on children of school age and the college subsidy for college students. The former we approximate as $5000\$$ per pupil based on UNESCO (1999-2005) data, as for example in Holter (2015). The latter is set to 0.388 of tuition costs, as in Krueger and Ludwig (2016).

As for the pension system, the payroll tax τ^p is set to the current legislative level of 12.4% and the pension benefit level relating average pension benefits to average net wages is endogenously chosen such that the budget of the pension system is balanced giving a replacement benefit level of $\rho^p = 0.18$.

3.8 Calibrating the Pandemic Induced Recession

The calibration of the average income loss for parental households during the lockdown is based on the aggregate output drop in the US in the first and second quarter of 2020 of 4.8% and 32.9% with respect to the previous year, respectively. Assuming, as an optimistic baseline calibration, full recovery in the third quarter this implies a reduction of output in our two year model by $(0.5 \cdot (4.8+32.9)/4)\% = 4.7\%$. According to Bick and Blandin (2020) the drop of hours relative to February 2020 during the six months March to August was 17.3%, which in our two year model under the optimistic scenario of full recovery in the third quarter translates into a reduction of aggregate hours by $17.3/4\% = 4.3\%$. Accordingly $4.3/4.7 \cdot 100\% = 92\%$ of the aggregate income reduction is due to the drop in hours and the 8% is due to a drop in productivity. This productivity reduction is calibrated by assuming a one-time increase of the probability of transiting from high to low incomes in our two stage approximation of the Markov process, π_{hl}. In turn, the hours drop is calibrated by reducing hours worked of those households who have low labor productivity, $\eta = \eta_l$, during the recession.

To also take into account the heterogenous distribution of this reduction of hours and productivity across education groups we resort to Mahnken (2020), who reports that the unemployment rate of workers with education of less than high school ($s = no$) increased by 14.4 percentage points, for workers with a high school degree but less than college ($s = hs$) it increased by 12.9 percentage points, and for workers with a college degree ($s = co$) only by 5.9 percentage points.[12] We approximate this in our income shock calibration in terms of relative losses of hours and productivity by factor $14.4/5.9 = 2.44$ for education group $s = no$ and by factor $12.9/5.9 = 2.18$ for education group $s = hs$. To account for this heterogeneity across education groups we specify the transition probabilities of the Markow process in the period of the lockdown as education specific and calibrate $\pi_{hl}^{ld}(s) = \pi^{ld}(\eta' = \eta_l \mid \eta = \eta_h, s)$ to match the share of 8% of the aggregate income decline and its distribution. We accordingly let

$$\pi_{hl}^{ld}(s = no) = 2.44 \cdot \pi_{hl}^{ld}(s = co)$$
$$\pi_{hl}^{ld}(s = hs) = 2.18 \cdot \pi_{hl}^{ld}(s = co)$$

and calibrate $\pi_{hl}^{ld}(s = co)$ to match the average income reduction by 4.7% taking as given the aggregate hours reduction (and its distribution) by 4.3% . This gives $\pi_{hl}^{ld}(s = co) = 0.0308$, thus implying $\pi_{hl}^{ld}(s = hs) = 0.0671$ and $\pi_{hl}^{ld}(s = no) = 0.0751$. Likewise, we distribute the aggregate hours reduction of 4.3% across the education groups who experience low productivity, $\eta = \eta_l$,

[12] See the figure entitled "Unemployment Rate by Educational Attainment (seasonally adjusted)" from which one can read off these exact numbers.

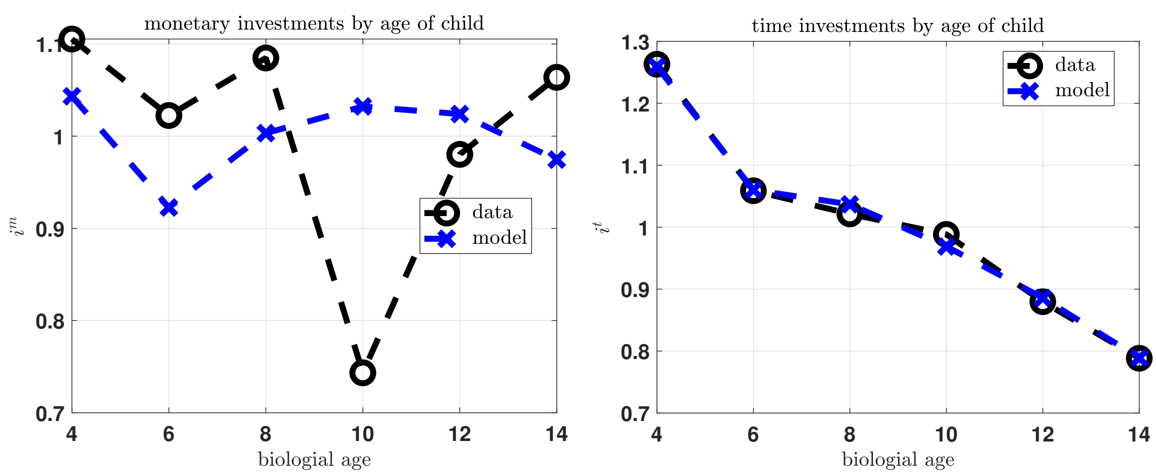
in the period of the recession which gives a an average reduction within education categories by 5.40%, 4.83% and 2.21% for $s = no$, $s = hs$ and $s = co$, respectively.

3.9 Evaluating the Model

Figure 3 shows average time and monetary investments in the model and the data by the age of the child. The good match of the model of time investments in Panel (b) is a consequence of calibration since this is largely a targeted profile through age dependent parameter κ_j^h and parameter κ_0^g. Monetary investments in Panel (a) are slightly downward sloping in the data, and we match the lower slope of monetary investments compared to time investments through the age dependency of κ_j^m.

Figure 4 shows the analogous output by parental education levels, all of which are not targeted in the calibration. The model matches well the positive slope of both types of investment in parental education.

Figure 3: Money and Time Investments by Age of Child



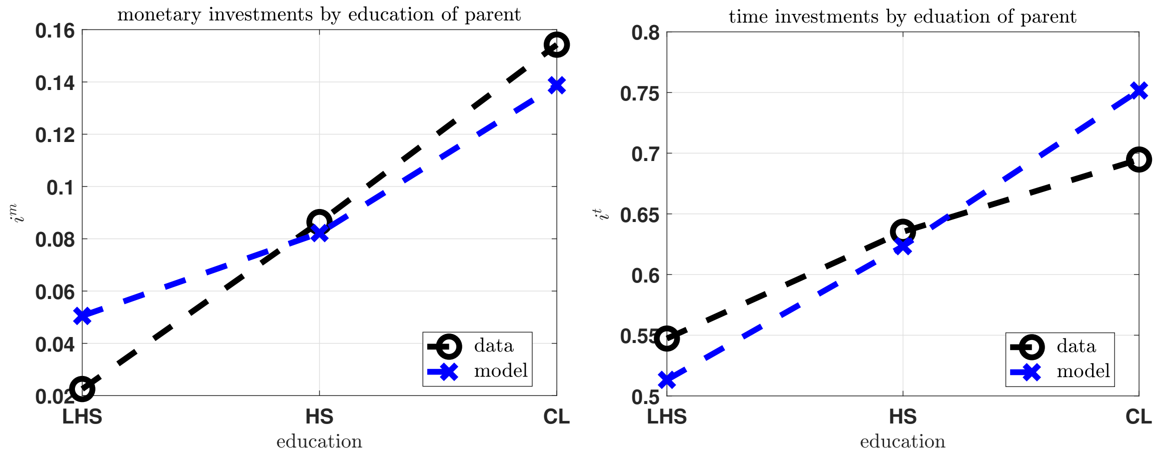
(a) Money Investments (b) Time Investments

Notes: Average money and time investments by children's biological age in the data (black circles) and model (blue crosses).

In our calibration we also do not target directly measures of inter-generational persistence. We measure inter-generational persistence of education by the regression coefficient β_1 in a regression of parental education on children't education,

$$s^p = \beta_0 + \beta_1 s. \tag{8}$$

Figure 4: Money and Time Investments by Education of Parents



(a) Money Investments (b) Time Investments

Notes: Average money and time investments by parent's education in the data (black circles) and model (blue crosses). LHS: less than high school ($s = no$), HS: high school ($s = hs$), CL: college ($s = co$).

In this regression we form two groups, non-college for $s \in \{no, hs\}$ and $s^p \in \{no, hs\}$ and college, for $s = co$, respectively $s^p = co$. Standard estimates of inter-generational education persistence according to this metric range from 0.4 to 0.5 and our (non-targeted) coefficient estimate of $\beta_1 = 0.41$ is in that range. Likewise, we measure inter-generational persistence of earnings. To this purpose we follow parents and their children over the life-cycle until the age before retirement at $j = j_r - 1$. We then perform a regression analogous to (8) based on log earnings. The resulting coefficient of inter-generational earnings persistence is 0.33, close to the standard estimate of 0.4 (Restuccia and Urrutia (2004)).

Our model implies a significant decline of incomes of children from the closure of schools. We are not aware of any empirical evidence on the effects of school closures on later in live outcomes for children in the US and therefore resort to the reduced form evidence of Jaume and Willén (2019) on the effects of teacher strikes in Argentina between 1983 and 2014 on long-run economic outcomes of the affected children. Their main estimates refer to the closing of primary schools by half a year, and they report that this leads to a reduction of wages at ages 30-40 by about $2 - 3\%$. In our model, we consider for our main experiment an exogenous reduction of investments by the government corresponding to a school closure by half a year. We can therefore directly compare the implied average wage loss at biological ages $30 - 40$ for children who were of biological age 6 in the period of the lockdown predicted in our model to the estimates by Jaume and Willén (2019). Our model predicts an income reduction by 1% for these children, which is around half the size of the estimates by Jaume and Willén (2019) (for a different country in a different time period).

Finally, our calibration implies that the time investment response of parents in our full experiment—where government education investments i^g are reduced and where parental households are both subject to a negative productivity shock and a negative hours shock calibrated as described above—translates to 2.04 hours per day of increased time investments into children during the period of the lockdown of schools. This coincides with the estimates provided on the basis of real time surveys by Adams-Prassl et al. (2020).

4 Aggregate Consequences of the School Closures and Income Recession

We conduct two main thought experiments. First, we study the impact of school closures that last for half a year, which in our two year model periods corresponds to a reduction of government time investments i^g by 25%. Second, in addition we subject parents to the asymmetric negative income shocks described in Section 3. Recall that this income shock is mainly driven by a reduction of hours worked and only to smaller extent by a decline of average labor productivity, and both reductions are more severe for parents with lower educational attainment. Table 10 summarizes the average consequences of our main experiments for human capital accumulation, educational attainment, earnings and welfare of children, and Table 11 displays the behavioral responses of parental time- and monetary investments as well as inter-vivos transfers to these shocks.

4.1 School Closures

4.1.1 Human Capital Accumulation, Educational Attainment and Earnings

The lockdown of schools leads to a decline in educational attainment when the children affected by the Covid crisis today make their tertiary education decisions at age 16. As the first panel of Table 10 (second column) shows, across all age cohorts the share of children that will end up dropping out of high school (i.e. choosing $s = no$) increases by 0.66 percentage points, and the share of college-educated children declines by -0.49 percentage points. While these shifts do not appear to be dramatic, they correspond to a 4.1% increase in the share of children without high school degrees, and a -1.7% decrease in the share of college educated children.

The reason for the reallocation towards lower final educational attainment is the reduction in the amount of human capital the average child arrives with at age 16, which falls by -1.46%.

Table 10: Aggregate Outcomes for Main Experiments

	baseline	Change for Children of Biological Age						
		average	4	6	8	10	12	14
Panel A: Lockdowns of Schools								
		change in %p						
share $s = no$	16.28	0.66	0.32	1.02	0.85	0.71	0.59	0.48
share $s = hs$	55.54	-0.17	-0.08	-0.33	-0.24	-0.17	-0.11	-0.08
share $s = co$	28.18	-0.49	-0.24	-0.69	-0.62	-0.54	-0.47	-0.40
		change in %						
av HK	1.00	-1.46	-0.72	-1.97	-1.78	-1.61	-1.43	-1.25
av earn	0.70	-0.96	-0.48	-1.31	-1.18	-1.06	-0.93	-0.81
CEV [in %]	-	-0.65	-0.34	-0.88	-0.79	-0.71	-0.63	-0.54
Panel B: Lockdowns of Schools & Asymmetric Income Shock								
		change in %p						
share $s = no$	16.28	0.62	0.32	0.97	0.80	0.65	0.53	0.42
share $s = hs$	55.54	0.14	0.19	-0.05	0.08	0.16	0.22	0.25
share $s = co$	28.18	-0.76	-0.51	-0.91	-0.88	-0.81	-0.74	-0.67
		change in %						
av HK	1.00	-1.46	-0.82	-1.98	-1.78	-1.59	-1.40	-1.21
av earn	0.70	-1.03	-0.62	-1.37	-1.25	-1.12	-0.98	-0.86
CEV [in %]	-	-0.75	-0.48	-1.00	-0.89	-0.80	-0.71	-0.62

Notes: share $s \in \{no, hs, co\}$: education share in respective education category $s = no$: less than high school, $s = hs$: high school, $s = co$: college; av HK: average acquired human capital at age 16; av earn: average life-time earnings; CEV: consumption equivalent variation. Columns for biological ages 4-14 show the respective percentage point changes of education shares, the percent changes of acquired human capital and average earnings, and the CEV expressed as a percent change, for children of the respective age at the time of the school closures. Column "average" gives the respective average response. The CEV is the consumption equivalent variation of the welfare measure (9).

As Table 11, panel A, first column, demonstrates, parents increase their private investments into children, both in terms of resources as well as in terms of time. However, as discussed in greater detail below, this reaction is not sufficient to fully compensate the loss of government inputs into human capital production in the form of schooling. Consequently, average human capital at age 16 is lower than without the Covid19 school closure shock, and the child cohorts affected by the shock choose on average lower educational attainment, which in turn implies losses in average life-time earnings by -0.96%, see the 5th row of Table 10. Thus, a very transitory shock of closing schools by half a year alone on average leads to a permanent reduction in long-term earnings by almost 1% for the affected children, even after taking parental adjustments into account.

The remaining columns of Table 10 show that there is considerable heterogeneity in the size of these effects by the age of the child at the time the schooling shock hits. Overall, the most

Table 11: Parental Decisions for Main Experiments

	baseline	%-Change for Children of Biological Age						
		average	4	6	8	10	12	14
Panel A: Lockdowns of Schools								
av mon inv	0.09	5.23	1.69	1.60	2.50	3.94	6.69	14.93
av time inv	0.63	3.01	0.88	0.93	1.42	2.24	3.84	8.76
av ivt	1.84	0.29	0.10	0.71	0.46	0.27	0.14	0.04
Panel B: Lockdowns of Schools & Asymmetric Income Shock								
av mon inv	0.09	4.99	1.05	1.24	2.17	3.69	6.57	15.21
av time inv	0.63	4.29	1.01	1.35	2.02	3.20	5.52	12.64
av ivt	1.84	-0.72	-0.88	-0.50	-0.53	-0.70	-0.81	-0.88

Notes: Columns for biological ages 4-14 show the percent changes of money investments, time investments, and inter-vivos transfers for children of the respective age at the time of the school closures. For money and time investments, these are averages of the percent changes of the respective investment over the remaining life-cycle, so, e.g., for a child of age 6 the percent change is the average of the percent changes of investments at ages 6-14 for this child. Column "average" is the raw average across the biological ages of children.

severely affected group are the 6 year old children, i.e., those at the start of primary school. For them, the predicted share of high school dropouts increases by 1 percentage point, the share of college educated decreases by -0.7 percentage points, and their average long-term earnings drop by -1.3%. Younger children are most affected by the school closures due to the self-productivity and the dynamic complementarity implied by the human capital production function: a decrease in human capital accumulation at younger ages due to the school closures translates into lower human capital and lower optimal investment in human capital in the future, as we will discuss in greater detail below.[13] Even though the adverse effect of school closures on human capital accumulation and future educational attainment is most severe for young school children, it is non-negligible even for the 14-year olds at the time of the Covid crisis. For this age cohort, the predicted share of high school dropouts increases by 0.48 percentage points, the share of college educated decreases by -0.4 percentage points, and their average earnings during the rest of their life fall by -0.81%, see the last column of panel A in Table 10. Note that 4 year old children are somewhat shielded against the negative effect of experiencing closures of day care centers and kindergarten, due to the lower importance of governmental inputs relative to parental inputs in the human capital production function at that age, i.e., $\kappa_0^g < \bar{\kappa}^g$.

[13] Note that children aged 4 are somewhat shielded against the negative effect of experiencing closures of day care centers and kindergarten, due to the lower importance of governmental inputs relative to parental inputs in the human capital production function at that age. Since private investment is especially productive at this age (see the value of κ_0^g versus κ_1^g in Section 3.5), relative to age 6 where publich schooling is more important in human capital accumulation, parents respond more strongly

4.1.2 Parental Responses to the School Closures

The previous section painted a fairly dire picture of the long-run outcomes of children impacted by Covid-19-induced school closures. We now document that these effects emerge *despite* substantial efforts of parents to take mitigating actions. In our model, parents have three principal means by which they can cushion the blow of the Covid-19-induced schooling crisis. They can expand their time investments and their resource investments into the children's human capital accumulation during the schooling ages, and they can facilitate attending high school and college by providing children with inter-vivos transfers. Table 11, Panel A, shows that they do all three. Specifically, the table reports the average percentage change of parental decisions for a child of a given age j. For a given age column $j \in \{4, 6, ..., 14\}$ the table entry gives the percent change of investments, averaged over the periods the child remains in the household, relative to the pre-Covid19 scenario. For example, for children aged 4 during the crisis, the table entry measures the average change in parental investments from age 4 to 14, while for children aged 14 during the school closure, it captures the change in parental investments only at this age (since it is the last age the child spends in the household). The first column displays the unweighted average, across all children ages, of the age-specific percent changes.

On average, parents increase their monetary investments into their children's education by 5.2%, their time investment by 3%, and their inter-vivo transfers to children by 0.3%. Thus, overall, parents respond to the school closures with positive and substantial additional investments into their children in all three dimensions, albeit significantly stronger with their direct human capital investments than with their transfers once the children are about to leave the household.

As the remaining columns of the table demonstrate, the exact composition of the parental adjustment depends on the age of the child. Parents increase their monetary investments more for the 4 year old children than for the 6 year old, but their time investment slightly less. Note, though, that for the 4 year old children, these are averages over the ages 4 to 14, but for the 6 year old children only over the ages 6 to 14, so that in sum the investment into 4 year old children are higher. The reason for this is that parental investments into human capital are especially productive during kindergarten age.

Average monetary and time parental investments increase the least for 6 year old children, and then increase in the age of the child.[14] For none of the children ages, parents find it optimal to completely offset the effect of the public school closure on human capital investment during the period of the crisis. Consequently, all children leave the crisis period with less human capital than they otherwise would have had. This reduction, due to the dynamic complementarities of human

[14] Note that these are average investments over the remaining childhood, and the investment horizon is therefore longer for a young than for an old child, and a lower average increase need not mean a lower total increase in investment over the remaining stay of the child in the household.

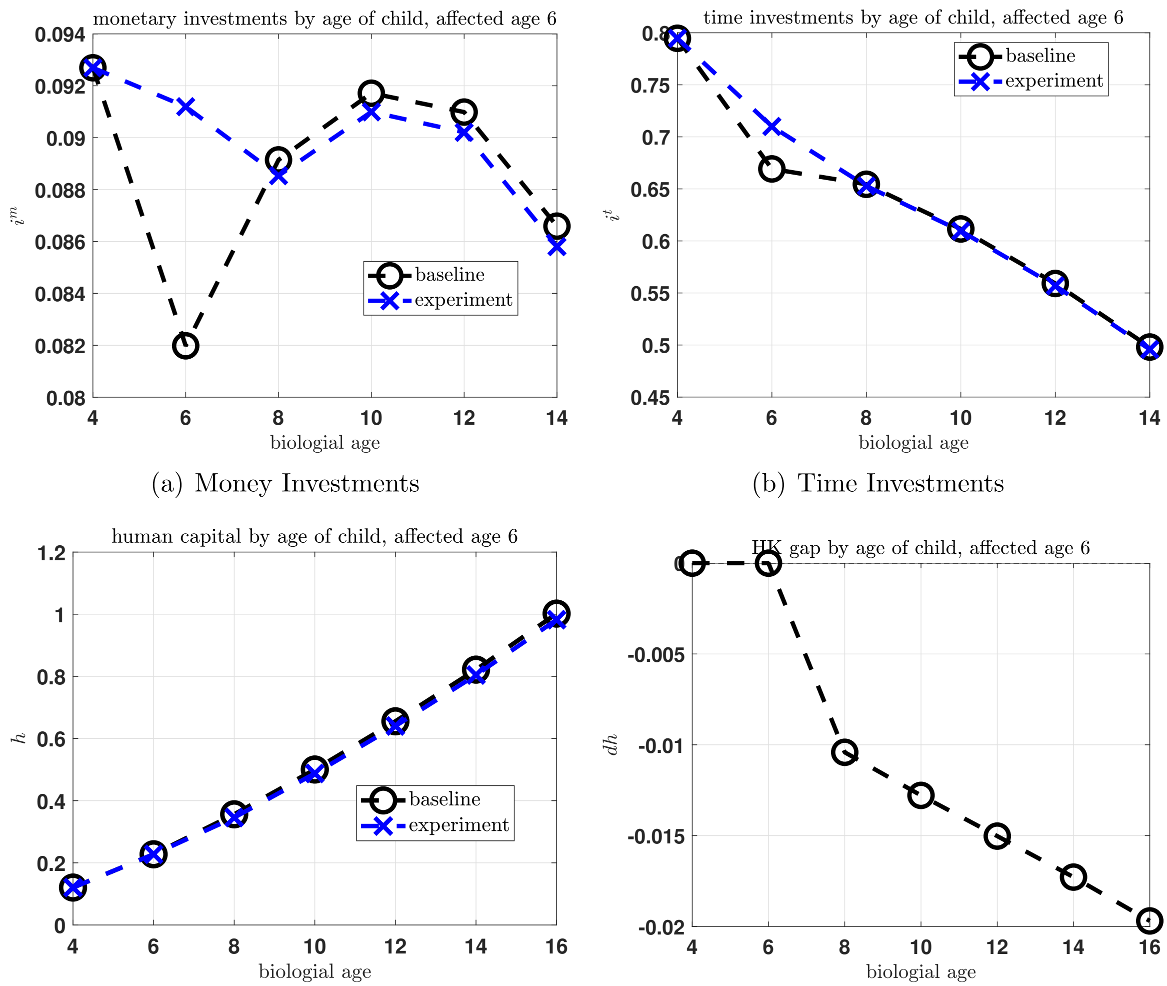
capital and investment, curbs the incentives for private parental human capital investments in all future periods. This effect is the more severe, the longer the human capital accumulation phase during school ages still is, that is, it impacts young children the most. Furthermore, even in the crisis period itself, monetary and time investments increase strongest for older children in percentage terms. For time investments, part of this comes from the fact that baseline time investment is highest for the youngest children. An additional reason lies in the decreasing importance of public investment relative to own human capital in the human capital accumulation process over the child life-cycle: this necessitates stronger investment responses by parents of older children to counteract the negative effects of the school closures. Last, parents of young children have on average lower incomes and assets than parents of older children, making it more likely that they are borrowing constrained. Thus, for them it is relatively easier to increase inter-vivos transfers, which are paid in the future, than current monetary investments into their children.

In contrast to parental monetary and time investments, inter-vivos transfers increase strongest for the 6 year old children, and then less for older children. At the end, parents care about the lifetime utility of their offspring, and not about the means by which they buffer welfare of the children against the adverse schooling shock. Given the dynamic complementarity mechanism explained above, it is relatively more efficient to support younger children more significantly through higher inter-vivos transfers, whereas for older school-aged children, human capital investments are the better option to smooth the Covid schooling shock for their children. Note, though, that even for the parents of 6 year old children, inter-vivos transfers increase by less in relative terms than monetary investments.

We display these dynamic adjustments of time- and monetary investments into human capital investments of children that are six years old at the time of the school closures in Figure 5. Panels (a) and (b) depict time- and monetary investments of parents of these children over their life cycle, both in the benchmark no-crisis scenario and in the presence of the school closures. Panel (c) of the same figure shows the resulting evolution of human capital as the child progresses through school ages following the Covid shock, and Panel (d), for better visualization, displays the absolute change of the human capital stock over the life cycle resulting from the Covid shock. In the period of the lockdown, at biological age 6, there is a substantial increase in parental investment. Private resource investments rise by 11% and time investment by 6%. This is not enough, however, to compensate for the 25% decline in government inputs during this two-year period (half a year of lost schooling), and thus at biological age 8 human capital is at a slightly lower level than in the no-Covid scenario.[15] In response, future human capital investment

[15]Note that panels (c) and (d) show human capital at the beginning of the period. Thus, the effect of the Covid-19 shock at age 6 on human capital only shows up at age 8.

Figure 5: Money and Time Investments and Human Capital over Remaining Child Life-Cycle for Children of Age 6



(a) Money Investments

(b) Time Investments

(c) Human Capital

(d) Change of Human Capital

Notes: Average investments for children of age 6 over their (remaining) life-cycle and acquired human capital at respective age. Black circles: baseline steady state, blue crosses: experiment. Panel (a): money investments, panel (b): time investments, panel (c): acquired human capital, panel (d): absolute change of acquired human capital. Since these are averages for children of age 6 in the period of the lockdown the initial points at age 4 for investments and at ages 4 and 6 for acquired human capital are identical in the baseline and in the experiment.

incentives are dampened (due to the dynamic complementarity feature of the human capital production function), both time- and especially resource investments are lower in subsequent periods (at older ages of the child), with the consequence that the child 10 years after the crisis (at age 16) enters the tertiary education phase with less human capital, therefore opting (on average) for lower educational attainment and the associated reduction in lifetime earnings.

4.1.3 The Welfare Cost of School Closures on Children

Thus far, we have documented that the Covid-induced temporary schooling crisis triggered a reduction in human capital accumulation and educational attainment when currently young cohorts of students enter high-school and college ages. Although the allocational consequences are of course interesting in their own right, they beg the question how significant the Covid shock is from a welfare perspective.

To answer this question we define as social welfare W_j of children that were of age j at the time of the crisis as expected lifetime utility these children obtain when the education decisions of all members of this group have been made and they have entered the labor market at age j_c,

$$W_j = \int V(j_c, s, \eta; a, h)\Phi(j_c, ds, d\eta; da, dh \mid j), \tag{9}$$

where $V(j_c; s, \eta; a, h)$ is the value function of children at age j_c after all education decisions are made (and children with education $s = co$ have completed college) and $\Phi(j_c, s, \eta; a, h \mid j)$ is the distribution of children at age j_c over the relevant state variables: education s, income realization η, assets a and human capital h. The distribution across these states at age j_c is conditional on age j; it is the cross-sectional distribution across young adults implied by the distribution of this cohort at age j just prior to the lockdown, and parental and child education decisions since then. In our household model with constant prices and constant government policy parameters, the value function of the children $V(j_c; \cdot)$ upon future, post-Covid labor market entry is not affected by the shock (which, at the time of labor market entry at age $j_c > j$, lies in the past for all cohorts under consideration), the welfare consequences of Covid school closures are therefore exclusively driven by changes in the distribution $\Phi(\cdot)$. The Covid schooling shock as well as the ensuing parental education investments and child tertiary education choices lead to a different (and typically worse) cross-sectional distribution of a given cohort j at labor market entry at age j_c, relative to the no-Covid scenario.

There are three dimensions along which the cross-sectional distribution for a given cohort deteriorates due to the Covid schooling shock. First, children reach a different human capital position h at age j_c; second, they receive different amounts of inter-vivos transfers from their parents and thus start their working lives with different assets, and third, they make different tertiary education decisions and thus start working life at age j_c with a different education distribution.

To quantify the welfare consequences of the school lockdowns we compute, for each child cohort j, the consumption equivalent variation (CEV) of the Covid19 schooling shock. That is, we calculate the uniform percentage increase in consumption such that the average labor market entrant of a cohort of age j is indifferent between the welfare consequences arising from the original cross-sectional distribution across states at labor market entry and its Covid-impacted counterpart. As the last row of Panel A in Table 11 shows, the welfare losses from the closing of schools are quite substantial, with a reduction of welfare as measured by the CEV by -0.65% on average. Thus, the highly temporary half-year lockdown of schools has strong long-run welfare consequences for children in the order of -0.65% worth of permanent consumption, and this despite the increased human capital investments by parents through home schooling and increased resource investments. For the least affected cohorts, children aged 4 at the time of school closures, the welfare losses are still -0.34%, and for the most affected cohort, children of age 6, welfare losses amount to almost -0.9%. We view these as substantial welfare losses, considering that the school closures are purely temporary shocks, and parents adjust their behavior optimally to counteract the adverse effects on their offspring.

4.2 School Closures and Income Recession

We now turn to the thought experiment in which school closures are accompanied by adverse income shocks stemming from the economic recession induced by the Covid-19 pandemic (or the economic policy response to it). The reduction of incomes is primarily attributed to lower average hours worked (92% of the total income decline) and, to a lesser degree, to lower labor productivity (accounting for 8% of the income decline). Thus, the pandemic-induced economic shock not only implies lower incomes, but at the same time leads to an increase in available time that can be used for investment into the children's human capital. As a consequence, as Panel B of Table 11 shows, parents shift their school closure responses away from increased private resource investments to higher private time investments into their children, relative to a world where the recession income shock is absent. The time investment response now amounts to 2.04 additional hours per day instead of 1.4 hours. The income shock induced by the recession also leads to a decline in inter-vivos transfers, rather than the increase observed in the pure school closure experiment.

As a consequence of the reduced parental investments, the long-term outcomes of the children further worsen, as Panel B of Table 10 displays. The share of college-educated children drops by -0.76 percentage points, rather than by -0.49 percentage points in the presence of only school closures, and long-term earnings drop by -1.03% rather than by -0.96%. The resulting adverse welfare effects are also somewhat larger than in the case of school closures alone. While the income loss of parental households in the Covid19 induced recession leads to additional welfare losses for children, the direct effect of the lockdown of schools with a welfare reduction of -0.65% is more significant for children than the additional effect of the recession, which increases the welfare loss to -0.75%. Importantly, it turns out that there are no sizeable interactions of the two experiments.[16] Even if we model the income shock purely as a shock to resources and not affecting parental time endowments, we still find that the utility loss from school closures alone amounts to around $1/2$ of the total welfare losses of the children. For the welfare of *children*, school closures during the pandemic thus matter more than the negative income shocks to their parents.

4.3 Children of Age 16

In the model, parental investments into children's human capital stop at age 16. Thus, the only margin at which parents can buffer the negative shock of the school closures for children at that age are inter-vivos transfers. At the same time, children cannot invest anything themselves into their human capital at age 16. These two features make children of age 16 quite different from the younger children in the model. In results documented in Appendix B, we consequently find that the average earnings of children aged 16 decrease substantially by -1.7% based on the school closures alone, and by even -1.9% once the pandemic-induced income shock is added, more than for any other age group. The associated welfare losses are thus also very large, with -0.87% and -1.12%, respectively.

5 Distributional Consequences: The Role of Parental Characteristics

The aggregate results presented above mask important heterogeneity by parental characteristics. We focus on two dimensions, parental education and net worth, and summarize the importance of the heterogeneous effects by the dispersion in the CEV welfare measure (9). The results concerning parental education are contained in Table 12, and Table 13 summarizes the results concerning parental assets.

[16] We also conducted an experiment where we only considered the asymmetric shock to parental incomes, and the effects in the whole experiment are almost identical to the sum of its parts—the sum of the effects from the reduction of government investments only and the effects of the parental income reductions only.

Focusing first on parental education, we observe that the welfare losses, as measured by the CEV's, from the Covid-19-induced school closures are largest (-0.7%) for children whose parents are high school dropouts, and smallest (-0.57%) for children of college-educated parents. Thus, higher parental education and the associated higher parental income partly shield children from the negative impact of the school closures through positive investments and increased inter-vivos transfers by their parents.

The differences by parental education become less pronounced if the economic shock of the pandemic is added to the school closures, and now the group of children from high school educated parents are the most affected (while children of college parents continue to be the least affected). The reason is the reaction of inter-vivos transfers of parents to the shock, as well as the induced educational decision of their offspring. With school closures and the income recession, parents in the lower two education groups ($s = no$ and $s = hs$) reduce their inter-vivo transfers to their children, which are by contrast increased for children of parents with a college degree. While the relative reduction is stronger for children of less than high-school parents, the strong absolute reduction of inter-vivos transfers both hurts children of high school educated parents directly (fewer assets) and also reduces the fraction of these children that decide to obtain a college degree. This later, indirect effect is largely absent in the group of children with parents of lowest educational attainment, because only a very small share of them would go to college even before the Covid-19 crisis. In this precise sense, this group of children has not much to lose from Covid-19-related school closures. This explains the non-monotonicity in CEVs. Stating this differently, children from college-educated parents suffer less than children from high school educated parents because their parents can successfully shield them by raising their investment into them and increasing their inter-vivos transfers. By contrast, children from less than high school educated parents suffer less than children of high school educated parents because they have less to lose from the start, given that few of them attend college, which is a major determinant of future income, and parental investments are small for them in the baseline.

Table 12: Welfare Consequences (CEVs) by Parental Education

Experiment/Parental Education	$s = no$	$s = hs$	$s = co$
Lockdown of Schools	-0.70%	-0.68%	-0.57%
Lockdown of Schools & Asymmetric Income Shock	-0.71%	-0.82%	-0.65%

Notes: CEV: consumption equivalent variation of the welfare measure 9 by parental education $s = no$: less than high school, $s = hs$: high school, $s = co$: college.

In Table 13, we delineate the distribution of the welfare consequences by parental net worth, measured at the time children are born into the adult household. Recall from our description in

Section 3 that this cross-sectional wealth distribution in the model is directly estimated from the data. Whereas the differences in the welfare losses between net worth quintiles 2 to 4 are not very pronounced, children of parents in the first wealth quintile experience quite strong welfare losses of -0.8% from the school closures alone and of -1% from the full Covid19 crisis. In contrast, the welfare losses of children in the highest asset quintile amount "only" to -0.54% and -0.63%, respectively. This suggests that low wealth holdings and borrowing constraints of parents are a strong impediment to parents trying to increase their private education resource investments into their children, in response to the reduced governmental investment associated with school closures.

Table 13: Welfare Consequences (CEVs) by Parental Assets

Experiment/Asset Quintile	1	2	3	4	5
Lockdown of Schools	-0.80%	-0.64%	-0.64%	-0.63%	-0.54%
Lockdown of Schools & Asymmetric Income Shock	-0.96%	-0.72%	-0.72%	-0.72%	-0.63%

Notes: CEV: consumption equivalent variation of the welfare measure 9 by parental asset quintiles $1-5$.

We thus far have documented that children of highly educated parents, and those with substantial net worth, experience lower relative welfare losses. What is thus the effect of school closures on measures of intergenerational persistence? In fact, in our experiment with school closures the coefficient in a regression of children's education on parental education, or children's log earnings on parental log earnings, barely change, and even slightly decrease from 0.41 without school closures to 0.40 with school closures in the education regression, or from 0.33 to 0.32 in the earnings regression, respectively. In general, the directional change of inter-generational persistence is ambiguous. On the one hand, we find that less well-off parents increase their absolute monetary investment into children substantially less than better-off parents in reaction to the school closures, which increases persistence. On the other hand, children of well-off parents have more to lose when it comes to attending college, the major factor determining future earnings, given that their initial college attendance rates are higher. Indeed, we find that college attendance drops by -0.9 percentage points for children of college-educated parents (whose baseline college attendance rate is 60.8%), but only by -0.1 percentage points for children of high-school dropouts (whose baseline college attendance rate is a mere 4.7%). This effect decreases intergenerational persistence. Thus, the effects on intergenerational education or earnings persistence are ambiguous. In terms of welfare effects, however, there is a clear ranking with children from better-off parents suffering smaller welfare losses than children with parents at the lower end of the socio-economic distribution. The key to this finding are inter-vivos transfers. Whereas well-to-do parents cannot completely offset the loss of human capital and thus the lower final educational attainment of their children caused by the school closures, they increase the inter-vivos transfers to the children as an additional channel to buffer their welfare losses.[17]

6 Inspecting the Mechanisms

6.1 The Role of Parental Re-optimization

To understand the importance of the reaction of parents to the lockdown of schools, we analyze the results of a model in which parental decisions are held constant. Thus, governmental inputs fall due to the school closures, but parental inputs remain unchanged. As a consequence of unchanged parental inputs, the pandemic-induced recession plays no role in this experiment. Results for this experiment on aggregate effects are summarized in Table 14, which should be compared to Panel A of Table 10. We observe that the aggregate effects are now substantially larger than in the scenario of school closures with parental behavioral adjustments. The share of college-educated children now decreases by -0.75 percentage points, rather than by -0.49 percentage points when parents react optimally to the school closures. Average earnings fall by -1.7% rather than -0.9%, and the CEV associated with the school closures is now on average -1.28% instead of -0.65%. Thus, by optimally adjusting their investment into children, parents mitigate the welfare losses of their children caused by the school closures by around one half. Of course, these adjustments are associated with (unreported) welfare losses to the parents.

We complement these results by additional decomposition analyses in Appendix B where we hold constant one decision at a time. This exercise shows that the welfare losses of children by parental education are lowest for children of college educated parents due to inter-vivos transfers. Holding these constant leads to highest losses for children of college parents and lowest losses for children of parents that themselves dropped out of high school. This also shows that the non-monotonicity of welfare results by parental education in the recession experiment—with highest losses accruing to children of high-school parents—disappears once the response of inter-vivos transfers is shut down.

6.2 The Role of Asymmetric Income Shocks

Suppose that instead of the asymmetric distribution of the income shock, it would be symmetric across households, i.e., the reduction in hours worked and the probability of transiting from a high to a low income is the same during the lockdown for all education groups. With such symmetric

[17] Note that the same main results—namely essentially no change in intergenerational persistence, but lower welfare losses for children of well-off parents—hold when the asymmetric income shock is added to the school closures.

Table 14: Aggregate Outcomes under Constant Decisions when Government Investments are Reduced

	baseline	Change for Children of Biological Age						
		average	4	6	8	10	12	14
		change in %p						
share $s = no$	16.28	0.70	0.46	1.00	0.87	0.74	0.62	0.51
share $s = hs$	55.54	0.05	0.10	-0.08	-0.02	0.04	0.10	0.15
share $s = co$	28.18	-0.75	-0.56	-0.92	-0.85	-0.78	-0.72	-0.66
		change in %						
av HK	1.00	-2.01	-1.45	-2.43	-2.29	-2.14	-1.97	-1.77
av earn	0.65	-1.67	-1.34	-1.95	-1.85	-1.75	-1.64	-1.51
CEV [in %]	-	-1.28	-1.07	-1.51	-1.41	-1.32	-1.22	-1.13

Notes: This table is the analogue to table 10 where parental money and time investment decisions and inter-vivos transfer decisions are held constant. This is computed by holding parental policy functions constant *and* by aggregating with a hypothetical distribution of children over human capital computed under constant decisions.

income shocks, we re-calibrate this probability to again generate a reduction of aggregate income by 4.7%, as in our baseline income shock calibration. As a consequence, the probability of transiting to the low income realization is now $\pi_{hl}^{ld} = 0.058$, which is thus higher for college households and lower for non-college households than in the baseline asymmetric income shock scenario. Likewise, the average reduction of hours is now 4.3% for all households.

Table 15—which is the analogue to the last row of Table 12—summarizes the welfare consequences of children by parental education. As a consequence of the lower incomes of parents with a college degree under the symmetric income shocks, CEVs of children of these parents display stronger welfare losses than in the asymmetric income shock experiment. Their average welfare loss now amounts to -0.79% compared to -0.65%. On the other hand, welfare losses for children of parents with a lower educational degree are slightly reduced compared to the previous results. As a result, children of parents with less than a high school education experience the smallest (absolute) welfare losses of all.

Table 15: Welfare Consequences (CEVs) with Symmetric Shocks

Experiment / Parental Education	pLHS	pHS	pCL
Lockdown of Schools & Symmetric Income Shock	-0.71%	-0.80%	-0.79%

Notes: This table is the analogue to the second row of Table 12 now assuming that the income shock is symmetric across all parents.

7 Are the Effects Non-linear? Prolonged School Closures & Recession

We now consider a prolonged closure of schools by one full year so that i^g drops by 50% in one model period. We also assume that the deep recession the US experienced in the second quarter of 2020 continues for two more quarters and then, at the beginning of 2021, the US economy experiences a full recovery. Thus, the we now assume a drop of hours by $17.3\%/2 = 8.65\%$ and an income drop of $((4.8\% + 3x32.9\$)/4)/2 = 12.93\%$. Table 15 summarizes the results on CEVs showing that the welfare losses are more than twice as large than under the respective previous experiments. With school closures alone, we now find a CEV of -1.44% compared to -0.65% from above, and in the recession scenario we obtain a total effect of -2.25% compared to -0.78%. This shows that the loss of human capital accumulation over the life-cycle not only has long-lasting effects, but also that the effects on welfare are strictly convex in the size of the shock.

Table 16: Welfare Consequences (CEVs) of Prolonged School Closure & Recession

	Lockdown of Schools	Lockdown of Schools & Asymmetric Income Shock
CEV	-1.44%	-2.25%

Notes: Column 1 shows the CEV if schools are closed for one year, column 2 if in addition the recession lasts for one year with full recovery in the next year.

8 Conclusion

In this paper, we analyze the long-term welfare losses of children caused by the school closures in the Covid19 crisis. We use a partial equilibrium model in which parents differ by marital status, education, income, and assets. The human capital production function of children incorporates governmental inputs through public schooling, as well as monetary and time investments by parents. The Covid-19 crisis is modelled as leading to unexpected school closures of half a year. We have three main results. First, the school closures alone lead to substantial reductions in children's welfare, with a consumption equivalent variation of on average -0.65%. Thus, these temporary measures have substantial permanent effects on the welfare of children. Secondly, for the affected children's welfare, the school closures themselves are a more important facet of the Covid-19 crisis than the negative shock to parental income: adding the negative shock to parental in come, the consumption equivalent variation rises in absolute terms to -0.75%. Last, there is substantial heterogeneity in the welfare effects, with children of well-off parents fairing better after the school closures than children of less well-off parents.

The results of this paper thus caution that the school closures have significant long-term consequences on the affected children and especially affect the welfare of children from disadvantaged households negatively. Note that our model only incorporates the main direct effect of the school closures on children that are caused by reduced public investment in human capital. There potentially exist additional dimensions along which negative long-term consequences are to be expected and which are not incorporated into our model. First, the lack of social contact during the school closures could directly affect children's welfare, but also their non-cognitive skills and thereby their long-term wages. Secondly, parents who have to take care of their children during the closures likely experienced increased stress that could affect the well-being of their children, and might face a higher risk of job loss or fewer possibilities for career advancement (Alon et al. (2020)), which could induce less investment into children in the coming years. On the other hand, we model the school closures as a complete loss in schooling, while many schools tried to maintain some form of schooling through distance learning and virtual teaching. Although these measures might have reduced the long-term impact of the school closures, they likely exacerbated the distributional consequences.

We conclude that school and child care closures should be considered as potentially very costly measures to avoid the spread of the Corona virus. However, we want to clearly acknowledge that we have not modelled the potential health benefits of these closures as this would require an explicit model that links disease transmission to school activity. We hope to have provided an informative model of the cost side and view the construction and quantification of such a comprehensive model as a next desirable step for future research.

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A Theoretical Appendix

A.1 The Dynamic Investment Problem of Parental Households

During age bracket $\{j_f, ..., j_f + j_a - 1\}$ parents solve the problem

$$V(j,s,m,\eta;a,h) = \max_{c,i^m,i^t,a',h'} \Big\{ u\left(\frac{c}{1+\zeta_c\xi(m,s)+\mathbf{1}_{m=ma}\zeta_a}\right) - v\left(\frac{\ell(m)+\kappa(j)\cdot\xi(m,s)\cdot i^t}{1+\mathbf{1}_{m=ma}\zeta_a}\right) + \beta\mathbf{E}_{\eta'|\eta}\left[V(j,s,m,\eta';a',h')\right]\Big\}$$

subject to

$$\begin{aligned} a' + c(1+\tau^c) + \xi(m,s)i^m &= a(1+r(1-\tau^k)) + y(1-\tau^p) - T(y(1-0.5\tau^p)) \\ y &= w\epsilon(s,j,m)\eta\ell(m) \\ a' &\geq -\underline{a}(j,s,k) \\ h' &= g(j,h,i) \\ i &= i(i^m,i^t,i^g) \end{aligned}$$

and

$$h' = g(j,h,i) = \left(\kappa_j^h h^{1-\frac{1}{\sigma^h}} + (1-\kappa_j^h) i^{1-\frac{1}{\sigma^h}}\right)^{\frac{1}{1-\frac{1}{\sigma^h}}} \tag{10}$$

$$i = i(j,i^g,i^p) = \bar{A}\left(\kappa_j^g\left(\frac{i^g}{\bar{\imath}^g}\right)^{1-\frac{1}{\sigma^g}} + (1-\kappa_j^g)\left(\frac{i^p}{\bar{\imath}^p}\right)^{1-\frac{1}{\sigma^g}}\right)^{\frac{1}{1-\frac{1}{\sigma^g}}} \tag{11}$$

$$i^p = i^p(j,i^m,i^t) = \left(\kappa_j^m\left(\frac{i^m}{\bar{\imath}^{m,d}}\right)^{1-\frac{1}{\sigma^m}} + (1-\kappa_j^m)\left(\frac{i^t}{\bar{\imath}^{t,d}}\right)^{1-\frac{1}{\sigma^m}}\right)^{\frac{1}{1-\frac{1}{\sigma^m}}}, \tag{12}$$

A.1.1 First Order and Envelope Conditions

Lagrange multipliers on budget constraint λ_b, on the borrowing constraint λ_a, on the law of motion for human capital λ_h, the investment aggregator λ_i and the private investment aggregator λ_p.

- Consumption c

$$\frac{u'\left(\frac{c}{1+\zeta_c\xi(m,s)+\mathbf{1}_{m=ma}\zeta_a}\right)}{1+\zeta_c\xi(m,s)+\mathbf{1}_{m=ma}\zeta_a} = (1+\tau_c)\lambda_b$$

- Assets a'

$$\lambda_b = \beta \mathbf{E}_{\eta'|\eta} V_a(x'; a', h') + \lambda_a$$

- Human capital h'

$$\lambda_h = \beta \mathbf{E}_{\eta'|\eta} V_h(x'; a', h')$$

- Human capital investment i

$$\lambda_i = \lambda_h \frac{\partial g(j, h, i)}{\partial i}$$

- Private Investment aggregate i^p

$$\lambda_p = \lambda_i \frac{\partial i(j, i^g, i^p, I)}{\partial i^p}$$

- Investment into private resources i^m

$$\lambda_b \xi(m, s) = \lambda_p \frac{\partial i^p(j, i^m, i^t)}{\partial i^m}$$

- Investment in private time i^t

$$v' \left(\frac{\ell(m) + \kappa(j) \cdot \xi(m, s) \cdot i^t}{1 + \mathbf{1}_{m=ma} \zeta_a} \right) \frac{\kappa(j) \cdot \xi(m, s)}{1 + \mathbf{1}_{m=ma} \zeta_a} = \lambda_p \frac{\partial i^p(j, i^m, i^t)}{\partial i^t}$$

- Envelope condition with respect to to a

$$V_a(x; a, h) = \lambda_b (1 + r(1 - \tau^k))$$

- Envelope condition with respect to to h

$$V_h(x; a, h) = \lambda_h \frac{\partial g(j, h, i)}{\partial h}$$

A.1.2 Optimality Conditions and Interpretation

The key choices parents make are a) how much to spend today and how much to save for the future b) how much of that spending to devote to assets and how much to human capital and

c) what is the optimal combination of time and monetary investment. The eventual question we are interest in are comparative statics with respect to a fall in i^g, a fall in income y and a change in the stochastic process for η.

Intertemporal Optimality The intertemporal Euler equation is standard and reads as

$$
\begin{aligned}
(1+\tau_c)\lambda_b &= \frac{(1+\tau_c)V_a(x;a,h)}{(1+r(1-\tau^k))} = \frac{u'\left(\frac{c}{1+\zeta_c\xi(m,s)+\mathbf{1}_{m=ma}\zeta_a}\right)}{1+\zeta_c\xi(m,s)+\mathbf{1}_{m=ma}\zeta_a} \geq (1+\tau_c)\beta\mathbf{E}_{\eta'|\eta}V_a(x';a',h') \\
&= \text{ if } a' > -\underline{a}
\end{aligned}
$$

Thus we can state the intertemporal optimality condition in terms of the marginal utility of wealth (ignoring the inequality coming from the borrowing constraint for a second)

$$V_a(x;a,h) = (1+r(1-\tau^k))\beta\mathbf{E}_{\eta'|\eta}V_a(x';a',h')$$

or in terms of consumption, given that education and marital status do not change for a given household

$$u'\left(\frac{c}{1+\zeta_c\xi(m,s)+\mathbf{1}_{m=ma}\zeta_a}\right) = (1+r(1-\tau^k))\beta\mathbf{E}_{\eta'|\eta}u'\left(\frac{c'}{1+\zeta_c\xi(m,s)+\mathbf{1}_{m=ma}\zeta_a}\right)$$

Note that if the utility function is logarithmic, then

$$\frac{u'\left(\frac{c}{1+\zeta_c\xi(m,s)+\mathbf{1}_{m=ma}\zeta_a}\right)}{1+\zeta_c\xi(m,s)+\mathbf{1}_{m=ma}\zeta_a} = \frac{1}{c}$$

and the Euler equation becomes

$$1 = (1+r(1-\tau^k))\beta\mathbf{E}_{\eta'|\eta}\left(\frac{c}{c'}\right)$$

As always, a constant consumption tax drops out of the intertemporal optimality condition.

Optimal Investment in Human Capital Combining the first order conditions with respect to

$$\frac{u'\left(\frac{c}{1+\zeta_c\xi(m,s)+\mathbf{1}_{m=ma}\zeta_a}\right)}{(1+\tau_c)(1+\zeta_c\xi(m,s)+\mathbf{1}_{m=ma}\zeta_a)} = \frac{\partial g(j,h,i)}{\partial i}\frac{\partial i(j,i^g,i^p,I)}{\partial i^p}\frac{\partial i^p(j,i^m,i^t)}{\partial i^m}\frac{\beta\mathbf{E}_{\eta'|\eta}V_h(x';a',h')}{\xi(m,s)}$$

The left hand side is the marginal cost of reducing spending on consumption goods by one unit, the right hand side gives the discounted benefits, per child, of one additional unit of the final good being spent on education, where $\frac{\partial g(j,h,i)}{\partial i}\frac{\partial i(j,i^g,i^p,I)}{\partial i^p}\frac{\partial i^p(j,i^m,i^t)}{\partial i^m}$ is the marginal benefit of that spending on human capital tomorrow, and $\mathbf{E}_{\eta'|\eta}V_h(x';a',h')$ is the expected marginal benefit of a smarter child.

Optimal Allocation between Time and Money Taking the ratio between the optimality conditions for time and money investment yields

$$(1+\tau_c)\frac{\kappa(j)(1+\zeta_c\xi(m,s)+\mathbf{1}_{m=ma}\zeta_a)}{1+\mathbf{1}_{m=ma}\zeta_a}\frac{v'\left(\frac{\ell(m)+\kappa(j)\cdot\xi(m,s)\cdot i^t}{1+\mathbf{1}_{m=ma}\zeta_a}\right)}{u'\left(\frac{c}{1+\zeta_c\xi(m,s)+\mathbf{1}_{m=ma}\zeta_a}\right)}=\frac{\frac{\partial i^p(j,i^m,i^t)}{\partial i^t}}{\frac{\partial i^p(j,i^m,i^t)}{\partial i^m}}$$

This equation simply states that the marginal rate of substitution between time and consumption times its relative price (the consumption tax rate) equals the marginal rate of transformation in the production of inputs for human capital production.

A.1.3 Functional Forms

Assume

$$\begin{aligned}
u(c) &= \log(c)\\
v(x) &= \frac{x^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\\
h' &= g(j,h,i)=\left(\kappa_j^h h^{1-\frac{1}{\sigma^h}}+(1-\kappa_j^h)i^{1-\frac{1}{\sigma^h}}\right)^{\frac{1}{1-\frac{1}{\sigma^h}}}\\
i &= i(j,i^g,i^p)=\bar{A}\left(\kappa_j^g\left(\frac{i^g}{\bar{\imath}^g}\right)^{1-\frac{1}{\sigma^g}}+(1-\kappa_j^g)\left(\frac{i^p}{\bar{\imath}^p}\right)^{1-\frac{1}{\sigma^g}}\right)^{\frac{1}{1-\frac{1}{\sigma^g}}}\\
i^p &= i^p(j,i^m,i^t)=\left(\kappa_j^m\left(\frac{i^m}{\bar{\imath}^{m,d}}\right)^{1-\frac{1}{\sigma^m}}+(1-\kappa_j^m)\left(\frac{i^t}{\bar{\imath}^{t,d}}\right)^{1-\frac{1}{\sigma^m}}\right)^{\frac{1}{1-\frac{1}{\sigma^m}}}
\end{aligned}$$

and thus the optimality condition for the relative inputs of time and resources is given by

$$(1+\tau_c)\kappa(j)\frac{[\ell(m)+\kappa(j)\xi(m,s)i^t]^{\frac{1}{\varphi}}}{(1+\mathbf{1}_{m=ma}\zeta_a)^{1+\frac{1}{\varphi}}}\cdot c=\frac{1-\kappa_j^m}{\kappa_j^m}\left(\frac{\bar{\imath}^{m,d}}{\bar{\imath}^{t,d}}\right)^{1-\frac{1}{\sigma^m}}\left(\frac{i^m}{i^t}\right)^{\frac{1}{\sigma^m}}$$

This implies that in a recession, as goods become scarcer (c falls), households should shift away from goods investment into human capital, and towards time investment. The impact of a reduction in public investment i^g induces households to increase private investment, with the optimal mix again given by the equation above

B Results Appendix

B.1 Children of Age 16

Table 17 shows the effects on inter-vivos transfers, acquired human capital at age 18 and average earnings for children who are of age 16 in the period of the lockdown when they are about to leave the adult household but parents may still decide on the inter-vivos transfers to these children. The table also reports the CEV of the welfare function (9).

Table 17: Effects on Children of Age 16

Lockdown of Schools			
av ivt	HK age 18	av earn	CEV [in %]
-1.30	-1.62	-1.73	-0.87
Lockdown of Schools & Asymmetric Income Shock			
-2.12	-1.60	-1.92	-1.12

Notes: Effects on age 16 children. Percent changes for inter-vivos transfers (ivt), acquired human capital at age 18, average earnings and the consumption equivalent variation of the welfare measure, cf. equation (9).

B.2 Decomposition of CEV

According to (9) the welfare of children of a given age is affected by changes of the distribution along assets, human capital and the resulting endogenous education decision of the children. Table 18 decomposes the CEV into these various components by subsequently switching off model elements, in the first part of the table for the experiment with the school closures. As a first step we hold constant inter-vivo transfers. Consequentially, the cross-sectional asset distribution at age $j_c > j$ is not influenced by this element and the education decision of children is altered. Comparison between columns 1 (which is the full model) and column 2 shows that the inter-vivo transfers play a crucial role in the model. With them being constant, children of college educated parents would lose in terms of welfare from the lockdown of schools. Also, with this missing adjustment elements, the size of the CEV increases (in absolute value) relative to the full model for all education groups. We subsequently switch off money and time investments by parents in column 3 and, finally, also hold constant the education decision by children in column 4 of the table. This shows that with each adjustment switched off the level of the CEV increases and the difference in the CEV between children from college parents and less than high school parents becomes larger. The last experiment shows that the exogenous reduction of time investments by the government into children and the induced human capital distribution has strong welfare implications. The only channel through which welfare is adjusted in this experiment is the fixed effects in earnings $\gamma(s, h)$. Since the slope of the fixed effect in h increases in the level of education, a reduction of human capital leads to the largest losses for those children with the highest education (when the education choice is held constant), which explains the strong education gradient observed in the last column of the table.

In the second part of the table we repeat this experiment for the additional recession scenario. In that decomposition the only relevant changes appear when holding the inter-vivos transfers constant because once those are constant the rest of the decomposition is identical to the one in the school closure experiment. We observe that once the inter-vivos transfers are held constant again the children of college youngsters experience the highest welfare losses and the non-monotonicity of welfare results (that the children of high school parents experience the largest welfare losses) is also gone.

Table 18: CEV Decomposition for Main Experiments

Lockdown of Schools				
	full model	ivt const	ivt, inv const	ivt, inv, edu const
$s = no$	-0.70%	-0.80%	-0.97%	-1.63%
$s = hs$	-0.68%	-1.12%	-1.29%	-2.08%
$s = co$	-0.57%	-1.36%	-1.54%	-2.66%
Lockdown of Schools & Asymmetric Income Shock				
$s = no$	-0.71%	-0.78%	-0.97%	-1.63%
$s = hs$	-0.82%	-1.13%	-1.29%	-2.08%
$s = co$	-0.65%	-1.37%	-1.54%	-2.66%

Notes: Decomposition of the CEV of welfare function 9. ivt const: inter-vivos transfers are held constant; ivt, inv const: additionally, parental investments through money and time are held constant; ivt, inv const, edu const: additionally, education decisions are held constant.